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THE COWLES COMMISSION FOR RESEARCH IN ECONOMICS

COLORADO SPRINGS, COLORADO

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THE COWLES COMMISSION FOR RESEARCH IN ECONOMICS is a not-for-profit corporation, founded in 1932 for the purpose of conducting and encouraging investigations into economic problems. A function of the COMMISSION is to encourage the preparation of, and to issue from time to time, papers and significant monographs of an econometric or economic-statistical nature without, however, assuming responsibility for theories or opinions expressed therein. The COMMISSION is affiliated with the ECONOMETRIC SOCIETY, an international society for the advancement of economic theory in its relation to statistics and mathematics.

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INTRODUCTION

The Annual Research Conferences on Economics and Statistics, held under the auspices of the Cowles Commission for Research in Economics, originated in a series of informal meetings during the summer of 1935 following the sessions of the Econometric Society at Colorado College on June 22-24 of that year. At these gatherings, a number of papers were presented and discussed informally by prominent economists who remained in the vicinity. These meetings were so successful that it was decided to continue them in subsequent years.

The Second Annual Research Conference was held at Colorado College from July 6 to August 8, 1936. A lecture, followed by a discussion period, was scheduled for each weekday beginning at 10:30 A.M. In addition to these technical sessions, a series of four public evening lectures, of popular interest, was presented by prominent speakers who were participating in the conference. The attendance at the scientific sessions ranged from thirty to seventy; at the popular ones, from two hundred to five hundred.

In addition to the scientific program, there was an interesting series of social events. Receptions for visiting guests were held on successive Wednesday afternoons at the home of President Davies of Colorado College, at the Colorado Springs Fine Arts Center, at the Fountain Valley School, at Bemis Hall, and at the residence of President Cowles of the Cowles Commission. There were also motor trips to the Garden of the Gods, including a picnic supper, to the summit of Pikes Peak, to the Cripple Creek gold fields and the Portland mine, and to the summit of Cheyenne Mountain, including a supper there. Informal dinners and swimming parties attracted numerous participants, and a number of Saturday afternoon hikes were arranged by Mr. Cecil Graves.

The following out-of-town participants were registered as in attendance: Mr. Kaoru Ando, Japan; Prof. R. M. Bourne, University of Wyoming; Mr. David Cook, Indiana University; J. J. J. Dalmulder, Holland; Prof. W. T. Dawson, University of Texas; Mr. Albert E. Dickens, Indianapolis; Prof. and Mrs. Edward L. Dodd, University of Texas; Miss Mary Doyle, St.

ABSTRACTS OF SEMINAR LECTURES

July 7 — INCOME IN THEORY AND INCOME TAXATION IN PRACTICE, I,

IRVING FISHER, Professor of Economics, Yale University.

Income was defined by me in *The Nature of Capital and Income* in 1906 as services received. The bulk, in value, of services received is in money—dividends from stocks, interest from bonds, etc.

Economics and accountancy are separated by a wide gulf which the above named book sought to bridge, as did John B. Canning's *Economics of Accountancy* in 1929.

In the present paper the subject is reviewed and extended by the use of mathematics. One object is to show the fallacies underlying our present income tax.

I agree with Canning that income is, without exception, the most fundamental concept of economic science; only by this concept can other economic concepts ever be fully developed and understood, and upon beginning with this concept depends the full fruition of economic theory in economic statistics.

Our income tax laws make capital levies masquerade as income taxes. They therefore lend themselves to tax evasion and avoidance, as when Raskob and DuPont reduced their taxable income by selling each other stock in order to register capital losses and then bought the stocks back.

In this lecture, and until further notice, I shall avoid the term "income" and discuss only money receipts, whatever these receipts may be or be called. The three following assumptions will be made: (1) Every money receipt (M) is a definite sum; (2) occurs at a definite time; and (3) is definitely certain to be paid.

From such sums, M_t, known to be forthcoming in the future, their present value C_0 may be obtained by discounting, using i as the rate of interest. The present value is

$$C_0 = \sum_{t=1}^{t=n} M_t\, v^t \qquad (1)$$

where M_t has the values M_1, M_2, $\cdots M_n$ (for times $t = 1, 2, \cdots n$) and $v = 1/(1+i)$.

Capital gain is defined as $G_1 = C_1 - C_0$. (2)

Earnings are defined as $E_1 = M_1 + G_1$. (3)

It may also be shown that $E_1 = C_0\, i$. (4)

Numerous other formulae may be derived connecting the quantities M, C, G, and E.

Capital value C_0 was found by discounting M_1, M_2, $\cdots M_n$. It cannot be found by discounting E_1, E_2, $\cdots E_n$. The nearest approach is equation (14), namely,

$$C_0 = \sum_{t=1}^{t=q} (M_t\, v^t + G_t\, v^q) + C_0\, v^q$$

in which it will be noted that E_t, namely, $M_t + G_t$, does not enter, though it would, if instead of "q" we could write "t". Only the qth term contains E; for then the parenthesis becomes $M_t\, v^q + G_t\, v^q$ which is $(M_t + G_t)\, v^q$, or $E_t v^q$.

But, in the special case of a perpetual annuity, E and M will be equal. For $M_1 = M_2 = \cdots = a$ and $C_0 = C_1 = \cdots$, so that $G_1 = C_1 - C_0 = 0$, and $E_1 = M_1 + G_1 = a + 0$. Yet even then, though E and M are equal, they differ; for instance, E accrues continuously, while M occurs suddenly and E has the effect, as it accrues, of increasing C, while M has the effect, when it occurs, of decreasing C.

A bond is like a segment of a perpetual annuity bought at the beginning of that segment and sold at the end — at par. A bond may be turned into a perpetual annuity by being refunded or by its principal, when paid, being re-invested — at par.

Perpetual annuities are taken by accountants as their ordinary norm. The owner of a security is counseled to reinvest enough to maintain capital value C_0 unimpaired and obtain incomes M, no greater than a perpetual annuity of $C_0\, i$.

But to *reckon* such reinvestment is not to make it. Nonrecurring items like a principal of a bond "ought" to be spread over time but may not be. Only a corporation whose life is supposed to be "perpetual" usually lives up to the accountant's norms of a perpetual annuity and of a capital which remains constant or, at any rate, unimpaired.

July 8—INCOME IN THEORY AND INCOME TAXATION IN PRACTICE, II,

IRVING FISHER.

The factor of chance will affect the equations in various ways; but the main principles still hold. For instance, capital value is still the discounted value of *expected* receipts M.

So far only money received from property has been discussed. Money from labor is subject to different norms. All money so received is commonly regarded as "earned" although the capital value of a man as a money earning machine is constantly diminishing. That is, the norm of maintaining a constant capital value is not applied to labor.

The algebraic sum total of the money valuations of all services received by Smith will be called I without any committal, as yet, that this sum is "income."

This grand total I includes (1) items already in money, such as dividends and proceeds of sales, the quantities M; (2) the quantities R, the money valuations of "real" services of consumption goods, such as the shelter from a house; and (3) the values Y which are the psychic services of Smith's "body-mind." In each case there are also negative items to be reckoned with.

The services may be classified according to sources, i. e., the assets which yield them. There are six classes:

(1) $\Sigma M_r - \Sigma M_d$, the money received from property less the money disbursed on it.

(2) $\Sigma M_w - \Sigma Y_n$, the money received from labor less the valuation of the "irksomeness" of labor.

(3) $\Sigma R - \Sigma M_s$ the value of the "real" services from consumption goods less the money spent on consumption goods.

(4) $\Sigma Y_p - \Sigma R$, the value of psychical satisfactions, less the value of the "real" services which these satisfactions cost.

(5) $\Sigma M_{cr} - \Sigma M_{db}$ (or, more briefly m), the money credited to (coming out of) the cash drawer less the money debited to (going into) it.

(6) U, any other items. U will be assumed to be zero; this assumption will be discussed later.

Since every money item occurs by double entry, once as a plus and once as a minus, the sum total of all money items is zero. Likewise the $+ \Sigma R$ cancels the $- \Sigma R$. The net result of the summation is evidently $\Sigma Y_p - \Sigma Y_n$. This final result may be called the psychic I.

But, though fundamental, this cannot be numerically estimated except indirectly; and even indirectly to calculate ΣY_n is hopeless. We therefore ignore ΣY_n and address ourselves to estimating ΣY_p. This may be called gross psychic income I_g, so that $I_g = \Sigma Y_p$.

In order to estimate I_g indirectly, we assume that the total psychic satisfaction ΣY_p is worth the same as the real services which they cost, ΣR. Our problem, then, reduces to evaluating ΣR in terms of money.

July 9—INCOME IN THEORY AND INCOME TAXATION IN PRACTICE, III,

IRVING FISHER.

We have reduced the problem of approximating I_g to measuring ΣR in terms of money. This is ΣM_s provided $\Sigma R - \Sigma M_s = 0$. If not we need to estimate the last expression. Let this estimate be c. Then ΣR is estimated as ΣM_s

$+ c$ or $\Sigma M_r - \Sigma M_d + \Sigma M_w + m + c$ which is the best expression we can get to I_g and is called I_{ba}. To calculate this we need only to consult Smith's accounts in terms of money. This will give us all the terms of his income except c. The value of c must be estimated separately.

To do this we assume that most of the consumption services, R, are equal to the money, M_s, spent for them. The only exceptions worth attention are three: house shelter, house furnishings, and automobile. The problem therefore reduces to calculating these three discrepancies, namely: (1) c_h, the discrepancy between the value of house shelter and its cost; (2) c_f, the corresponding cost-use discrepancy as to house furnishings; and (3) c_t, the corresponding cost-use discrepancy as to automobile.

To estimate these we need only three appraisals, the appraisal of the year's house shelter, what Smith's shelter is reasonably worth or what he would pay if he paid rent for it, the appraisal of the use of his house furnishings or what rental he might reasonably pay for them, and likewise as to the automobile.

From these respectively must be subtracted the sums of money actually paid during the year for house, furnishings, and automobile, including any outright purchases or part payments. If Smith rents a house, furnished and provided with the use of an automobile, it should be presumed that their rental cost to him is equal to the value of their use so that the cost-use discrepancy is zero.

If, on the other hand, Smith buys a new automobile he pays more than the value of one year's use and the cost-use discrepancy for automobile is negative. If he lives in his own house, fully paid for before the current year, the cost-use discrepancy for housing is positive.

The above appraisals of the money value of house-use, furnishings-use, and automobile-use can be made by accountants on the basis of original cost and a percentage thereof to represent the one year's use. Here is the only place where the accountant's art need be employed.

There remains the term U which is zero since it can only contain terms which cancel each other. It cannot contain any

M, *R*, or *Y* terms since these have already all been included. The only terms left are services in kind prior to the final uses *R* of consumption goods; and all prior or anticipatory services are "interactions" to be entered doubly as positive and negative.

The net *I* of a corporation or other artificial person must be zero, because all items must be double entried; there can be no terms *R* or *Y*; but only terms *M*. A corporation can only have *I* when considered as consisting of stockholders—real persons. They receive its dividends. But the corporation, as such, does not receive these dividends; it pays them.

Is *I* to be called income? Yes; for two reasons. *I* plays the most fundamental role in this analysis, and the most fundamental factor ought, it would seem, to bear the name of "income." We have seen, for instance, that *C* is the capitalization of the quantities *M*, i.e., of *I* items but not of the *E* items. If we were to call *E* income, *C* would have no such relation to "income."

A second reason is that if *I* were not to be called income but *E* were, the latter would have two names while *I* would have none.

We may, however, compromise by calling *I* income realized and *E*, income *earned.*

July 10—INCOME IN THEORY AND INCOME TAXATION IN PRACTICE, IV,

IRVING FISHER.

We have seen that real income is "consumption," though "use" would be a better term. Consumption implies destruction. While this destruction is often an incident of use as in the case of food, it often is not, as in the case of a house.

An income tax should, then, be a tax on "consumption," or "use," or "service;" for income is service.

There are three practical ways of measuring income for taxation purposes. One is to measure it as the "spendings" for, or on, consumers' goods, namely, M_s, plus c, the correction on account of the three cost-use discrepancies. This requires an exact accounting of thousands of items. The second way is to substitute for said "spendings" an exactly equal figure, namely the total of all disbursements of money during the year (which total is usually on record without need of special computation), less the disbursements for business or property, namely investments and business expenses. The third and best way is to substitute for said spendings another exactly equal figure found as follows: (1) the money received from property and work less the money going into business, plus the use, or depletion, of cash during the year. This, plus c as formerly computed, is Smith's income.

There is little or no ambiguity as between spendings, M_s, and business disbursements, M_d. The former include food, clothing, shelter, house operation and the many miscellaneous items in any householder's budget. But "investments" and "savings" are not to be included. Also there should be certain deductions for dependents, charity, contributions, taxes, and life insurance.

The tax schedule for such an income tax will differ from the present tax schedule in several important particulars.

The items here excluded but which are entered in U. S. tax schedules are: "capital gain and loss" (although all sales and purchases *within the year* are included), "losses by fire, storm, etc.," "bad debts." Moreover, the item "from private business or profession, partnership, syndicates, pools, less money put in" differs from the item now usually entered "income (or loss) from partnerships, syndicates, pools, etc." by excluding any allowance for "bad debts," "depreciation," "obsolescence and depletion," and by excluding the two inventory items (for the beginning and end of the year). Only the actual net money taken out of the business should be used in the income tax schedules (though all the above items are extremely useful in the bookkeeping of the business itself).

In the same way only the actual money received from fiduciaries is included, with no bookkeeping adjustments.

The money items here included but not entered in existing tax schedules are: money borrowed and lent, money disbursed for investments, gifts, bonuses, bequests, and the net cash used (m).

It has been shown that in computing income we must not add or subtract any mere changes in capital valuations, G. Just as a *rente's* capital value may go up and down without increasing or decreasing the income, so capital gain or loss of any kind may be great or small without affecting the income of the year. Only actual payments M, monies taken in or paid out, are to be counted in the year in which these payments occur. If there is capital gain this merely symbolizes future income. When that income comes will be the time to tax it. Smith should not be required to pay more tax this year because of more income next year.

And no allowance is made, or should be made, for losses from shrinking in value of property, even when it shrinks to zero, by fire, flood, drought, storm, shipwreck, earthquake, or anything else. Such loss merely symbolizes lack of future income. If Smith's income in future years will be lessened because his factory burns up this year, he will then—in those future years— have correspondingly less taxes to pay. He need not have any less to pay *this* year on account of less income *next* year.

We may illustrate the fallacy and absurdity of taxing a person's capital gain by a simple example. Imagine three brothers each inheriting $100,000 and each investing it at 5 per cent. They differ only as to re-investing. Each of the three has earnings the first year from his property of $5,000. The first brother spends all of his $5,000 and continues to do so every year. The second spends none of the earnings of his capital but re-invests it all and continues to do so for 14½ years when his capital will have doubled—after which the earnings will be $10,000 a year, all of which he thenceforth spends. The third spends $20,000 every year until his fortune has been eaten up—in six years.

Under such an income tax as is here proposed, the tax (if at a uniform rate) would correspond to the spendings and all three tax-series would have the same present value. If the tax is 10 per cent, the first brother would pay $500 a year indefinitely, the present value of which, at 5 per cent, is $10,000, or 10 per cent of his fortune. He could "compound" for his future taxes by paying a lump sum of $10,000.

The second brother would pay nothing for 14½ years, after which he would pay $1,000 a year indefinitely. The present value of this tax, beginning 14½ years in the future, will also be $10,000 and could be prepaid by a lump sum of $10,000.

The third brother would pay $2,000 a year as long as his $20,000 a year of realized income lasted, namely 6 years, and then nothing. Again, the present value of this series of tax payments is also $10,000.

But under the U.S. tax laws the three would fare differently. They could "compound" for their taxes as follows: the first brother $10,000, the second $17,100, the third $1577.

Is there not something wrong with such an income tax? This could all occur under our present income tax laws if the brothers sold and re-bought each year—if the forms of property earning 5 per cent were of the right sort.

The present capital-gain tax is a haphazard tax occasionally just, usually unjust and seldom self-consistent. It does not even tax capital gain as it steadily accrues. It is not a true capital gain tax nor a true income tax—neither fish, flesh, nor fowl, but a monstrosity. It is even worse than haphazard, for it is subject to manipulation by clever tax-evaders or tax-avoiders, since only realized capital gain is taxable and not the capital gain which merely accrues. The taxpayer often has it in his power to choose when, if at all, he will make the sale and take the capital gain, and to choose what to sell. Naturally, he will choose so as to make the realized capital gain as small as possible and even negative if possible—a capital loss. That is, there is an impulse to sell after a fall of price, thus increasing the fall, and a reluctance to sell after a rise, thus increasing the rise. Consequently, among other faults of the present system, we find that it intensifies booms and intensifies depressions,—tends to "boom the booms and bust the busts." It was doubtless one of the contributory causes of the stock market boom of 1929 and the depression following.

Again, the loss-taking reduces the taxes at the very time that the government most needs income.

Ogden Mills, when congressman and before he was Secretary of the Treasury, proposed the nearest approach yet made to a true income tax. His bill was HR 7856, July 20, 1921. The main section read: "That on and after January 1, 1922, ... there shall be levied, collected, and paid for each calendar year upon the spendings during the calendar year of every individual, a citizen or resident of the United States, a tax equal to the following: - - - - ".

The British income tax, though not as near to a true income tax as Mills's proposal, is much nearer than the income tax of the United States.

The United States Government has printed a "Summary of the British Tax System" from which the following is quoted: "The British conceive income to arise from all occupied real property whether or not such income is actually received in the form of money or money's worth. If the property is occupied even by the owner, income is computed on the basis of the average rental value. This is entirely different from the conception in the United States. In the latter country, if a man owns and occupies a $10,000 house, which might normally rent for $1,000 a year, he is considered to receive no income from the house; on the other hand, in England, in a like case, he will be assessed on the annual value; that is, he will include $1,000 in his schedule A income.

"The British do not consider income to arise in the case of gains arising from the sale of capital assets, unless the taxpayer makes transactions in such assets his trade or business. There is no depletion deduction on account of the exhaustion of natural resources, such as mines."

July 11—MONETARY FACTORS IN TRADE CYCLES, PRICE DISTURBANCES, AND DEPRESSIONS,

CARL SNYDER, Formerly of the Federal Reserve Bank of New York.

Beyond question, no economic problem has so deeply troubled the human spirit, in all times and in the United States especially, from long before our War of the Revolution (to date), as the everlasting "money problem" which, in our own day at least, has become chiefly a credit problem. It is painful to record that on these subjects there are many economists as divided in their basic ideas and as to methods of reform proposed, as the lay public. Slight wonder then that the business man, the banker, and the lay writer should be slightly bewildered before questions seemingly so complex, and by this sometimes violent divergence of economic views.

But within the last fifteen or twenty years there has become available, especially in the United States, a wealth of new data which has for the first time provided adequate material for quantitative studies of the relations of money, credit, deposit velocity, volume of trade, and price changes, and the role of these in our familiar booms and depressions. The World War, and later years, have, with their inflations and deflations, provided a kind of ready-made economic laboratory on the grandest scale. Building on this recent material it has been possible to go back for 60 years or more into the older data, and to at least have some idea of the more fundamental relations, going back a century and more. This has been due largely to new measurements of the rate of growth of our great basic industries and latterly of industry and trade as a whole, and their variation, and likewise to much broader indexes of the general level of prices than the familiar indexes of commodity prices at wholesale.

From this it has been possible to establish a remarkable similarity between the variations of industry and trade from their long time and very persistent trends of growth and the variation of the velocity, or rate of turn-over, of bank deposits and especially of demand deposits. Data as to the "currency" and money and credit go a long way back, and bank deposits especially can be traced continuously to the beginning of the last century, that is, for over 130 years.

These measurements have greatly widened our knowledge of fundamental economic relationship. It is a remarkable fact that in a very broad way trade and credit (or bank deposits) have tended to rise at very much the same long time rate, but with wide divergence at times, as, for example, in our Civil War, in our World War, and in the last seven years. When the indexes for these two factors, credit and trade, are divided one into another you have what I have termed the Trade-Credit ratio.

This, on classical theory, should be in effect the General Price Level. But when this Trade-Credit ratio is compared with the best indexes we have of commodity prices at wholesale there are often wide and serious discrepancies, a fact that has been duly pointed out by opponents of proposals for monetary stabilization. This at once suggests that at times these indexes of commodity prices are not a close or accurate index of the broad general level of prices and values; and so in their early attempts at a statistical test of Newcomb's well-known Equation of Exchange, Professor E. W. Kemmerer and Professor Irving Fisher twenty-five or thirty years ago recognized this possibility and sought a wider base.

But, in this early time at least, it was difficult to find a sufficient variety of price indexes to construct such a broad index of the General Level. Such data have only been available, in an adequate way, in the last fifteen or twenty years. Utilizing these new data it has been possible by correlation methods to carry back an index devised in the statistical department of the Federal Reserve Bank of New York to 1860. And more recently Mr. Rufus S. Tucker has by the same methods sought to extend this index back for another 70 years or so.

The result is that we now have fairly good measures of all the factors in the Equation of Exchange back for nearly a century. When these new measures are compared with the Trade-Credit ratio, there is found, in general, a remarkable correspondence, so remarkable indeed that many inquirers in this field have felt some suspicion, not unreasonable, as to the indexes themselves. But proof of their validity and a fair degree of accuracy is to be found in the relationship between the variations of the Volume of Trade and of Deposit Activity from their long time trends. As these two factors tend to correspond pretty closely over a period of 60 years and more for which such data are available, it would follow from this that if there exists a close and definite relationship between the volume of credit and the volume of trade, in the long run, then the *variations* in the two factors would tend mutually to cancel and we should have the Newcomb formula of the Equation of Exchange, $\frac{MV}{T} = P$, reduced to a simple relationship of M and P, (Money, or Credit, and Prices) allowing always for the fairly steady long time trend of growth of trade itself. In other words, if we were to ignore entirely the question of deposit velocity and likewise of the variations in trade, then simply by dividing the measure of the volume of credit by the index of the long time growth of trade, we should have approximately the General Price Level.

And this is what we find. This would seem to establish, therefore, a simple quantitative relationship upon which a theory and procedure for monetary stabilization may rest. And since it is inconceivable, at least by any present means available, to control the long time growth of trade as a whole (so enormously complex and diversified as trade and industry have become) it would follow that the problem reduces to the simple question of a control of Credit, not in any qualitative sense but simply as to actual volume. It seems now well established that if credit expands at a uniform rate corresponding to the very steady long time growth of trade and production as a whole, we should have a fairly stable general price level. This does not remotely imply a fixed and invariable level of commodity prices at wholesale, any more than it would imply stable prices for cough drops or cottage cheese. The indexes of commodity prices at wholesale are inevitably weighted heavily by the speculative commodities, such as cotton, corn, wheat, copper, and the like, and it follows from this that commodity price indexes, in any way that they are constructed, often vary widely from the theoretical, or any actual measure of what I have termed the broad General Price Level.

Furthermore, it has been found that relative to almost all other available price measures, such for example as the familiar Cost of Living index, commodity prices tend pretty steadily to decline, except in periods of inflation; precisely as good theory would suggest. The desideratum, therefore, is not a "Commodity Dollar."

On the other hand, a fair approach to general price stability seems possible if the expansion of credit is kept reasonably close to the steady growth of Trade. For a simple reason. Speculation can only thrive or become disturbing through a free use of credit. Always the speculator, whether in land, stocks, or commodities, can afford to pay far higher rates of interest than ordinary business. What does the speculator care for a 7 per cent bank rate if he feels assured of a 30 or 50 per cent rise in shares or, say, wheat? But if the volume of credit expansion is always strictly limited to the long term rate of growth of trade, it follows that speculative activity when it becomes dangerous will inevitably force higher interest rates, to the discouragement of legitimate trade, and thus quickly defeat itself.

To the objection that business itself would suffer from such attempts to limit credit, the answer may be made that speculation is far more sensitive to a sharp rise in bank rates than is business, so that it is possible to provide a sharp check to speculation before general trade has felt any serious injury. And it has been shown further that any such check to speculation usually outlasts a year before it can gain headway again. If, therefore, credit policy be directed towards attaining precisely this result, it can quickly correct the balance and restore any lagging activity in trade by suitable measures looking towards adequate credit expansion, and, naturally, lower interest rates.

This would appear to be, then, the technique of monetary stabilization. This technique seems theoretically sound and in this country it has been successfully in effect at various times, both before and since the establishment of the Federal Reserve System, in two conspicuous instances, as after the panic of 1907 and the crisis of 1921, with conscious intention, and in two later instances, as in 1924 and 1927, with conscious design.

(The statistical evidence for the thesis here presented is summarized in a paper entitled: "The Problem of Monetary and Economic Stability," *Quarterly Journal of Economics*, February, 1935.)

July 13—THE CAPITAL SUPPLY AND NATIONAL WELL BEING, CARL SNYDER.

Almost everyone knows nowadays that in this country the consumption of comforts, enjoyments, and luxuries—what we call "goods"—is on the average per person far above that of any other nation that ever existed—probably twice that of the next highest nation like Great Britain; probably three or four times the average for Europe; probably ten or twenty times, at the least, for the vast populations of China and India.

How has all this come about? It is not through the increased fertility of the fields; the average yield per acre of farm products has risen very little in one hundred years and more. Neither is it due to the increased skill of the workers. On the contrary, it is probable that the average mechanic or worker of a hundred years ago, was handier with his tools, better able to make things and fix things than the average worker of today. What then has wrought this economic miracle? The answer seems to be: chiefly, if not wholly, the increased use of machinery, and new discoveries and new processes.

For a very striking example we may consider the farms. One hundred years ago three-fourths or more of the population of the United States lived on farms. Today much less than one-fourth. In other words, it now takes less than one-fourth of the population to produce all the grains, cotton, fruits, vegetables, and the like needed for the nation where it took three-fourths of the population in the earlier period. If the skill of the workers is no greater and the average yield per acre no more, then it must be that in some other way one man can produce as much on the farm as three could have done one hundred years ago. This seems almost wholly due to the increased use of farm machinery.

All this machinery and equipment costs money, that is to say, in the narrower sense, capital. But consider how marvelously fruitful it has been. We have no data going back of about 1850, but since that time the investment per acre in farm machinery has multiplied seven times. It is still only about $3.50 per acre. And it has increased very slowly, only at the rate of about 4 per cent per annum. But this extraordinarily small investment has released about half the population of the nation from the business of raising food and the like, and made possible the extraordinary advance in manufacturing, which has made the United States now by far the greatest maker of goods in the world.

A wondrous example of this release of workers from the business of maintaining the food supply has been witnessed in our own generation in an almost fabulous story. Thirty years ago a motor car was a rare sight; today, on the average, twenty millions or more, that is two out of three families in the United States, own a car. Our factories have turned out in a single year more than five million cars, and the total expenditure on new cars and the maintenance and enjoyment of those already in existence has been estimated to exceed the total value of all the products of the farms in the nation. This is a sample of the industrial progress of the United States.

This astounding product has been made possible almost wholly through the development of machine tools and other mechanical devices, so that the number of cars turned out, per worker in the industry, per year, is probably now ten or twenty times what was possible thirty years ago.

All this machinery and equipment likewise requires a tremendous investment of capital, amounting now to billions of dollars. Where did it come from? From the "savings" of the people? Practically none at all, except as it was contributed unconsciously by the buyers of the cars. The capital for the manufacture of automobiles has been derived almost exclusively from the industry itself. A perfect example is the Ford Motor Company, a story often told. It started with only about

$28,000 of capital and only half of this in cash. In thirty-two years the total value of its products (sales) have run close to 13 *billions* of dollars. The total dividends paid out has amounted to only 125 millions and the actual investment in plants has been at least four times this. That is, very largely the earnings, profits, have been "plowed back" into the industry.

Very much the same is true of the whole motor car industry, and likewise all the other great industries, the steel industry, the chemical industry, farm implements; practically all of them tell much the same story. The total value of all the industrial plants of the United States, including the railroads, the mines, and all, is somewhere near a hundred billions of dollars. This represents a total investment of maybe two or three times this sum, for it is little realized how great the losses in industry actually are; and especially in new enterprises, a large part of which always fail. This is why, in general, only rich people can afford to invest in new enterprises. People of moderate means cannot take the risk. This is why those few who have gone into new enterprises and been successful have been the chief suppliers of the new capital that has been needed for the continued industrial growth of the country. This is to say that this new capital has been largely derived from the industries themselves and chiefly from the newer industries in which the profits have been high.

Now it has been found that, going back one hundred years and more, the industry of the country has grown at something like four to five per cent per annum, nearer five per cent in the last century; nearer four per cent in this century. And about the same with the capital employed in industry. Here the rate of increase has been just a little higher, between five and six per cent; rarely more. This implies a very striking fact; that is that, speaking very broadly, a dollar of capital invested in industry one hundred years ago produced about the same amount of goods as it does today. In other words, the need for capital, and especially for new capital, to maintain the continuous growth of industry and the well-being of the nation is just as vital today as it was a century or more ago.

The idea, widely prevalent, that somehow "the country has just growed," like Topsy, has, like most illusions, little basis in fact. The fact is that our wondrous wealth, our well-being, our comforts, our enjoyments, which are the envy and astonishment of the whole wide world have come from a very definite mechanism which is little understood. Its essentials are, as indicated: high profits, use of these profits in large capital expenditure in machinery and plant, encouragement of invention and discovery, bold adoption of these inventions; hence a steady increase in the product per worker, and a steady cheapening of every kind of "goods." No other way.

This accounts also for the fact that "real wages," that is the average of what a day's work will buy, are far higher in the United States than in any other country, and these real wages have been steadily rising throughout the century or more for which we have accurate records. These high wages could only have been paid by reason of the increased product per worker; there is no other conceivable way.

What now has been the price of all this advance in national wealth and well-being? Something like five per cent of the national annual income. This at least has been the net realized gain per year in the reproductive capital employed in industry; probably not over half or a third of the total national savings, the larger part of which has gone into creative comforts, better houses, fine roads, motor cars, and the like. But so vital, like the seed of the farmer, indispensable to his livelihcod.

Such, as I conceive it, is the mechanism by which this industrial miracle which we call the United States has come into being, and now the singular part is that millions of people, beneficiaries of this miracle and this mechanism, seem to wish, unconsciously I do not doubt, to destroy this mechanism and thus bring the miracle to an end.

To the observer of human affairs, their futilities and bizarre aberrations, this indeed seems one of the strangest of all. It is in no wise new. It seems due largely to these feelings of envy and jealousy of the rich which have been manifest throughout all the ages since mankind became a half civilized being. Always talent, business ability, and shrewdness have been objects of suspicion, jealousy, and attack. It was the same a hundred, a thousand, and perhaps ten thousand years ago. And in spite of it what we call the capitalistic system has continuously grown and advanced, to flower at last in our own generation, into this modern marvel which we call industrial America, with the widest diffusions of culture, comforts, and general well-being of a great people that the world has ever known.

(The factual and statistical bases of this argument are given in a paper in the *American Economic Review*, June, 1936; with 8 charts).

July 14—THE ANCIENT LINEAGE OF CAPITALISM,

CARL SNYDER.

Only humanity itself has a lineage more ancient than that of Capitalism. Out of prehistoric man's most basic urges, first to survive and second to reproduce himself, was born his most determining characteristic, the yearning for security. Out of what agonizing repetitions of trial and error, pre-historic man crept painfully upward to the knowledge that stored food meant security and that weapons meant more food and protection, we can only vaguely surmise. But of that long and obscure travail, Capitalism was born.

The record of the progress and development of Capitalism is the record of the progress and development of humanity; of its institutions, religious, civil, legal; of its culture both in living and in art. Analysis of the wondrous industrial advance in the United States in the last century and more reveals that it has been achieved by the corresponding increase in the amount of Capital. Analysis of the periods of depression shows the close relation of these regressions to the misuse and mismanagement of capital, money, and credit.

It is of the most challenging necessity that human society should, from the endless experiments of its past, evolve a knowledge and a philosophy of economics and monetary policy that will insure continued progress, and prevent the disastrous regressions which almost all countries have experienced in recent years. This, indeed, means simply the understanding of the relation of capital supply to national progress and the maintenance of that proper balance of credit and trade that means well-being.

To aid modern man in solving this challenging problem, there is the amazing disclosure of the last hundred years of investigations into human origins. It is astonishing to think how little the wisest and greatest minds of a century and more ago could have known of the early history of the world in which they dwelt, and the race of which they were a part and product. Within this time have come such revelations as have never been vouchsafed to man. In a fascinating volume De Burgh observes: "The last century has witnessed the gradual unveiling of the shroud that hid the remote past. Imagination is dazzled as scholars have revealed cycle upon cycle of past history, stretching back at least into the fourth millennium B.C., a history of art and culture comprising a series of rich civilizations hitherto unsuspected by mankind."

With a widening wonder investigators have learned that the calendar, practically as we now use it, was in use in Egypt by 4241 B.C. and that an equally elaborated and compensated calendar was perhaps later in use among the ancient Incas and Mayas. With a kind of stupefaction we learn that they had in this earliest day some rude means for astronomical observations and knew well the stars and planets; that they were composing grammars and dictionaries of their language; that they were developing codes of laws strangely resembling our own, relating to land tenure, land transfers, business contracts of every description, control of the liquor traffic, a rude form of banking, laws as to a wife's debts, and legal rights of women and children and slaves; and that soon they were to have school books and therefore some kind of schools; that they had a great number of the lighter luxuries of the present time. Their titled ladies painted their faces, blacked their eyebrows, crimsoned their fingernails, had magnificent jewelry, probably bobbed their hair, and otherwise conducted themselves much as the favored beauties of today.

These ancient peoples searched for gold and other metals, they had developed a high form of government; they had vast palaces and marvelous gardens, like the famous Hanging Gardens of Babylon; they had commerce upon the waters and trade upon the land, and caravans and voyages that covered thousands of miles. Our soaring skyscrapers had their forerunners in huge constructions like the Tower of Babel. In brief, save for the mechanical and constructional marvels of today, there seems astonishingly little to differentiate that once mythical epoch from our own.

It is, then, against this background, once so bewildering as to seem to the most habituated a kind of dream of unreality, but now emerging in such clearness that its institutions and culture appear as modern as our own, that we can envisage our present economic, commercial and financial origins. This has been a development of the past twenty years or so. A writer still living has ventured, in the depths of our ignorance, to speak of "modern" capitalism as dating back at most not more than three or four centuries. We now know that in all its essential details the economic system under which we live, and under which the more favoured people have risen to an almost god-like power—in some details, at least, approaching a kind of omnipotence—has existed not for hundreds but for thousands of years. We may say this because at the dawn of what we familiarly call "recorded history," this economic organization which we broadly term the capitalistic system steps forth full-bodied.

In the service of this capitalistic system we now have mighty vessels that can traverse the ocean at a thousand miles per day, where the early mariner could move but a few leagues; we have factories that can turn out in a single day thousands or millions of products and devices of which our forebears scarcely dreamt. We have wonders of steel and copper, and the alloys of other metals, which would have been beyond the comprehension even of our grandfathers. But the essential nature or structure of our industrial system has not materially changed. The system of private property, of the right to the products of one's own handicraft, the exchange of these "goods" and benefits and enjoyments and luxuries, is older than civilized man. It is older than the tribal organization which we call "barbarian." Its roots lie far back in remote epochs that we carelessly regard as "savage," forgetful that much that was characteristic of these savage races survives, but little changed, in our day.

This is indeed one of the paradoxes of racial evolution. The "savage" and the "barbarian" do not die out; they are with us today in full strength. We are sometimes forgetful that these savage and barbarian elements still make up a large part of the most advanced aggregations, in earlier times like those of Greece and Rome, and now Great Britain and our United States; they have not only added power but increased numbers. It is the moron of today and the vast surging underworld of the mentally undeveloped, the essentially uncivilized, which make possible our fantastic "dictators," our savage wars, our "revolutions."

And so we find a capitalistic system of organization which has achieved this miracle of our present day industrial world, which has created our science and to a large extent our culture, the greater part of our enjoyments and diversions, is again and again threatened with a kind of vast resurgence of this underworld or the under-mind, under the seductive delusion that somehow the whole body of the population could be suddenly lifted to a plane of some imagined affluence—as though by a wave of the hand, by a stroke of the pen, we might achieve an indefinite increase in the present production, itself the wonder of all the ages.

All this we know is not new. It may have happened over and over again; and this may explain much that is obscure and perplexing in the economic history of the early world. Almost the earliest laws we know of, dealing with economic questions, was the famous "Seisachtheia" of Solon, "the lifting of the burdens," i.e., the usual debasement of the currency, a moratorium for mortgages and debts, and the effort to "bring about a more equitable distribution of wealth." This was 600 B.C. What was the immediate effect of this we do not precisely know, but we do know that within a few years Greece had come under the sway of a great and enlightened statesman, apparently one of the world's great rulers, the famous Peisistratus, the first Tyrant of Athens, a man who had gained a large fortune in the silver mines of Laureion. It was he who laid the foundations of the commercial and financial supremacy of Athens which subsisted for some centuries. And it was this wealth whose creation he began which made possible the wondrous Age of Pericles, and all its creations of art and literature.

Most of all Peisistratus gave, alike to government, to trade and to the currency, a new stability, so that industry and commerce could thrive and wealth be accumulated. The capitalistic system is very old; but for centuries, nay for millennia, it could not thrive, it could only creep. The great enemies to the accumulation of capital and to the development of commerce, that is to say the great enemies of civilization, were then, as now, wars, piracy, brigandage, in all their myriad forms. So long as property was insecure, so long as vessels setting out were liable to be seized by pirates, or towns and cities could be raided and a large part of the population thrown into slavery, homes destroyed, and enterprise ruined, neither capitalism nor civilization could thrive.

The wondrous development of the last three or four hundred years was directly due to the establishment of *security,* legal restraints, justice, and freedom from the dangers and harassments of invasion, pillage, and destruction. Until the human race could have these priceless possessions there could be little accumulation of capital, which makes possible the discovery and development of all the myriad inventions which have arisen within the slender period since the discovery of America.

We seem only just beginning to learn the nature of economic evolution; and so to dispel the incredible ignorance even of the more instructed among mankind of the forces, the mechanisms and the institutions which have created modern civilization with its comfort and wealth. It must have been a philosopher who said that man differs from the animals in that the animals can learn from experience. It is sadly evident from the few centuries whose economic history we now know in detail, that even a rudimentary knowledge of economic phenomena, alike among our people and our rulers, as yet scarcely exists. But with the larger knowledge which we now have of the long foreground of present day institutions perchance we shall come, in time, to a clearer understanding and a more enlightened world.

July 15—THE ACCUMULATION OF THE PRECIOUS METALS IN INDIA AND CHINA,

DICKSON H. LEAVENS, Research Associate, Cowles Commission.

The future of the precious metals in world currency systems is uncertain. Nevertheless it is probable that gold will continue to be used in some way; the monetary use of silver, on the other hand, is declining, except for the great accumulation in the United States Treasury. As one factor affecting the availability of the precious metals for the monetary uses of Western countries, it seems worth while to consider the past history, present status, and future prospects of gold and silver in India and China, which traditionally have absorbed immense quantities of both metals and which only in the last few years have re-exported them to any extent.

Gold has played no part in the currency of China and only a minor part in that of India. Both countries long were on the silver standard. India abandoned that in 1893, and adopted a gold exchange standard, but continued to coin and use large amounts of silver rupees of full legal tender. China abandoned the silver standard for a managed currency in 1935. In India much larger quantities of both metals have gone into non-monetary uses than into the currency system. These uses have included ornaments for the women of all classes, household articles for the wealthy, paraphernalia of temples and courts, and liquid hoards in the form of gold bars and foreign gold coins. In China, on the other hand, the use for ornaments and utensils is limited to the well-to-do classes; gold is held to some extent as a store of value and as a means of speculation.

The available data for this study are incomplete. Customs figures for the seaborne movements of the precious metals are available since 1834 for India and since 1889 for China. For prior years, and for unrecorded exports and imports in the pockets of travelers, by smugglers, and over land frontiers, it has been necessary to use various estimates of other writers, supplemented by estimates or guesses made for this study. Although the large element of estimate makes the absolute figures somewhat uncertain, it is possible to indicate current trends with considerable confidence.

The total accumulation of gold in India reached 150,000,000 fine ounces in 1931; since then the high price of gold has caused the export of 30,000,000 ounces. The existing stock of 110,000,000 ounces (after allowing 10,000,000 ounces for wastage over the centuries by abrasion, loss, manufacture of gold leaf, and other destructive uses) consists of 7,000,000 ounces held in the currency reserves, perhaps 20,000,000 ounces held for investment and speculation, and 83,000,000 ounces in semi-permanent use as ornaments and the like. It seems probable that gold will continue to flow out of India at a decreasing rate for the next few years, and that, at the present high price, the Indian people are not likely ever to absorb gold again to the extent that they have in the past.

The total accumulation of gold in China reached an estimated 33,000,000 fine ounces in 1929; since then the high price of gold has caused the export of 8,000,000 ounces. Estimated wastage of 8,000,000 ounces leaves a stock of 17,000,000 ounces, part of which is held speculatively, and part as ornaments and the like. At present prices, gold is more likely to flow out of China than in, although the Government may continue to acquire modest quantities abroad for currency reserves.

The accumulation of silver in India reached over 5,000,000,000 fine ounces in 1931. Since then Government sales of about 35,000,000 ounces annually, derived from melting surplus rupee coins, have more than balanced the net demand for silver for ornaments and have slightly decreased the total. Allowing 700,000,000 ounces for wastage, the existing stock of 4,300,000,000 ounces is represented by nearly 1,000,000,-

000 ounces in the currency system and over 3,300,000,000 ounces in ornaments and the like. The Government may continue to sell silver at about the present rate of 35,000,000 ounces per year for several years. The import of silver for the ornament trade, however, has increased greatly since the price of silver fell in January, 1936. The Indian people, at the present low price of the white metal, are likely to buy silver extensively and not to dispose of old ornaments except in cases of severe need. It is, therefore, not unlikely that India will show a net absorption of a few tens of millions of ounces per year, provided the price remains low and agricultural and business conditions are fairly favorable.

The accumulation of silver in China reached 2,550,000,000 fine ounces in 1931, but by the end of 1935 had been reduced by exports of 450,000,000 ounces, largely as a result of the American silver purchase policy. Allowing 300,000,000 ounces for wastage, the existing stock of 1,800,000,000 ounces is probably divided roughly as follows: 500,000,000 ounces in the Government banks, 800,000,000 ounces of money in the hands of the public in spite of the nationalization order, and 500,000,000 ounces in ornaments and the like. Since the Chinese Government has definitely abandoned the silver standard for a managed currency and is continuing to dispose of silver reserves in exchange for gold and foreign exchange, it seems likely that China will be an exporter rather than an importer of the white metal in the future. This prospect, however, might be changed by internal economic or political disturbances or by foreign aggression.

In summary, it would seem that any plans for the restoration of some form of gold standard in the West need not allow for any large drain of the yellow metal to the East. As to silver, India and China together are not likely, on balance, to absorb any large part of current new production.

July 16—SOME OBSERVATIONS ON THE CHOICE OF FUNCTION IN CURVE-FITTING,

C. C. GROVE, Professor of Mathematics, College of the City of New York.

At the risk of "bringing coals to Newcastle" or of being classed with the pastor who lectures the faithful regular attendants of his flock on the unfaithful absentees, I am venturing to present to this audience *Some Observations on the Choice of Function in Curve-fitting,* because our Secretary and others of the Council felt that even in *this* presence it may be valuable to emphasize the importance of careful consideration of the elements, factors, and variables involved in the problem and data under consideration.

Dr. Harry H. Laughlin, Director of the Eugenics Record Office of the Carnegie Institution of Washington, Cold Spring Harbor, Long Island, N. Y., has repeatedly expressed a deep need of a book giving a wide range of graphs with the equation of each, a gallery of curves from which to select one that looks like the distribution under consideration. Something like but much more extensive than the book by Prof. Theodore R. Running: *Empirical Formulas,* is what he has at times almost persuaded me to write.

Even with such a gallery at hand, the worker must remember the effect of—

The choice of class interval,
The choice of scale,
The kind of coordinate system

to be used, as well as any peculiar physical properties or relationships in the material involved. Any one or all of these may change appearances.

The data to be used first are from a study on straight line radio condensers made in October and November, 1930. The ratio of the computed to the observed capacity of a certain condenser for each dial reading is the series to be fitted. Further, it was a part of our study to determine the nature of the coefficient P in the expression of capacity as given by Lowenstein, $C = Pe^{nx}$, whether it is an absolute constant or a function of the dial reading.

In the complete paper there comes at this point some consideration of the general relationships connecting inductance, capacity, frequency, wave-length; the logarithmic equations that give straight line graphs on semi-logarithmic paper; the type of function of the dial reading necessary to use as a factor to bring a specified condenser and circuit into actual straight line form.

With Lowenstein, take the capacity of condenser, alone or with the circuit, as given by

$$C = a\left(\frac{b}{a}\right)^{\frac{x}{d}}$$

where C is the capacity in microfarads, a is the capacity for dial reading zero; b for 100; x is the dial reading; and d is the number of divisions on the semi-circular dial.

By taking logarithms of both members of the above equation we get

$$\log C = \frac{x}{d} \log\left(\frac{b}{a}\right) + \log a;$$

thus $\log C$ is of the form $mx + \varkappa$. Likewise, if we use the former Lowenstein equation for capacity we get

$$\log C = nx + \log P;$$

so, in either case, $\log C$ is a linear function of the dial reading.

Applying the readings of capacity of Radiola 16 to both of the formulas, we obtain respectively,—

$$C = 41.1 \left(\frac{301.9}{41.1}\right)^{\frac{x}{d}},$$

$$\log_{10} 301.9 = 2.47986,$$
$$\log_{10} \;\; 41.1 = 1.61384;$$

$$\therefore \log \frac{301.9}{41.1} .86602.$$

Then for $x = 10$,
$d = 100$,

$$\log_{10} C = \log 41.1 + .086602 = 1.700442,$$

making capacity at 10 equal 50.63, but it is to be 51.2. So we get true zero reading by writing a in the equation and solving for a.

$$\text{Thus } a\left(\frac{301.9}{a}\right)^{.1} = 51.2.$$

By logarithms,

$$\log a + .1(\log 301.9 - \log a) = \log 51.2.$$

In numbers this is—

$$.247986 + .9 \log a = 1.70927,$$
$$\log a = 1.62365, \text{ and}$$
$$a = 42.04 \text{ really}$$

without edge effect. Thus the equation to calibrate Radiola 16 is

$$C = 42.04\left(\frac{301.9}{42.04}\right)^{\frac{x}{d}}.$$

For $x = 10$, $C = 51.2$ and
$\log_e C = 3.93573.$

For $x = 100$, $C = 301.9$ and
$\log_e C = 5.71010.$

$$\text{Whence, } 100n + \log_e B = 5.71010,$$
$$10n + \log_e B = 3.93573,$$
$$90n = 1.177437,$$
$$\therefore n = .0197152.$$

Then, $\log_e P = 3.93573 - .197152 = 3.738578$. And therefore $P = 42.0381$, and the true Lowenstein capacity readings for this instrument are given by the equation

$$C = 42.0381\, e^{.0197152x},$$

after noting that, due to edge effect, the zero reading is not the true zero reading. This equation is then used to calibrate the instrument for true straight-line readings.

The respective capacities by either calibration follow:

Dial	Capacity	Dial	Capacity
0	42.04	50	112.65
5	46.39	55	124.34
10	51.20	60	137.2
15	56.50	65	151.4
20	62.36	70	167.1
25	68.82	75	184.4
30	75.95	80	203.5
35	83.81	85	224.6
40	92.50	90	247.8
45	102.08	95	273.5
		100	301.9

A Study of the Ratio of Lowenstein Capacity to Total Capacity of Radiola 16.

Careful study of the series of capacities of Radiola 16 suggested that these readings may be brought to the true Lowenstein, "straight line," readings by the use of a multiplying factor that is a function of the dial reading. So we got the ratios and studied the series of them, by finite differences, showing that no parabolic fitting would be good. The graph of the ratios suggested a skew catenary but we still tried a third and a fourth degree parabola as well as an ordinary parabola enough to get variations of more than ten per cent. The deviations from the catenary $r = A + Be^{n} + Ce^{-n}$ were, on the other hand, so satisfactory, even within instrumental errors in reading, that this seemed the ideal function. Further, it separated a constant part from the total value of each ratio, and this information was a part of the investigation.

The procedure towards fitting the ratios to the equation is as follows:—

First, from the logarithms, already found in a former table and from which the above readings of capacity were secured, we subtract the logarithms of the corresponding Radiola 16 capacities. The anti-logarithms of these differences in order give us the successive values of the ratio for each dial reading.

The Method of Least Squares yields these general normal equations,—

$$N\,A + \Sigma e^{n}\,B + \Sigma e^{-n}\,C = \Sigma r\ ,$$

$$\Sigma e^{n}\,A + \Sigma e^{2n}\,B + \quad N\,C = \Sigma e^{n}\,r,$$

$$\Sigma e^{-n}\,A + \quad N\,B + \Sigma e^{-2n}\,C = \Sigma e^{-n}\,r\,.$$

Substituting the corresponding summations, we get the specific normal equations, whose solution gives the following values,—

$A = .8171204392, \qquad B = .0014646597, \qquad C = .0034247541.$

To facilitate the computations we changed the independent variable, x (dial reading), into a new independent variable n, so that dial reading 50 will become the new origin and that the interval of 10 dial readings becomes the new unit.

Accordingly, $x = 10n + 50$, or $n = .1x - 5$.

Thus, the catenary has the equation:

$$r = .8171204392 + .0014646597e^{n} + .0034247541e^{-n},$$

or in terms of x, after substituting the value of n and simplifying,

$$r = .8171204392 + .000009868730e^{.1x} + .5082785747792e^{-.1x}.$$

The following results of the fitting here given are interesting:

Dial	n	Per Cent ratio Difference to True	Const. is per cent of Instrumental Coefficient
10	—4.0	.41%	81.38%
15	—3.5	.85%	87.81%
20	—3.0	.98%	92.23%
25	—2.5	.28%	95.13%
30	—2.0	.96%	96.97%
35	—1.5	1.85%	98.12%
40	—1.0	2.36%	98.81%
45	—0.5	2.62%	99.22%
50	.0	2.59%	99.41%
55	.5	2.09%	99.45%
60	1.0	1.30%	99.36%
65	1.5	.24%	99.11%
70	2.0	1.04%	98.64%
75	2.5	2.17%	97.83%
80	3.0	2.70%	96.51%
85	3.5	3.90%	94.39%
90	4.0	3.32%	96.08%
95	4.5	1.19%	86.10%
100	5.0	3.45%	78.99%

In addition, the fitting of a surface to the probabilities of inheritance of racing capacity, as described by Dr. Harry H. Laughlin, was also discussed.

July 17 and 18—PSEUDO-SCIENTIFIC ECONOMIC DOCTRINE,

JOSEPH MAYER, Formerly Head of the Department of Economics and Sociology at Tufts College and Graduate School and Director of Research for the National Industrial Conference Board.

Dr. Mayer's two lectures may be comprehended under the title given above. In these lectures he extended his analyses of scientific method and social study undertaken previously, in an effort to clarify certain long-standing preconceptions in classical and neo-classical economic doctrine which still persist and which, in the light of 20th century methodology, can be given no other label than that of pseudo-scientific. These preconceptions revolve primarily about the ideas of "value," "cost," "utility," and "price," ideas which have suffered little change up to the present day and which even early writers recognized as being vague and ambiguous.

The first part of the first lecture dealt with the nature of the classical cost doctrine of price determination; with certain fallacies that are inherent in it; with its rise and decline; with the Austrian objections to it; and with its present status. The cost argument was epitomized as follows: first, it was conjecturally assumed that, in early society, embodied labor controlled exchange ratios; second, under the hypothetical conditions thus assumed, embodied and commanded labor and commanded money would evidently be equal, hence they came to be regarded as equal in modern society; third, scarcity and monopoly were thought of as virtually non-existent, hence it was concluded that market price equals normal price and that these are determined by embodied or commanded labor.

Recapitulating the fallacies and inadequacies in order: (1) A primitive stage of society in which embodied labor measured exchange value or price appears never to have existed. Traditions of forced rather than free exchange, and of predatory power, monopoly, and tribute, seem to have been the starting points of the "values" and "prices" inherited by more civilized peoples, who in slow stages eliminated the worst of the earlier political and social inequalities. (2) By means of the "alternative" sophistry and the erroneous assumption of homogeneous units of labor and waiting, the doctrinal superstructure of cost as the measure of price was nevertheless erected. (3) Its crumbling began as scarcity and monopoly conditions came to be recognized more and more as the rule rather than the exception, and qualitative differences in labor and waiting had to be admitted to account for decreasing per capita costs from generation to generation. (4) Its downfall was finally completed when a comparison of consumption with production factors brought to light an ever-mounting surplus or savings under modern production and indicated the increasing importance of ability to pay as against cost of production as a determinant of price. The cumulative recognition of these fallacies and deficiencies spelled the doom of the classical cost structure, although the "alternative" sophistry and other phases of the fallacious cost dialectic still continue to persist.

Turning to contemporary utility theory, Dr. Mayer analyzed in some detail its main contentions, namely, that diminishing utility explains the downward slope of the demand curve and that the resultant margin determines price. His analysis was summarized as follows: (a) diminishing utility is not a universal phenomenon, since increasing utility is probably just as prevalent, and a uniform utility is undoubtedly even more prevalent; (b) as Marshall indicated, the alleged determinative significance of the margin in utility analysis is without foundation; (c) the negative slope of the demand curve is better accounted for by unequal incomes, differing desires, and prudential expenditure, thus eliminating the need for a universally-diminishing-utility fiction. Utility, therefore, does not determine market price—first, because the universally-diminishing-utility and marginal assumptions are largely inapplicable, second, because the special ratio-of-equality argument is contrary to fact, and third, considering analyses based upon customary price, pecuniary choice, and the bounties of nature in supplying utilities to free and economic goods alike, because there is no correlation between desire or utility and price offers. Thus both major contentions of the utility school would seem to be clearly fallacious, while other more realistic hypotheses are available to explain the relation, or lack of relation, between so-called utility and price.

Whereas in the first lecture Dr. Mayer dealt with classical cost and utility theory on its own plane, he undertook in his second lecture to analyze the more general philosophical, psychological, and logical suppositions underlying it, dealing with such subjects as: obstructions to free competition from points of view of demand and supply; the lack of homogeneity and commensurability in utility and disutility; the normalistic, hedonistic, and rationalistic assumptions criticized by such men as Veblen and Mitchell; the dogma of unfailing mutual gain in exchange; mechanistic and organismic pseudo-analogies, especially those of economic organismic systems criticized by Schumpeter and Davenport.

Toward the end Dr. Mayer contrasted the rationale of the classical point of view, interpenetrated as it is with 18th century philosophy and theology, with a more modern point of view in which Evolution has been substituted for Perfection; Relativity in truth, beauty, and goodness for Absolutism; Becoming for Being; Immanence for Transcendence; and experiment and control in human affairs for "letting things take their course," which latter view has usually meant letting an unscrupulous minority take advantage of the pious majority.

His concluding words were as follows:

That modern business competition is far from being "free," either on the side of demand or supply, is finally coming to be more fully recognized. Competition in matters economic there always has been and doubtless always will be. But competition was of a decidedly predatory nature until modern social legislation (representing the forces of community co-operation) stepped in to "free" it somewhat of caveman influences. Prize fighting was once more predatory than it is today. Marquess of Queensbury rules have helped to make for a more equitable competition there. In a similar sense, co-operative action, in setting the rules of the economic game, lifts competition to a higher plane of freer and more equitable operation. In the economic and social revolution through which the United States now appears to be passing, a laudable effort is being made to formulate a still fairer code of business rules.

The new emphasis upon human welfare, as against the old emphasis upon the inviolability of vested property holdings, is wholly in line with the dissipation of the misapprehension that utility, cost, and value have economic meaning only in terms of consumption, production, and exchange practices, whatever their character. When deliberate psychological appraisals, which the marginalists erroneously assume to have been constantly made in the past, actually come to be undertaken in any discerning manner, it is found that the real and the ideal are often poles apart and that a normative standard in terms of public benefit is urgently needed to render reasonably significant terms that otherwise make economic nonsense.

The focusing of attention upon the achievement and maintenance of a fairly invariable purchasing power and a reasonable standard of living for the different groups of a society, rather than upon the encouragement of further wild orgies of frenzied speculation under the guise of business prosperity booms, is another illustration of the more modern point of view. It inferentially repudiates the anachronistic assumption that we still live in a barter economy. As a matter of fact, even business men are not interested in an exchange of goods for goods, but rather in securing a net profit on their transactions; and, in their broader rôle of human beings, they are still less concerned with mechanical swapping operations. They and others are interested in exchanging their services (given with a maximum of enjoyable effort) for sufficient purchasing power to maintain what they regard as a fair standard of living. The effort is often too painful and the standard of living attained not always the desirable one. But at any rate these are the exchanges in which human beings are primarily interested—not in a crude long-since-superseded barter of goods for goods. . . .

Such is the inertia of long-standing social preconceptions, authoritatively promulgated, that in economic thinking, where the classical ideas have taken firmest hold, utility-disutility dogmas are still widely followed, when what we actually need are scientifically-drawn hypotheses about value (based upon an up-to-date knowledge of human nature) and about economic institutions (based upon realistic studies of their actual functioning), such as are already being formulated by a few private commissions, bureaus, and institutions, and latterly by the Federal government itself. As it is, most economists appear to remain content with marginalism and its corollaries, even though some of them recognize that other ways of economic thinking have taken form.

The many die-hards among the classicists and neo-classicists, though perhaps admitting certain logical and psychological shortcomings in cost and utility theory, continue to insist that in the form of marginalism it nevertheless enjoys an indispensable probity and that it will not and should not totally disappear. It is presumed to possess "essential integrity," to bear a considerable "resemblance to the facts," and to be assured permanence, to quote Prof. John M. Clark, through "its pedagogical compactness, its logical coherence and availability, and its large measure of pragmatic truth."

It is the chief contention of these lectures that this "older economics" possesses "integrity" and is "logical" only on the score of pseudo-scientific assumptions and fallacious preconceptions, that for the most part it bears no resemblance at all to the facts of modern economic life, and that its very availability and pedagogical compactness are a snare rather than a help in the training of future economists except possibly by way of an object lesson in fallacious dialectic. Should this appraisal be correct, even though certain historical achievements in economic analysis may be retained in their proper setting and some of them utilized in the projection of more adequate hypotheses, the classical structure as such, together with its neo-classical refinements, must, I submit, be discarded in its entirety if a truly scientific approach to the problems of modern economic society is to be achieved.

(Further details bearing on Dr. Mayer's lectures will be found in the July and October issues of *Philosophy of Science*.)

July 20—SOME COMMENTS ON THE PRACTICAL SIGNIFICANCE OF TESTS OF SIGNIFICANCE,

WALTER A. SHEWHART, Statistician, Bell Telephone Laboratories.

(No abstract available).

July 21—THE NATURE OF REGRESSION FUNCTIONS IN THE CORRELATION ANALYSIS OF TIME SERIES,

HERBERT E. JONES, Research Fellow, Cowles Commision for Research in Economics.

The nature of the regression function in the correlation analysis of time series is shown to depend upon the type or general form of the series being correlated. The series are classified with respect to two general criteria. One, a criterion of "steepness" and the second, a criterion of "skewness."

Deviations from an adequately fitted trend are used, and the cycles divided into two groups, that portion of the cycles above the trend being called the positive parts of the cycles, and that below the trend being called the negative parts. The coefficient of variation, i. e., the standard deviation divided by the mean, is taken as the criterion of "steepness" and denoted by v. This coefficient is computed for each part of the cycle separately, the coefficient for the positive part designated v' and that for negative part, v''.

If two series, X and Y, are positively correlated, the ratio of their criteria of "steepness" indicates the general form of the regression function. If the ratio of v_y to v_x is taken, then: $v_y/v_x \gtreqqless 1$ as the slope of the regression curve tends to increase, remain constant, or decrease, respectively. This ratio is computed for the positive and negative parts of the cycle and when *both* are either greater than one or less than one the regression is S-shaped, when one ratio is greater than one but the other less than one the regression tends toward a hyperbola or parabola, if the ratios equal one the regression is linear. When the series are negatively correlated one must remember to compare the positive half cycles of one series with the negative half cycles of the other.

The criterion of skewness, designated by j, is the distance from the centroid of the area under the cycle to the midpoint of the half cycle; it is negative if the centroid lies to the left of the midpoint and positive when it lies to the right. It is seen that, when $j_y - j_x \neq 0$, the regression curve will be in the form of a double loop, i. e., of figure 8 shape. If the series are positively correlated and the difference of the criteria of skewness is negative it indicates that the points on the regression curve travel in a clockwise direction while if positive they travel counterclockwise. If the series are negatively correlated, these rotations are reversed. When the difference in skewness is large two regression curves are necessary, one showing the relation during the upward swing of the cycle, the second for the relation during the downward swing.

If the series are out of phase, i.e., when one series lags behind the other, an open loop will be formed in the scatter diagram when successive points are connected. If the series are positively correlated a clockwise rotation in these loops means that the Y series precedes the X series and counterclockwise rotation means that the X series precedes. If the series are negatively correlated, these rotations are reversed. By this method the limits for the optimum lag can easily and quickly be found by connecting successive points in a scatter diagram; if there is a consistent rotation in one direction we lag one of the series according to the above rule until the rotation is reversed. When we find a position where the lagging of one more item is enough to reverse the rotation, then we know the optimum lag lies between these two points.

Although the above analysis was illustrated by using hypothetical, smooth, periodic curves, four applications of actual time correlations showed the practical use of such analysis, the lag being easily found by means of the scatter diagram and the correlation coefficients being increased by using nonlinear regression functions where the ratio of the criteria of steepness indicated curvature.

July 22—MEASURES OF THE RELATIVE IMPORTANCE OF INDEPENDENT VARIABLES IN CORRELATION ANALYSIS,

E. J. WORKING, Associate Professor of Agricultural Economics, University of Illinois.

To many users of correlation analysis, how to measure the relative importance of the independent variables is a perplexing problem. Various measures are used. Among them are coefficients of separate determination, of partial correlation, of part correlation, and the Beta coefficients. These measures always differ somewhat in their relative size. Sometimes they differ very widely. As an example, we may take a table which appears in Ezekiel's *Methods of Correlation Analysis* (page 184), and compares for a certain multiple correlation problem the coefficients of partial correlation, part correlation, and the Betas. To that table I have added the coefficients of separate determination, with the following results:

	Partial correlation coefficients	Part correlation coefficients	Betas	Coefficients of separate determination
X_2 (Acres)	.27	.67	.402	.003
X_3 (Cows)	.80	.90	.926	.631
X_4 (Men)	.22	.54	.282	.173

Why do these measures vary so widely? Which is the best measure, and what does each really mean?

Elsewhere I have discussed the conditions under which regression equations may be expected closely to approximate the true causal connection between the independent and the dependent variables. When they do accurately express that connection we may speak of the importance of the independent variables in affecting the dependent variable. Otherwise we must speak of their importance in affecting the estimated independent variable.

A helpful approach to the problem is through a consideration of the trigonometry of correlation and vector analysis.

The coefficient of correlation between any two variables corresponds to the cosine of an angle between two forces. If we can solve the problem of the relative importance of two forces which contribute to the movement of an object we have by analogy virtually solved the problem of measuring the relative importance of independent variables.

Consider a man rowing a boat at right angles to the current of a river. If the current moves 6 ft. per second and the man would row 8 ft. per second in still water; then, due to the two causes combined the boat will move 10 ft. per second. What is the relative importance of the current and the rowing in contributing to the 10 ft. per second movement of the boat. Some will hold that we must have two figures which will add together to equal a resultant. In this case we must use;

$$(8/10)^2 + (6/10)^2 = (10/10)^2$$

or an equivalent expression wherein the squares of the two separate absolute or relative contributions add to the square of the resultant. Most people would, I believe, prefer to measure the importance of the rowing and the current in terms of the relative movement each would contribute by itself, rather than in proportion to the squares of these figures. Hence they would say that the relative importance of the rowing and the current were respectively as 8/10 is to 6/10 or perhaps as 8 is to 6.

The use of coefficients of separate determination corresponds to using the squares of the two forces acting on the boat as a measure of their relative importance. The Beta coefficients on the other hand correspond to .8 and .6 as measures of the relative importance of rowing and current in contributing to the movement of the boat.

The analogy of correlation and vector analysis may be carried out for any number of variables and for any relationships of inter-correlation between the variables. It should be noted, however, that ordinarily the coefficients of separate determination are not equal to the squares of the Betas. They are always equal to the Betas multiplied by the correlation coefficients of the dependent variable with the independent variables in question. ($d_{12.34} = \beta_{12.34} \cdot r_{12}$). On terms of vector analogy the Betas are equal to the length of the lines which represent the effect which each force would have separately, whereas the coefficients of determination equal the products of the lengths of those lines multiplied by the projections of the resultant upon them. The Beta coefficient for a particular variable always equals the ratio between the standard deviation of the dependent variable estimated (according to the multiple regression coefficient) from that variable alone, and the observed standard deviation of the dependent variable.

Partial correlation coefficients constitute a measure of the extent to which the coefficient of multiple correlation is increased by the addition of the variable in question to the analysis. The square of the coefficient for a given variable is equal to the ratio of the difference between the unexplained variance with and without that variable to the unexplained variance without the variable.* Like the other coefficients this too may be represented graphically in vector analysis.

A coefficient of part correlation is the coefficient of simple correlation between the independent variable in question and a "corrected" dependent variable—the dependent variable being corrected to remove the variation "due" to the other independent variables as indicated by the regression equation. It may also be defined by saying that its square is equal to the ratio of the variance "due" to the variable in question to that variance plus the variance due to "unexplained" causes.†

All the above mentioned measures as well as the first order correlation coefficients are of use if properly understood. Nevertheless, if we have a multiple regression equation which we believe to represent the approximate causal connection between variables there is little occasion to use coefficients of partial or of part correlation. The Betas are the best meas-

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* $$r^2_{12.34} = \frac{(1 - R^2_{1.34}) - (1 - R^2_{1.234})}{1 - R^2_{1.34}}$$

† $${}_{12}r^2_{34} = \frac{(b_{12.34}\sigma_2)^2}{(b_{12.34}\sigma_2)^2 + (1 - R^2_{1.234})\sigma^2_1} = \frac{(b_{12.34}\sigma_2)^2}{(b_{12.34}\sigma_2)^2 + \sigma^2_z}$$

ures for general use since they measure the relative importance of the independent variables in terms of the "strength" of each by itself contributing to variation in the estimated dependent variable.

July 23—EFFECTS OF INTERNAL STRESSES ON THE PRICE LEVEL,

E. J. WORKING.

Two matters of definition are here involved, namely, what is meant by internal stresses and what is meant by price level. By internal stresses I mean those things which affect the price level directly rather than primarily through other elements of the "equation of exchange." The statistics which I shall present deal with wholesale prices and hence it may be considered that I refer to the wholesale price level and do not include other elements of the "general price level." However, I shall use wholesale prices alone merely because of the greater adequacy of statistics relating to them. All the principles involved may appropriately be extended to other elements of the "general price level."

One of the most common methods of approaching the problem of price level, or value of money theory, is through the equation of exchange, $MV = PT$, or an equivalent expression. This approach leads to what is usually styled the "Quantity Theory of Money." While there are many different forms of the quantity theory, there are two primary points of agreement among "quantity theorists." First P (the price level) is a dependent variable—dependent upon the other elements of the equation. Second, the variation in P is largely dependent upon M (the volume of money)—including both currency or credit.

The equation of exchange itself is beyond dispute if its elements are properly defined. The important question is: How are the various elements of the equation related as to direction of causation? The equation of exchange is analogous to the equation expressing "Boyle's Law," $PV = RS$. Both equations give an important functional relationship between the various elements involved but they tell nothing of the *direction* of causal connection when these elements vary. The direction of causation is dependent upon circumstances quite apart from the equations themselves and will be different under some circumstances from what it is under others.

Sometimes changes in the amount of money (including credit) are responsible for price level changes. More frequently, I believe, the reverse is true or else the volume of trade, the price level, and the amount of money are dependent upon decisions of entrepreneurs and workers as to the amount of goods and services which will be produced. Furthermore, a general curtailment of production and hence of the volume of trade does not lead to a rise of prices as might be assumed from the equation of exchange. Instead it leads to a fall of prices and a reduction in the amount of credit.

Price level theory is essentially value theory, if by value we mean relative value. While I am disposed to admit that there may be such a thing as absolute value, such a concept is of no use except in the most abstract theory. Relative value is the only sort which is susceptible to measurement; and our theory should be in terms of it if that theory is to be based upon the facts of the economic world and is to be subject to any objective tests to substantiate it.

Many attempts have been made to deal with the value of money in substantially the same way as theorists have dealt with the value of commodities generally. Theories of the cost of production, marginal utility, supply and demand have all been applied. Since these different types of value theory represent different points of view and degrees of emphasis rather than essentially contradictory appraisals of the forces of value determination, it will suffice for our purposes to consider only the supply and demand theory as applied to the value of money. On the supply side changes in the cost of production of gold can affect total monetary stocks only very slowly. The supply of credit, on the other hand, can be changed very rapidly. However, changes in the supply of credit appear to be closely associated with changes in the demand for it. Furthermore, there is difficulty with the concept "the demand for money." For the most part money is useful to people only because it will purchase goods and services. Hence its usefulness and the demand for it depend upon its value. How then explain its value in terms of supply and demand?

The way to make price level theory consistent with the value theory is, I believe, through the study of the prices of individual commodities rather than through an attempt to consider money or gold as a commodity and subject to the same sort of influences as are commodities.

We have many statistical studies which support the traditional value theory and which show, in an approximate way at least, how values (or prices) and quantity demanded, and values (or prices) and quantity supplied are related. But the traditional theory treats of values (and prices if the price level is stable) as *depending* upon supply and demand. Our price studies show that over considerable periods of time prices of some commodities should not be treated as dependent variables. Over considerable periods prices are fixed by administrative decision, legislation, custom or inertia and the quantity produced is adjusted in accord with the changing conditions of demand. For many other commodities price is truly a dependent variable over short as well as long periods of time.

Statistical price studies also show that "real" demand both for individual commodities and for groups of commodities fluctuates with the volume of production of goods and services. This is in accord with traditional economic theory.

Under a system of partial price rigidity combined with inertias which prevent free flow of the productive agents from one employment to another we find reason to expect price level changes *due* to "internal" stresses of the price structure. Suppose that on account of war, tariff, or other changes there is need for a major readjustment of productive effort within a country. If the need is for a contraction of production of those industries whose prices are freely flexible the result will be a decline in the unit value of the products of such industries. If prices (in terms of money) of other products are fixed there must of necessity be a fall in the average level of prices. Then unit *values* of the commodities with fixed prices will be higher, smaller quantities of them will be demanded, less will be produced and there will be a still further decline in the *real* demand for commodities generally. This will cause a further fall in the *values* of commodities whose prices are not fixed and so the vicious circle will continue until there is a sufficient reallocation of the agents of production or breakdown of price rigidities to stop it.

We have many statistical evidences of internal stresses developing in the price structure. One way of showing them is by frequency distributions of price relatives. Taking the wholesale price relatives as published by the Bureau of Labor Statistics (1926 = 100) we find that ordinarily the distributions are fairly normal and that the dispersion of the price relatives increases gradually as the time interval from the base year increases. But as we come into the depression period the dispersion increases rapidly and the distributions become markedly skewed, indicating a marked change of internal stresses. These changes coincided with the decline in the price level. Following the devaluation of the dollar in 1933 the dispersion of the price relatives decreased markedly and the frequency distributions became more nearly normal, indicating a reduction in what may be called the abnormal stresses of the price structure.

We may think of the price structure as a system of balls each connected to the others by springs whose tensions change. The whole structure rests on a level surface. As the tensions of the various springs change some move north and some move south *relative* to others. But as long as all the balls are of the same sort and rest upon a homogeneous surface, the average position of the mass will not change. Suppose, however, that some of the balls are smooth and slide easily while others are rough and less easily moved. Suppose further that under certain conditions the majority of the rough ones tend to be moved north by the springs while the majority of the smooth ones tend to be moved south. Under these conditions the average of the mass will move to the south—under the influences of stresses between the balls combined with the "stickiness" of the balls which the springs tend to move to the north.

Similarly, the price level tends to move up and down under the combined influence of internal stresses in the price structure and the rigidity or "stickiness" of many prices of commodities and services.

July 24—REMARKS ON THE THEORY OF FOREIGN TRADE,

D. I. VINOGRADOFF, Manager of Foreign Engineering Department, Westinghouse Electric and Manufacturing Company.

(Read by Professor Charles F. Roos in the absence of Mr. Vinogradoff.)

Business men have little use for the classical theory of foreign trade, since it is not applicable to the actual facts and conditions of present day business. It seems to me that the time has come when we should seriously consider the necessity of a new approach to this problem. We should recognize the facts as they are, not as they were. We should discard some of the very foundations of the classical theory, namely, the immobility of productive agents. We should take into consideration the export of mental energy and the results of its expenditure in foreign countries.

At the present time there is a very pronounced desire of all countries to be self sufficient. Each country has begun to produce everything it can, and imports, particularly those of finished products, become more and more competitive with domestic products. In studying competitive exports we have to distinguish between conditioned and unconditioned or free markets. By conditioned exports we mean the existence of certain business relations between the buying and selling groups of different countries. Under such conditions the law of supply and demand is not applicable.

In the case of conditioned exports we have various factors, such as contracts, concessions, affiliations, branch factories in foreign countries, financial dependence. Owing to these factors the price may differ greatly from the open market price and goods may move from the higher price level to the lower. Even in the case of free markets such factors as the time element, preference given to experienced companies, effective servicing, the fad for foreign goods, and ease of credit, influence exports in spite of the higher price of the product which they cause.

These factors are known to specialists but they do not seem to be sufficiently appreciated; and I wish to emphasize that with the present industrial progress the classical theory of foreign trade fits the facts less and less and very soon will be in striking discord with them.

In suggesting the necessity for a new approach to the problem of building up a new theory of foreign trade, I am fully aware of the difficulties. A great deal of research into the various classes of exports, their destination and purpose, will be necessary; but this research will be very valuable independently of whether or not it will result in building up a theory. The data will be appreciated not only by theorists but by practical men engaged in foreign trade.

July 24—INSTABILITY IN COMPETITION BETWEEN TWO SELLERS,

TORD PALANDER, Rockefeller Fellow, Stockholm, Sweden.

This lecture is a part of a more extensive study of different types of competition between two enterprises, selling one good in a market. The purpose of the study is to show that "duopoly is not one problem but many." With respect to the multiformity of the problem the general point of view is thus the same as that taken by Chamberlin (*The Theory of Monopolistic Competition*, Cambridge, 1933) and other recent authors. I want, however, to stress that a deductive analysis of all the situations, where two sellers compete on a market with free competition between the buyers, must take into consideration many more factors than is generally assumed.

In the lecture only the relatively simple "Cournot case" (assuming a perfect market, the quantity offered by the sellers as the sole "parameter of action," and "autonomous action") is treated. It is shown, that even here a definite equilibrium will be the outcome of the competition only in certain cases.*

In Cournot's case of duopoly the assumptions are: a) a perfect market, i. e., only one price is possible in the market. Thus no preferences to deal with a certain seller, due to good will or differences in location, exist from the side of the consumers.

b) The quantity offered is the sole "parameter of action," which each enterprise attempts to adjust in the most favorable way (intending a maximum of profit). It has to be observed that this is the right and only possible assumption if the products of both sellers are sold by a common selling agency or a product exchange, which during each period of time exactly disposes of the total output at a common price, determined by the demand function of the market. When these market institutions prevail the sellers can not fix their prices. They can only fix the offered quantity and must accept the price, which is determined by the market with respect to the whole quantity offered (that is, offered by both sellers).

c) Each competitor acts "autonomously," that is, he believes at every time that the quantity at the moment sold by his competitor will remain unchanged, even if he himself changes his own quantity. The competitor's quantity is in other words regarded as a *constant* in the problem of maximizing his own profits.

The problem stated in general form. If the demand function of the market is

$$P = f(D_A + D_B) \tag{1}$$

*Other cases of duopoly (assuming: an imperfect market; the price, the local price policy, or the location of the enterprise in the market as "parameters of action;" and "heteronomous action," i.e., that one or both of the enterprises take the reactions of its competitor on its own actions into account) has been treated in Palander: *Beiträge zur Standortstheorie*, Upsala, 1935.

and the enterprise A offers the quantity D_A (the quantity offered by the competitor being D_B), the income or profit of the enterprise A will be:

$$I_A = D_A \cdot f(D_A + D_B) \ , \tag{2}$$

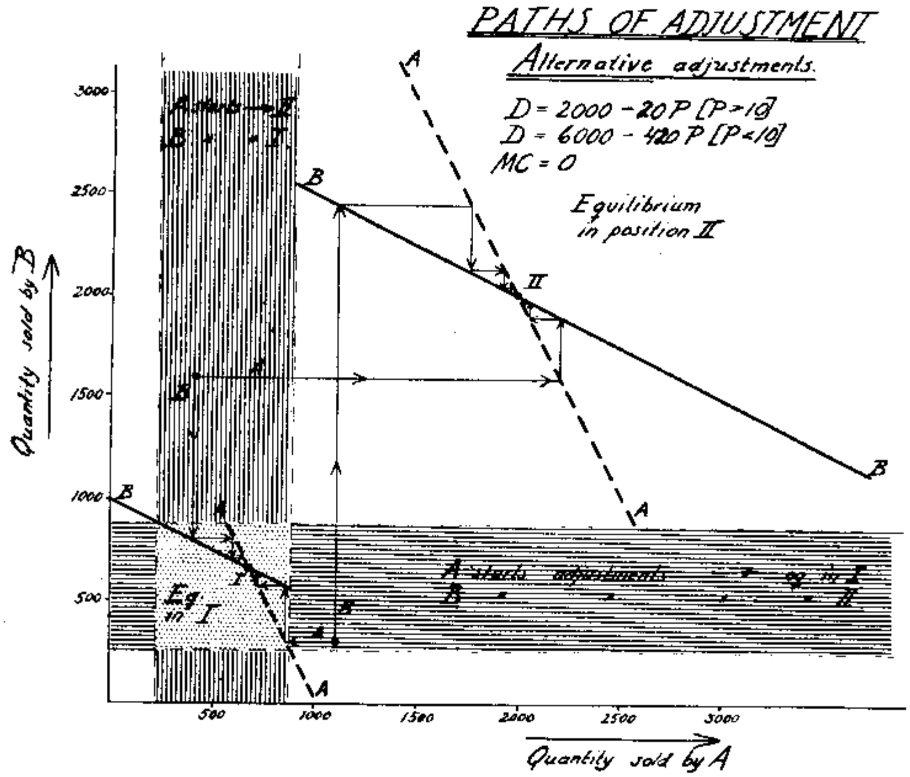


FIGURE 1

with a corresponding expression for B's income.

Equation (2) shows how A's income depends on two variables, D_A and D_B. For every magnitude of I_A a "profit indifference curve" can thus be plotted: it will prove to have a maximum point. For every given value of D_B (represented by a horizontal line in the diagrams) there must thus exist a certain output D_A, which maximizes A's profit. Geometrically this output D_A is determined by the point where one of A's profit indifference curves has a horizontal tangent at the given value of D_B. The *locus* of all points where A's indifference curves have horizontal tangents will thus show us which output, D_A, A must choose for every possible output, D_B, offered by B. It is a line, which shows how A adjusts his own quantity, when B offers a certain quantity in the market. The line is therefore called A's "path of adjustment."

It is mathematically derived by partial differentiation of (2) with respect to D_A:

$$f(D_A + D_B) + D_A \cdot f'(D_A + D_B) = 0 \qquad (3)$$

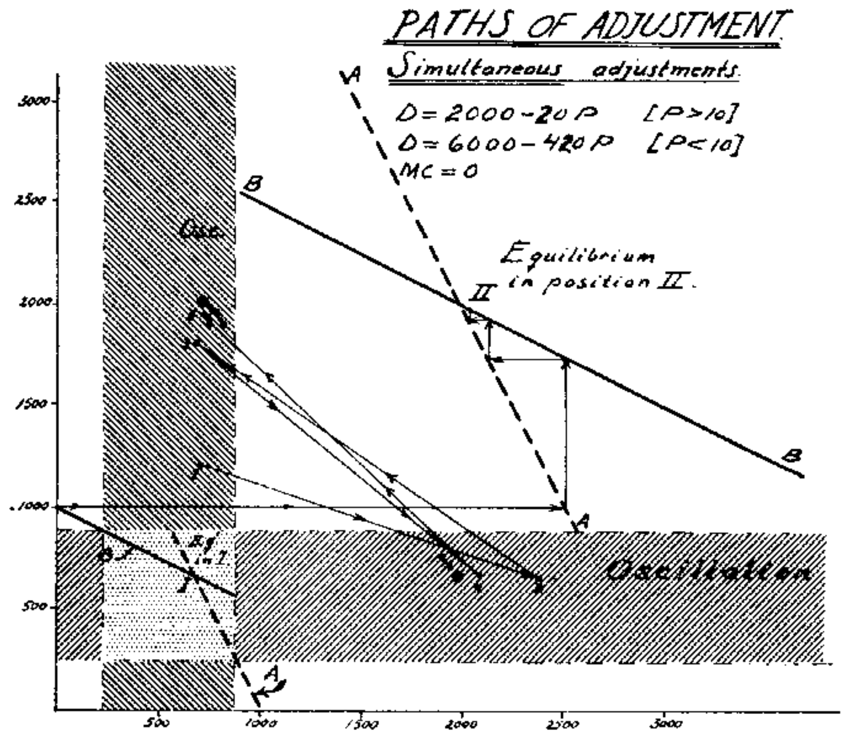


FIGURE 2

In a similar way we can define B's path of adjustment.

$$f(D_A + D_B) + D_B \cdot f'(D_A + D_B) = 0 \qquad (4)$$

The only combination of quantities, D_A and D_B, offered by two competitors, at which neither of them has any inducement to further adjustments, is represented by the intersection of the two paths of adjustment.

The foregoing was Cournot's solution of the problem. All later authors have followed Cournot in the belief that an equilibrium is always reached as a result of a series of adjustments of the offered quantities. The results can, however, be very divergent, as is shown in the following examples.

For cases, where the cost of production is equal for both enterprises and proportional to the output, the competition will lead to an equilibrium for linear demand curves and for such curved demand curves as have a steadily falling elasticity. But when the elasticity first falls, then rises, and then falls again, the outcome can be very divergent.

An example of such a case, which is mathematically easy to handle, is the following. Assume in a market two different classes of buyers (say two income classes), both having linear demand curves:

$$D_1 = 2000 - 20\,P \tag{5}$$

and

$$D_2 = 4000 - 400\,P, \text{ respectively.} \tag{6}$$

The demand of the second class of buyers disappears for prices above $P = 10$, but for lower prices its demand is very elastic. The *total demand* will be expressed by (5) for $P \geqq 10$; for lower prices it is the sum of the demand of both classes of buyers, that is

$$D = 6000 - 420\,P\,. \tag{7}$$

If the paths of adjustment for the two enterprises are computed, using (3) and (4), and plotted, we get the two lines AA for A's path of adjustment (Figs. 1 and 2) and the two lines BB for B's path of adjustment. The inner parts of the lines represent the paths of adjustment when the quantities offered by the competitor are small (and thus prices are high); the outer lines show the adjustments, when the quantities offered are big (and prices low). In the diagrams the lines are drawn only for the values of the competitor's output, for which they have the economic meaning of paths of adjustment, i.e., for which they indicate the position of an *absolute* profit maximum. An adjustment to the inner lines gives an absolute maximum of profit when the output of the competitor does not exceed 885. If the competitor's output is higher, the outer lines will represent the paths of adjustment.

The diagrams show that two points of equilibrium are possible. The final result depends, however, (1) on the *order of adjustment* of the outputs, (2) on the *starting point* (that is on the initial situation, from which the adjustments of the quantities start) and (3) on the *starter* (that is, which enterprise adjusts its quantity first).

a. *Alternative* adjustments of the quantities (Fig. 1). An equilibrium is always reached, but it can be either in I (low output, high prices) or in II (high output, low prices). We have the following possibilities:

1) The starting point within one of the four *white* fields in the diagram: equilibrium in II, irrespective of starter.

2) The starting point within the *dotted* field: equilibrium in I, irrespective of starter.

3) The starting point within the *horizontally shaded* areas:
if A starts adjustments: equilibrium in I.
" B " " " " II.

4) The starting point within the *vertically shaded* areas:
if A starts adjustments: equilibrium in II.
" B " " " " I.

b. *Simultaneous* adjustments. (Fig. 2). Three different results of the competition are possible: equilibrium in either I or II or continuously oscillating quantities without any possibility of reaching an equilibrium position. The outcome is determined by the starting point.

1) The starting point within the four *white* areas: equilibrium in II.

2) The starting point within the *dotted* area: equilibrium in I.

3) The starting point within the four shaded areas: a continuous oscillation of the quantities.

In cases like the above mentioned, the paths of adjustment will have at least one point of intersection. If the cost situation is asymmetrical, that is, however, not always the case. If the unit costs are 6 for one of the enterprises and 0 for the other (and the demand curves the same as before), the paths of adjustment will not intersect, and an equilibrium is impossible.

Even if the demand curve has falling elasticity and the cost situation is symmetrical, it is, however, possible, when the marginal costs are falling at certain outputs, that the paths of adjustment are broken and never intersect. The form of the demand curve alone, as well as the form of the cost curve alone, can thus be a sufficient cause for a lack of equilibrium. In reality it is probable that both these causes together can effect such a distortion of the paths of adjustment, that an equilibrium is impossible. Such is the case, e. g., when the demand function is

$$(D+2)(P+1)=100$$

and the cost function for each enterprise:

$$\text{total cost} = \frac{D^3}{45} - 0.8\,D^2 + 10\,D.$$

In this example not only is an equilibrium position impossible, but the situation is incompatible with the coexistence of two producers in the market: the competitors will successively throw each other out of the market.

To sum up: What is generally called Cournot's problem of duopoly turned out to be not one problem but many. The outcome of the competition is not always an equilibrium. It can depend on such factors as (1) the order of adjustment of the quantities offered by the competitors (simultaneous or alternative); (2) the position, from which the adjustments start; and (3) which of the competitors first start the adjustments. Only when the demand and cost curves fulfill certain conditions will the competition always bring about (a) an equilibrium. In other cases can (b) many equilibria, (c) a continuous adjustment from both sides of the output, or (d) a repeated change for each enterprise between monopoly, competition, and a total abandonment of the production, be possible results.

(Note: Because of delay in the mails it was not possible for the author to read the proof of this abstract).

July 25—SOME ECONOMIC EFFECTS OF WAGE REGULATION ON INTERSTATE TRADE,

CHARLES F. ROOS, Director of Research, Cowles Commission.

Wage differences among geographical regions of the United States have persisted for long periods of time and have not as sometimes held arisen during the present depression: they have developed over several decades as a result of differences in various factors. In particular, variations in the most important of these factors, labor productivity, have been caused by (1) migrations of the more efficient help to population centers, (2) varying degrees of mechanization attributable to a variety of causes, and (3) climatic and health conditions. Other factors determining wage differentials are (1) variations in the efficiency of management and (2) the degree of compactness of markets. With regard to the last, it has been pointed out that only in the large population centers can mass production methods be used effectively, and that, in less densely populated areas, transportation charges are so high as to reduce the amount of operating revenue which can be given to workmen.

Moreover, sizeable wage differentials have also persisted among cities of different sizes in the same region, and the factors described above have operated to cause them. When the effects of size of city are taken into account a large part of the North-South differentials disappears. More technically as shown by Henry Pixley, (*Econometrica*, July, 1935), the pre-NRA differentials can be reconstructed to a high degree of accuracy by a regression depending upon rent, which varies directly with the size of the city, and mean temperature.*

Variations in the cost of living, especially in rent, have enabled workmen in the small towns and in the cities of the South to accept lower money wages with little reduction in real wages. And what little difference has existed in real wages has been a result of lower productivity in the small town.

*Mean temperature is correlated with house rent, but has also an independent effect.

Despite these variations in regional wage rates and those by size of city, the NRA frequently set only one minimum wage for an entire industry. Even when multi-minima were established, they were usually set so as to lower materially pre-code differentials. As a result of this policy, the NRA placed economic pressure on employers in small towns and in the South, and discriminated among the states: some employers actually moved their plants to larger towns and cities and even across state lines. In other instances, minimum rate zones introducing abrupt changes in the minimum induced manufacturers to migrate into the low rate areas. In particular, Baltimore, Maryland, often placed in the Southern zone, received several Northern plants operating under higher minima, which would not have located there under *laissez faire* wage conditions. Thus, in a country as non-homogeneous as the United States, the introduction of either a single minimum wage or multi-minima should be expected to cause migration of industry and population.

In addition, a minimum wage benefits some classes but imposes hardships on others. It definitely causes displacement of both the young, inexperienced worker and the old one whose productivity has decreased. Therefore, it is not surprising that, following a year of the NRA, the United States has been forced to provide funds for keeping the young in school and for old age pensions. And yet to have substituted government control over exemptions for apprentices and handicapped workers would have led to a huge bureaucracy with dictatorial powers. Truly the problems involving minimum wages are enormously difficult!

As regards the effects of the NRA on wages above the minimum, we must conclude that since many of the NRA code provisions for adjusting the higher wages were vague, it will never even be fully known whether they were accepted in spirit. In fact, no one knows just what the spirit of these adjustments really was. As a rule, whenever employees did not complain, the wage adjustments were assumed to be satisfactory. Viewed a little differently, some Administration leaders apparently hoped that labor would force manufacturers and merchants to make "equitable adjustments," and were content to permit the matter to rest there. Thus, the manufacturer and the worker were frequently expected to assume the responsibility of deciding just what constituted equitable adjustments. And although some codes contained a provision authorizing investigations to determine what adjustments should be made, these studies were rarely undertaken and their results were often inconclusive.

Available data on wage distributions indicate that after the NRA codes became effective an abnormally large number of workers received wages near the minimum. Such concentration in general was almost complete in industries where the minimum was set higher than the wage formerly obtained by at least one-third of the workers. There was more concentration at the minimum among the women than among the men, and for each sex this was greater in the small than in the large cities. But, in general, industry did not reduce average weekly earnings below pre-code levels despite greatly curtailed hours, and in many instances weekly earnings were actually increased. In only a few cases has the weekly differential between earnings of skilled and unskilled workers decreased while the hourly differential has been maintained or even increased.† Therefore, the concentrations of workers at the minima are to be explained chiefly by increases in wages of workmen previously receiving lower wages: the groups for which pre-NRA average hourly earnings were increased by the greatest percentage show, in general, the most unsatisfactory post-code wage distributions, at least in the sense of a high percentage of employees near the minimum. In some few cases manufacturers' cost was increased while weekly earnings decreased: the wage of the employed skilled worker was divided with the unemployed.

†For instance, the paint and varnish industry reported an hourly differential increased sixteen per cent and a weekly differential decreased twelve per cent.

July 27—THE SIGNIFICANCE OF REGRESSION COEFFICIENTS,

R. A. FISHER, Galton Professor, University of London, and Director of Galton Laboratory.

In the theory of large samples, the variances and covariances of the simultaneous sampling distribution of a system of estimated partial regressions is given by a matrix reciprocal to the information matrix, and, without the use of matrix algebra, derivable from it by the solution of a set of simultaneous linear equations.

Knowing the variances and covariances, those of any chosen linear compounds of the regressions, such as may be valuable in reviewing the data under different aspects, are immediately available.

It is, therefore, in general, preferable, when dealing with partial regressions, not to solve the regression equations directly, but to derive, from the coefficients, the covariance matrix. This also simplifies the procedure when it is desired to omit one or more of the variates originally employed.

Significance so tested is measured against the residual variance of the dependent variate, which ideally is due wholly to errors of observation, but is often in fact due not to actual errors of observation, but to causes unknown. The "residual error," therefore, is often a measure of residual ignorance, and improvement in the form of the regression function chosen, or the introduction of new independent variates, may reduce its value.

The first possibility may often be excluded by using the variance within arrays to test the fitness of the regression formula chosen. If the fit is good no alternative form can do appreciatively better. For the introduction of new variates we should rely at best on theoretical considerations. If sociological or economical theory suggests that certain new variates may be relevant they should be tried, but we can never exclude the possibility that others which have not been tried may in fact be more important.

With respect to the sampling error of a predicted value, use should always be made of the principle developed by

Working and Hotelling, whose method, though given several years ago, has so far been scarcely appreciated.

The method of partial regression is due to Gauss, and has been used extensively in many fields under the title "The Method of Least Squares." Gauss, however, developed only the large sample theory. For small samples we may with exactitude use the t test for the significance of regressions, and the deviations of theoretical from experimental values, and the z test for the goodness of fit of the prediction formula.

One general limitation should be noted. The formula chosen must be linear, not necessarily in the variates, but in the adjustable coefficients. If this is not so, the fitting of a single constant may reduce the residual sum of squares by much more than the corresponding fraction of the degrees of freedom, and so tend to give apparently highly significant values, without a causal basis.

July 28—TESTS OF SIGNIFICANCE IN HARMONIC ANALYSIS,

R. A. FISHER.

For use in harmonic analysis a number of tests of significance have been developed, but their application is capricious owing principally to the vagueness with which the term "harmonic analysis" has been applied. Schuster's test is entirely rigorous for a particular harmonic component chosen in advance, this being one of the Fourier sub-multiples appropriate to the length of the series analyzed, provided that the variance of individual values is known a priori. These are serious limitations. In particular, it is usually desired to pick out the largest term or terms from a harmonic series and to test its significance, allowing for the fact that we have chosen the largest out of all those available. A formula which makes this allowance correctly has been given by Sir Gilbert Walker.

If we wish also to make allowance for the fact that our knowledge of the sampling variance has been derived from the data themselves, we must consider the problem of how frequently the largest component of the series contains more than a given fraction of the total. This may be solved by a construction in Euclidean hyperspace, and leads to a very simple solution for an unselected term with a somewhat more complex formula when we have chosen the largest term of the series.

It is important theoretically, and not in any way inconvenient in practice, to confine attention to the exact sub-multiples of the Fourier series. Without this restriction it would certainly be in general possible to explain a somewhat larger fraction of the total variance, using only the one adjustable constant designating length of period or the three constants specifying also the amplitude and phase. Even within the restriction the solution shows that the one degree of freedom specifying period is capable of diminishing the residual sum of squares by a fraction of the whole larger than would ordinarily be judged significant. The mathematical reason for this is that using harmonic series with variable period the expectations of the recurring values are non-linear functions of the adjustable constant.

July 29—INVERSE PROBABILITY,

R. A. FISHER.

The first serious attempt to find an exact rational basis for inductive inference was made by Thomas Bayes in a posthumous paper published in 1763. By the introduction of a special postulate Bayes' method, which soon became known as the method of inverse probability, seemed to bring within the domain of the theory of probability an immense number of cases in which inductive inference appeared intuitively to be possible.

Bayes' method rapidly gained acceptance on the Continent and was employed by Laplace somewhat uncritically in the definition of probability given in his *Théorie Analytique.*

The principle was criticized cogently and tentatively by Boole in 1856 and towards the end of the nineteenth century was rejected more definitely by Venn and Chrystal. These writers, however, supplied no alternative method and attempts have been made either to demonstrate or to defend the method in quite recent years.

If we frankly reject the claim to prior knowledge made in Bayes' postulate, the data are still found to supply a mathematical function of the unknown parameter or group of parameter by which the population sampled is specified. This function, known as the likelihood function, appears at first sight to supply an order of preference among the possible parametric values. As a method of estimation the choice of the value having a maximum likelihood has been shown to possess all the advantages which we should intuitively require of an estimate, namely, that it is always Consistent, always Efficient, and when a Sufficient Statistic exists this also will be obtained by the method of maximum likelihood.

The Sufficient Statistics are characterized by the fact that they alone contain the whole of the information supplied by the sample. When the Sufficient Statistics do not exist we may compare different possible estimates by means of the measurement of the Quantity of Information that they contain. The estimate of maximum likelihood is then found to lose less information than other possible estimates. The information lost has been shown to be recoverable in certain important classes of cases by using what is called the Ancillary information supplied by the sample. Estimation may thus be made exhaustive even when no Sufficient Statistic exists.

One of the most characteristic developments of the modern period in mathematical statistics has been the appearance of rigorously exact tests of significance. When these are known the exact sampling distribution of some quantity calculable from the data is rigorously determined by the form of the sampled population. When this population contains a variable parameter this may enter into the expression for the test quantity and its sampling distribution be inferred from that of the latter. This line of argument was first employed by practical experimenters and was entirely unknown to the earlier writers on mathematical probability. It supplied, however, probability statements respecting the value of an unknown parameter or group of parameters entirely independent of any assumed prior knowledge. These are known as statements of Fiducial Probability and the distribution which summarizes them as the Fiducial Probability Distribution.

Values of Fiducial Probability may differ numerically from the corresponding values of inverse probability and since in any particular case these latter also may be true it is evident that the two statements differ in logical content. It is useless, therefore, to adjust the supposed distribution *a priori* as has been attempted in order to make the two forms of statement agree numerically.

Two important limitations should be noted. From discontinuous distributions no Fiducial distributions can be inferred and the corresponding probability statements are no longer exact, though they may be given the form of mathematical inequalities. In the second place, statements of Fiducial Probability are only valid when the statistics employed are capable of supplying exhaustive estimation.

July 30—OPPORTUNITIES FOR THE USE OF STATISTICAL METHOD IN INDUSTRIAL RESEARCH,

ANSON HAYES, Director of Research, The American Rolling Mills Co.

The discussion is concerned with material which is intended to illustrate the use of certain statistical methods. In industrial research the process of developing new products or improving old products usually consists of the following procedure:

1. The solution of a statistical problem which usually involves creating an experimental setting which provides the necessary degree of assurance of being able to distinguish significant values in quality.

2. Determination of mechanism by which improvement in quality may be obtained.

3. Development of laboratory tests for methods of making them which makes possible rapid and extensive exploration of the effects of new compositions and new methods of processing. These explorations usually are necessary to discover principles which may be followed in realizing improvements. These principles also result in great savings in volume of experimental work.

4. The experimental production of the improved product and the testing of it in service.

The statistical point of view that has been resorted to in the work done in the author's laboratory is largely that developed by W. A. Shewhart and involves the isolation of the so-called constant chance cause system and the use of statistical criteria for the establishment of such a statistical system. The discussion is illustrated by a detailed exposition of the use of statistical methods in the development of a method of test for corrosion resistance of metals in atmospheric and in immersion services. The use of these methods in the establishment of controlled practices in the manufacture of steel products is also presented. An additional section consists of the presentation of that part of statistical theory and its use which is involved with the error of averages of samples of various sizes.

In the section devoted to the development of methods of testing for corrosion resistance, the use of these statistical methods resulted in the development of a laboratory test which gives controlled data. For such information the effects of such variables as kind of base metal, time of exposure, and a number of others were investigated. A number of interesting results in the immersion tests were obtained:

1. The scattering of observations becomes greater as the loss of weight increases. More specifically the standard deviation increases as the loss of weight increases.

2. Initially iron loses weight in proportion to time.

3. The duration of initial period decreases as the velocity is raised and is probably a hyperbolic function of the velocity.

4. After the initial period, during which the loss of weight is proportional to time, the experiments lose weight in proportion to the logarithms of time under the conditions of these experiments.

5. Iron experiments exposed to water and high velocity lose more weight for a time than those exposed at low velocity. After a month or so the logarithmic curves are quite likely to cross, so that in the long run the experiments exposed at the higher velocities will probably lose less weight than those exposed at the lower velocities.

In the section involving the study of the error of averages, solutions to three important problems are given, and these solutions are applied to fields of corrosion testing of materials, and of a number of tests in connection with concrete road building. The three problems dealt with are:

1. What is the probability P that the average of a sample of size n drawn from a constant system of chance causes will lie between the limits

$$\bar{X}' + x \quad \text{where} \quad x = \bar{X} - \bar{X}'$$

2. What is the value of x such that an average X of n observations will have a given probability P' of lying between the limits $X \pm x$?

3. What sample size must we use in order that there will be the probability P' of the average lying between the limits $X' \pm x$?

In these sections charts are developed from which may be read the three related quantities, size of sample, error of average, and the standard deviation of the universe. This discussion is closed with a recital of the present committee organization in the American Society for Testing Materials for the promotion of the use of statistical methods throughout this Society. The American Society for Testing Materials has now published a manual and a number of supplements to it which deal with the problem of presentation and interpretation of engineering data.

July 31—METHODS OF ELIMINATING THE INFLUENCE OF SEVERAL GROUPS OF FACTORS,

CORRADO GINI, Professor of Statistics, University of Rome, President of the Italian Institute for the Study of Population Problems, and Editor of *Metron.*

The quantitative treatment of collective phenomena admits of several stages or degrees:

1. Quantitative description of the phenomena;

2. Quantitative description and qualitative explanation of the phenomena;

3. Quantitative description and qualitative explanation, supported by a quantitative illustration of the influence of the circumstances (or groups of circumstances) allegedly involved;

4. Quantitative description and quantitative explanation, providing the measure of the influence of the various circumstances (or groups of circumstances) involved.

This measure, which characterizes the most advanced stage of the quantitative treatment, cannot, however, be obtained without some knowledge, or at least without some hypothesis, concerning the manner in which the effects of the several circumstances (or groups of circumstances) combine when these act simultaneously.

The simplest scheme is the *summatory scheme,* according to which the total effect of several circumstances acting simultaneously is equal to the *sum* of the effects of the single circumstances acting separately. This scheme is basic to the multiple correlation method as well as to the method of residues of Stuart Mill. It is equally adopted in applied mechanics under the name of "principle of superposition of effects." Its application in the field of collective phenomena is certainly justified in some cases, as when we calculate the net emigration of a country by subtracting the actual increase (derived from the excess of births over deaths) from the total increase (calculated by the difference between the figures of two censuses). But in many other cases the summatory scheme evidently does not fit the facts and it is more convenient to apply the *multiplicatory scheme,* according to which the total effect of several circumstances acting simultaneously is equal to the *product* of the effects of the single circumstances acting separately.

The multiplicatory scheme is basic to the so-called "ideal formula," adopted for the computation of index numbers of prices and advisable also in other fields for measuring the part of the total variation of a collective phenomenon, from time to time or from a locality to another, which is due to the influence of a certain group of factors, eliminating the influence of another group of factors.

Until the present time, however, the methods of elimination through the ideal formula have been generally applied to the simplest case of phenomena resulting (or considered as resulting) from the combined effect of two factors (or groups of factors); prices and quantities of different commodities; death rate and size of different age classes; and so on. Often, on the contrary, we meet phenomena which result from the action of several factors. For such cases is it possible to indicate formulae based on the same scheme and having the same properties as the so-called ideal formula for the case of two factors? An answer to this question is the main object of the paper. The case of three factors— indicated by the letters a, b, c—is considered, but the results may easily be generalized to any number of factors whatsoever.

The formulae are different according to the different types of interrelations which exist among the three factors. Three types of interrelations are taken in consideration:

Type A. The effect T is the resultant of the action of the factor a, on the one hand, and of the joint action of the two *separate* factors, b and c, on the other. Example: The general death rate may be considered as the resultant of the specific death rates for every category of a given age and of a given sex, on the one hand, and, on the other, of the composition of the population by age and by sex. The problem is that of constructing an index number of the variations of the specific death rates by eliminating from the variation of the general death rate the influence of the sex and age composition of the population.

Type B. The effect T is the resultant of the action of the factor a, on the one hand, and, on the other, of the joint action of two factors, b and c, one of which is *supplementary* to the other. Example: The annual salary of an employee may be considered as the resultant of his salary per hour, on the one hand, and, on the other, of the number of working days per year and of working hours per day. The problem is that of constructing an index number of the variations of the hourly salary by eliminating the influence of the variations of the number of working days per year and of the number of the working hours per day. Such an index may be reduced to an index of the variations of the hourly salary by eliminating the variations of the number of working hours per year.

Type C. The effect T is the resultant of the *symmetrical* action of the three factors, a, b, and c. Example: the volume of the chest may be considered as the resultant of the product of the two diameters (transverse and antero-posterior) of the thorax and of the sternal height. The problem is that of constructing an index number of the variations of one of the three linear dimensions by eliminating the variations of the other two.

The formulae obtained for the three types of researches by eliminating the influence of the two factors b and c are compared with those obtained by eliminating the influence of only one factor (for example b), and the differences between the former and the latter are explained.

The formulae attained for type B are applied to the index number of prices as a special case. A demonstration is given of the conclusion reached by the author some years ago

(*Metron*, 1924), that, if the index number of the economic utility of the commodities is equal to unity, the formulae of Laspeyres and of Paasche for the index number of prices give the limits (upper and lower, respectively) between which the index number of the economic purchasing power of money must lie.

(The above paper will be published in full in *Econometrica* in 1937.)

July 31—ON THE MEASURE OF CONCENTRATION WITH SPECIAL REFERENCE TO INCOME AND WEALTH,

CORRADO GINI.

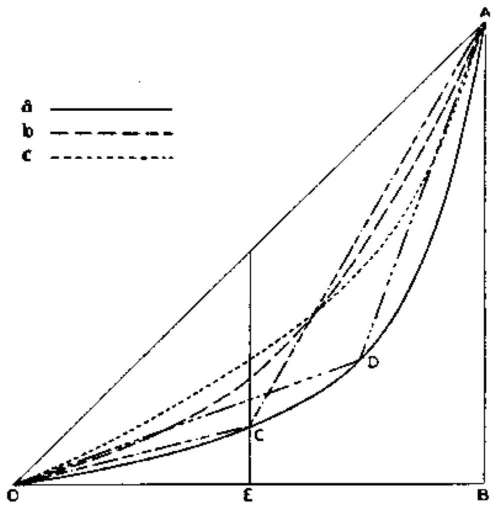
The shape of the distribution of individual incomes attracted the attention of several scholars in the course of the last century. Among them, R. Dudley Baxter (1868) deserves especial mention. However, the most important advance was made by Otto Ammon (1895) who thoroughly discussed this subject in connection with the distribution of individual abilities studied by Francis Galton (1870), and noticed the similarity of the curves of taxed income derived from the fiscal data of various states. Two years later, V. Pareto (1897) confirmed this similarity for different states as well as for different epochs and presented the formula $\log N_x = K - \alpha \log x$, (where K and α are constants and N_x indicates the number of taxpayers with an individual income greater than x), which became known as "Pareto's law." According to this formula the distribution of incomes in a double logarithmic scale is fitted by a straight line. α furnishes an inverse index of the concentration of incomes. The values found by Pareto for α were close to 1.5; so that Pareto concluded that there was a similarity, not only in the shape, but also in the degree of the concentration of incomes in various countries and ages. The above formula, however, represents only the first approximation given by Pareto for the income distribution; second and third approximations are represented by the formulae $\log N_x = h - \alpha \log(x + a)$, $\log N_x = t - \alpha \log(x + a) - \gamma\, x\, a$, where h, t, a, α are constants. These formulae, however, do not have the advantage of the first and simpler formula—that is, of furnishing an index of the concentration of incomes—as the parameters α and a, or α, a, and γ may vary in opposite senses. For the same reason little attention has been paid by statisticians to other formulae proposed by various authors for describing the distribution of incomes with more complicated expressions, containing 4, 5, and even 6 parameters.

If, instead of considering the number N_x of taxpayers with an income *above* the limit x, we consider the number n_x of taxpayers *with* an income x, we obtain the formula $\log n_x = s - \varepsilon \log x$, where s and ε are constants. Theoretically $\varepsilon = \alpha + 1$, a relation which also in practice is approximately true. In practice, it is, however, advisable to consider the integral (as Pareto did) rather than the differential distribution, because the statistics do not give the number of taxpayers n_x for the single incomes x, but only the number of taxpayers for categories of income, often very large.

Pareto noticed that his first formula fitted only the distributions of total incomes, not those of labour incomes (which are convex toward the axis of the abscissae) or of incomes from capital or of fortunes (which on the contrary are concave). For these it was necessary to have recourse to the second or third formula. But for such purpose also the statisticians, for the reason mentioned above, avoided the use of the second or third formula, looking for other formulae which might furnish an index of concentration. Benini applied to fortunes (1906) and Gini to inheritances (1914) the formula $\log N_x = K' - \alpha'(\log x)^2$. This formula has the inconvenience that the values of the constants K' and α' are not independent of the unit in which x is expressed, and therefore its use is advisable only when no better formula is applicable.

A substantial improvement in the formulas for measuring the concentration of incomes has been made by taking account not only of the distribution of the taxpayers, but also of the amount of their incomes. Gini (1908) noticed that, if A_x indicates the amount of the incomes above the limit x, the data may be fitted by the formula $\log A_x = r - \beta \log x$, and from this, and from the first Pareto formula, derived the other, $\log N_x = p + \delta \log A_x$. This formula may be also determined directly from the data and it has been used in numerous applications by Gini himself and other statisticians. Theoretically it is $\beta = \alpha - 1$; $\delta = \frac{\alpha}{\beta} = \frac{\alpha}{\alpha - 1}$, but in practice these equalities do not correspond exactly to reality. The parameter δ furnishes a direct index of concentration of incomes. Various advantages of Gini's formula and consequently of the index of concentration δ as compared with Pareto's formula and the corresponding index α have been recognized. 1. Gini's formula takes account of the taxpayers, often very numerous, of the lowest class for which generally only the above limit is given. 2. The data are generally better fitted by Gini's formula. 3. Gini's formula satisfactorily fits also the distributions of labour incomes and of rents, which is not the case for Pareto's formula. For rents it has been possible to apply Gini's formula with good results, not only to the values assessed by the fiscal administration, but also to the real values. Advantages (2) and (3) have surprised some statisticians, but really there was no reason for such surprise, because it is not possible to pass from Gini's to Pareto's formula, or vice versa, without some hypothesis, one of these being that the maximum value of x is infinite, which is evidently far from reality, especially for rents and labour incomes. 4. The index δ has a precise meaning which has also been useful for giving an appropriate interpretation to the small variations observed in the values of α. Gini's formula may, as a matter of fact, also be written $\frac{N_x}{N_0} = \left(\frac{A_x}{A_0}\right)^\delta$ which shows that δ is the exponent to which we must elevate the fraction of the total amount of incomes represented by the incomes above a certain limit in order to obtain the corresponding fraction of the taxpayers who possessed it. It has been found that the values of δ, including corporations, vary between 2.4 (Württemburg, 1905) and 6.3 (Cape of Good Hope, 1907-08), and between 1.6 (Cape of Good Hope, 1907-08) and 4.6 (Canton of Basel-Stadt, 1891), if only physical persons are considered. These results show that the concentration of income differs enormously from time to time and from place to place. As a matter of fact, a variation of δ between 2 and 6 means that one-half of the total income is possessed by a fraction of the tax-payers which varies between 1/4 and 1/64. Pareto arrived at the opposite conclusion because of the very limited sensitiveness of α, which he did not perceive. In fact, he found values of α ranging from 1.9 (Prussia, 1852) to 1.1 (Hamburg, 1891). Theoretically these values would correspond to values of δ ranging from 2.1 to 8.6, and hence are far from justifying Pareto's conclusion about the similarity in the degree of concentration of income in various countries and ages. For the reasons explained above, the Gini formula is preferred to Pareto's formula when the available data permit its application, that is, when the statistics give not only the number of taxpayers but also the amount of their income for every category of incomes.

The field of application of Gini's formula, even if wider than that of Pareto's, is always limited. As a matter of fact it is limited to total incomes, labour incomes, and rents. A general measure of concentration is given by the curves of concentration independently proposed almost at the same time by different authors, Lorenz (1905), Gini, Châtelain, Séailles. The curve of concentration gives the percentage (q_i) of the amount of a character as a function of the percentage (p_i) of the individuals which possess it, the individuals being generally graduated according to the increasing intensity of the character. For total incomes, labour incomes, and rents, the curve of concentration has the following expression: $p_i = 1 - (1 - q_i)^{\delta}$, which is another form of the above mentioned Gini formula. When the analytical expressions of the curves of concentration are not found, it is, however, possible, as Lorenz has proposed, to compare the concentration of two or more distributions on the basis of whether the convexity of the curves is more or less accentuated. The concentration is greater for the curve more convex toward the axis of abscissae (for example, in the figure, for curve a in comparison with curve b). Lorenz himself noticed the incompleteness of this method, as it is not always possible to decide what curve corresponds to a greater concentration when the curves cross each other (as, for example, curves *b* and *c*). Gini then proposed (1914) to take as a measure of the concentration (*concentration ratio*) the area (area of con-



centration) located between the line OA (line of equidistribution) and the curve of concentration, divided by the area of the triangle OAB. He demonstrated at the same time that this concentration ratio corresponds to the arithmetic average (*mean difference*) of the $n(n-1)$ differences (taken in absolute value) which may be constituted between the n terms, divided by its maximum possible value (equal twice the arithmetic average of the n terms). The mean difference had already been proposed by Gini (1912) and used by him and other statisticians as a better measure of the variability when the average has no typical value. Successively, Pietra (1915) demonstrated that the ratio of the mean deviation from the average to its maximum possible value corresponds to the ratio of the maximum triangle (ODA) inscribed in the concentration curve to the triangle OBA; while the ratio of the mean deviation from the median to its maximum possible value corresponds to the ratio to the triangle OBA of the triangle OCA having the line OA as its basis and the intersection of the curve of concentration with the perpendicular to the median point E as its vertex. Thus the relations between the theory of concentration and the theory of variability were clearly established. Several statisticians have been interested in developing more rapid techniques for calculating the concentration ratio and the mean difference, the determination of which, using appropriate devices, is not much longer than that of the coefficient of variability based on the standard deviation.

Later, Gini (1930) demonstrated that the triangle OAB represents the maximum that the area of concentration may attain, only when the value of the character admits no maximum (so that its total amount may be possessed by only one individual) and no minimum (so that its value may be $= 0$), and gave the new limits for the cases in which these hypotheses are not satisfied.

Other contributions to the theory of mean difference and of concentration ratio have been given by various statisticians, especially by Galvani (1931), and Castellano (1933), among the Italians, and by Czuber (1914), Gumbel (1928), and Wold (1935), among those of other countries. The curves of concentration and their relations with the frequency curves have been studied particularly by Gini (1932) and Castellano (1933). The applications of the concentration ratio have been numerous, especially in Italy, by Gini (1910, 1914), Corridore (1911), Porru (1912), Pietra (1914), Savorgnan (1912-1915), Vinci (1918), Saibante (1926, 1928), Orlandi (1933). So the different degree of concentration of total incomes, labour incomes, incomes from capital, fortunes, inheritances, land property, etc., has been exactly measured and compared with the degree of concentration of anthropologic, biologic, and demographic characters. Gini (1910, 1914), has measured the demographic component of the concentration of wealth, that is, the concentration of wealth which will be found in a generation, due to differences in number of children, on the hypothesis that all the parents of the preceding generation had left identical inheritances.

SELECTED BIBLIOGRAPHY

V. PARETO: Cours d'économie politique. Lausanne, Rouge, 1897. Livre III, Chap. I.

Aggiunta allo studio sulla curva delle entrate, Giornale degli Economisti, 1897.

O. AMMON: Die Gesellschaftsordnung und ihre natürlichen Grundlagen. Dritte Auflage. Jena, Fischer, 1900. (Erste Auflage, 1895).

R. DUDLEY BAXTER: National Income. The United Kingdom. London, Macmillan, 1868.

F. GALTON: Hereditary Genius. An inquiry into its laws and consequences. New York, Appleton, 1870.

M. O. LORENZ: Methods of Measuring the Concentration of Wealth. Publications of the American Statistical Association, 1905.

C. GINI: Il diverso accrescimento delle classi sociali e la concentrazione della richezza. Giornale degli Economisti, January 1909. (Paper presented to the Florence meeting of the Italian Association for the Advancement of Science, 1908.)

Indici di concentrazione e di dipendenza. Biblioteca dell' Economista. Series V., Vol. XX, 1910.

Variabilità e Mutabilità, R. Università di Cagliari, 1912 (To be reproduced in a volume under press: Memorie di Metodologia Statistica. R. Università di Roma, Istituto di Statistics).

Sulla misura della concentrazione e della variabilità dei caratteri, Atti del R. Istituto Veneto di Scienze, Letters ed Arti, 1913-14.

Contributions of Italy to Modern Statistical Methods, Journal of the R. Statistical Society, July, 1926.

Sul massimo degli indici di variabilità assoluta, etc., Metron, Vol. VIII, No. 3, 1930.

Intorno alle curve di concentrazione, Metron, Vol. IX, No. 3-4, 1932.

Di una estensione del concetto di scostamento medio, Atti del R. Istituto Veneto, 1917-18.

E. CZUBER: Beitrag zur Theorie der Statistischer Reihen; Versicherungswissenschaftliche Mitteilungen. Nelle Folge, 9 Band, 2 Heft., Wein 1914.

F. CORRIDORE: Relazioni tra affitto reale e valore locativo fiscale nel Belgio; Roma, Loescher, 1911.

E. PORRU: La concentrazione della richezza delle diverse regioni d'Italia. Studi economico—giuridici della R. Università di Cagliari, 1911-12.

F. SAVORGNAN: La distribuzione dei redditi nelle provincie e nelle grandi città dell' Austria, Pubblicazioni del Museo Commerciale di Trieste, Trieste, 1912.

Intorno all' approssimazione di alcuni indici della distribuzione dei redditi; Atti del R. Istituto Veneto, 1915.

G. PIETRA: Delle relazioni tra gli indici di variabilità; Atti del R. Issituto Veneto, 1914-15 (two notes).

F. VINCI: La concentrazione dei capitali delle nostre società ordinarie per azioni, Rivista delle Societa commerciali, Anno 8, Fasc. III, 1918.
L. AMOROSO: Reserche intorno alle curva dei redditi. Annali di Matematica pura ed applicata, Serie IV, Tomo II, February, 1925.
M. SAIBANTE: I profitti delle società per azioni e la concentrazione dei capitali industriali, Metron, Vol. VI, N. 1, 1926.
La concentrazione della popolazione; Metron, Vol. VII, N. 2, 1928.
E. J. GUMBEL: Das Konzentrazionsmass. Allgemeine Stat. Archiv, 1928.
R. ROY: La demande dans ses rapports avec la répartition des revenus. Metron, Vol. VIII, N. 3, 1930.
L. GALVANI: Contributi alla determinazione, etc.; Metron, Vol. IX, N. 1, 1931.
D. B. YNTEMA: Measures of the inequality in the personal distribution of Wealth or Income. Journal of the American Statistical Association, December, 1933.
V. CASTELLANO: Sulle relazioni tra curve di frequenza; Metron, Vol. X, N. 4, 1933.
S. ORLANDI: Su la distribuzione dei redditi mobiliari in Italia. R. Università di Roma, Istituto di Statistica, 1933.
H. WOLD: A study on the Mean Difference, Concentration Curves, etc., Metron, Vol. XII, N. 2, 1935.

August 1—THE EFFECTS OF SHORT SELLING ON STOCK PRICES,

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Part I of this paper deals with the flat contradiction between the claims of economic theorists that short selling stabilizes stock prices and the statements made by some of the practical short sellers at the Congressional Hearings on short selling. Economic theorists base their conclusions upon an assumption as to how short sellers act—on how it is reasonable and logical to expect them to act. The answer to the question of whether or not short selling stabilizes stock prices depends upon how short sellers really act. Instead of deciding the question on the basis of the reputation or ability of the two groups of men it seems better to analyze the data available.

Part II is a study of the relationship between the total short interest and the Standard Statistics stock price index as revealed by simple correlation, and a critical examination of a similar study by the late J. Edward Meeker. Several limitations and a serious contradiction in Meeker's work become apparent, but the most important conclusion of this part is that simple correlation is inadequate, for other important variables affecting both the short interest and stock prices must be included in the study before it is possible to learn the nature of the relationship between these two important variables. Even by using this simple technique it is not possible to prove that short selling has a stabilizing effect upon stock prices.

Part III is a study of the short interest, stock price index, and additional variables such as volume of sales, indexes of industrial activity, brokers' loans, etc., by using multiple correlation. Monthly data are analyzed, but it is not possible to establish any significant or reliable relationship between the short interest and price. It is possible to explain 90 per cent of the variance in the index of stock prices without considering the short interest, for the coefficient of multiple correlation between the index of stock prices and other variables is .95. The failure to establish a significant relationship between the short interest and the stock price index may be due to the fact that a month is too long a period between observations, and important shifts in the short interest are missed, or to the fact that the movements in stock prices and the short interest in individual stocks are in different directions at the same time or vary to a different degree. It is advisable to study the data for several stocks separately.

In Part IV the common stocks studied are United States Steel, American Telephone and Telegraph, Union Pacific Railroad, and American and Foreign Power. These four stocks represent different types of industries, products, and services rendered to the public. Both weekly and daily data are considered. The coefficients of simple correlation between the short interest and the price are:

	Weekly Data	Daily Data
United States Steel	.4491	.0956
American Telephone and Telegraph	—.5635	—.4048
Union Pacific Railroad	—.5326	—.7555
American and Foreign Power	.6329	.5125

For weekly data all the coefficients are significant; however, when other variables are included in the multiple regression equation and adjustments are made for longer-period shifts of the variables in time, the significant relationship between short interest and price vanishes in the case of the first three stocks but remains as a small positive value for the last stock. The non-significant values may very well be due to the small number of observations, for the data available for individual stocks are limited to 24 weeks.

For daily data the shifts over the entire period are well defined, and it is necessary to use a cubic in time to eliminate these shifts, which disturb the picture of the day-to-day relationships. When the cubic in time is included in the multiple correlation the net relationship between price and the short interest is negative and significant for all four stocks in spite of the fact that two of the coefficients of simple correlation are positive and one of them is significantly positive. If the use of a cubic in time for daily data is objectionable, it is possible to establish the same conclusions by using other variables instead, such as volume of sales of the particular stock, volume of sales of all stocks, and brokers' loans (both N.Y. member account and out-of-town) in the multiple correlation. The use of the cubic is merely a summary way of describing the shifts which have taken place in other important factors which disturb or obscure the short-time relationship being studied—that between price and the short interest.

In order to stabilize stock prices the short interest must be positively correlated with the price of a stock. When the price is too high the short interest must be large in order to force the price to drop, and as the price drops too low the short interest must gradually decrease or vanish in order to curb the decline. So far as the stock market as a whole is concerned, this statistical analysis by simple and multiple correlation does not reveal any significant relationship between the total short interest and an index of stock prices. It would appear that both the economic theorist and the practical short seller have over-stated the case for and against short selling. So far as the prices of individual stocks are concerned, it is possible to find some evidence that the weekly fluctuations of the short interest of one stock are positively related to the price fluctuations and thus establish a stabilizing influence, but for the other three stocks considered, no significant relationship seems to exist. When daily data are examined, negative correlation between the short interest and price is found to exist for all four stocks after allowing for the effects of other important variables. This means that short selling increases daily price fluctuations and throws serious doubt upon the stabilizing influence which is supposed to exist according to almost all economic theorists.

August 3—POPULATION ANALYSIS,

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The various species of living organisms that inhabit our globe are mutually interdependent, so that, strictly speaking, it would be impossible to make a well-rounded study of any one species without taking into account a number of other species that influence it in one way or another.

However, the internal factors (birth rate, death rate, rate of natural increase, etc., etc.) of a population of living beings are connected by a variety of inter-relations which permit, and which in fact call for separate study, without necessitating at each step explicit reference to the influence of the other species inhabiting the same territory.

The study of a population thus singled out for separate consideration constitutes a separate body of research, to the brief consideration of certain examples of which we shall here turn our attention, with special regard to the human population.

Now it is to be noted that the demographic and sociological study of the human species covers a number of relations which, for other living species, either do not exist at all, or play a subordinate role. Thus, for instance, certain statistical and sociological problems peculiar to the human species arise out of the essentially monogamous laws of civilized peoples, and out of the retarded adolescence of the human individual. Thus statistics of the marital state, of sterility and fecundity in marriage, of the family, of widowhood, orphanhood, etc., and the problems which arise from such statistics, exist practically only for our species, although fundamentally they are a particular aspect of the general phenomenon of biological reproduction. Limited to one species as are the applications of certain of these "family" problems, the central position which the human race occupies for us in the scheme of nature lends to these applications prime importance. On the present occasion, if only on account of limitation of time, our discussion will be restricted practically entirely to Population Analysis as applied to the human species.

Population studies can be carried on to some extent on a purely *empirical* basis, as when we investigate, by merely collecting and collating data, the relation (if any) between death rate and population density. Our concern here will not be with such empirical relations, that is relations the physical causes or logical reasons for which are unknown to us. We shall here be concerned, not with empirical, but with *necessary* relations between the characteristics of a population. It is this body of *necessary* relations to which the term *Population Analysis* is here to be applied.

It might seem at first sight as if the study of necessary relations must be a sterile occupation, inasmuch as it can never reveal to us any fact of which the truth is not already contained in the premises. To this we shall reply that a truth which for our mind remains "implied" without being recognized is without practical value; and further, that our intellect is more exacting: It asks not merely to be taught a body of facts, but to be shown their inter-relation. And it is precisely this inter-relation that constitutes the subject of study in *Population Analysis,* that is in the investigation of general demography by the deductive method.

As for those who see the justification of science solely in its practical applications, no apology to them is called for on behalf of the study of Population Analysis along deductive lines. For this study enables us, on occasion, to obtain indirectly information for which the direct observational data have not been, or for one reason or another can not be, obtained. And it is hardly necessary to recall that a whole industry of prime importance—life insurance—bases its computations and its practice on relations closely akin to, if not actually borrowed from Population Analysis.

In this lecture a number of illustrative examples of more recent developments in this field will be given, which from their nature are not adapted to reproduction in abridged form.

August 4—SOME ELEMENTARY MEANS AND THEIR PROPERTIES,

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The means in common use are with few exceptions generalizations of the *arithmetic mean* or *common average.* In forming such means an averaging process is actually performed, not on the given items, indeed, but on a derived set of items obtained by using some specified function of the items. In forming the harmonic mean, for example, we average not the given items but their reciprocals; and then we take the reciprocal of this result. However, it may be noted that two of the well-known means are not generalizations of the arithmetic mean; viz., the mode, the item or size of greatest frequency when there is such a size, and the median or middle item when the items are arranged in the order of their magnitude.

The arithmetic mean of $x_1, x_2, \cdots, x_n$ in its original or *symmetric* form is $A = (x_1 + x_2 + \cdots + x_n)/n$. What is substantially an arithmetic mean appears also in the *weighted* or *unsymmetric* form: $A' = (c_1x_1 + c_2x_2 + \cdots + c_nx_n)/c$, with $c = c_1 + c_2 + \cdots + c_n$. All the means related to the arithmetic mean have their weighted or unsymmetric analogues.

Since a mean is in some sense a *function* of the given items or measurements, we may use the language of function theory and distinguish between means as *unique* or multiple-valued; *continuous* or discontinuous, etc. Reserving for a later consideration the notion of *internality*, some rather important characteristics of means—beyond what are noted above —are covered by the following:

Definitions

A mean m of n variates or items $x_1, x_2, \cdots, x_n$ is:
Increasing, if m increases when one variate (taken at pleasure) increases, the others remaining constant;
Homogeneous, if the mean of $kx_1, kx_2, \cdots, kx_n$ is km; that is, k times the mean of $x_1, x_2, \cdots x_n$, where k is a constant;
Translative, if the mean of $x_1 + k, x_2 + k, \cdots, x_n + k$, is $m + k$;
Transitive, if the mean of means of sets of variates—each set containing the same number of variates — is the mean of all the variates considered individually;
Associative, if the mean m remains unchanged when in any subset of its elements or variates (taken at pleasure) each variate is replaced by the mean for that subset;
Potentive, if the mean of $x_1^t, x_2^t, \cdots, x_n^t$ is m^t.

As noted before, most means in common use are related to the arithmetic mean through some function, call it $\psi(x)$. This function is continuous and is also either an increasing function of x or a decreasing function of x. Such a function has a continuous inverse $\psi^{-1}(y)$ — the solution for x of the equation $y = \psi(x)$,—and the mean M of $x_1, x_2, \cdots, x_n$, with weights, $c_1, c_2, \cdots, c_n$, as formed from ψ and ψ^{-1} is

$$M = \psi^{-1}\{[c_1\,\psi(x_1) + c_2\,\psi(x_2) + \cdots + c_n\,\psi(x_n)]/c\}$$

where

$$c = c_1 + c_2 + \cdots + c_n$$

The corresponding simple or symmetric form is obtained by taking each $c = 1$:

$$M = \psi^{-1}\{[\psi(x_1) + \psi(x_2) + \cdots \psi(x_n)]/n\} \ .$$

The arithmetic mean, geometric mean, harmonic mean, exponential mean, root-mean-square, root-mean-cube, root-mean-fourth-power are then obtained in succession by setting $\psi(x)$ equal to x, $\log_b x$, $1/x$, b^x, x^2, x^3, x^4 $(0 < b \neq 1)$. The latter three means are in general use in statistics for measuring dispersion, skewness, and kurtosis, respectively. These are, indeed, special cases of the power mean

$$P = \{[x_1^p + x_2^p + \cdots + x_n^p]/n\}^{1/p} \ .$$

Here p may be any real number not zero, provided the x's are positive. Again, for all real x's not zero, this definition applies if p is an odd integer or its reciprocal. A more general expression for a *signed power* mean may be written with the aid of the function signum t, usually written *sgn* t, which takes on the values —1, 0, or +1, according as t is negative, zero, or positive. Then

$$P = |[\,|x_1|^p \operatorname{sgn} x_1 + |x_2|^p \operatorname{sgn} x_2 + \cdots + |x_n|^p \operatorname{sgn} x_n]/n\,|^{1/p} \operatorname{sgn} [\ \ ] \ ,$$

Corresponding to this, a weighted form may be written.

The means thus far mentioned may appear in various combinations. Thus, if R is the root-mean-square, then

$$R^2 = (x_1^2 + x_2^2 + \cdots + x_n^2)/n \ ;$$

and the *contraharmonic* mean may be written R^2/A, where A is the arithmetic mean. This contraharmonic mean is almost as old* as the "classical means," the arithmetic, geo-

*See *Handbook of Mathematical Statistics*, H. L. Rietz, Editor in Chief; Chapter I, by E. V. Huntington, p. 5.

metric, and harmonic means, known to the Greeks.

Let us consider now the following means: the arithmetic, geometric, harmonic, contraharmonic, power, exponential means, and also the median, mode, and average of least and greatest measurements of a set. Most of these are increasing means; but the contraharmonic mean is not, and the average of extremes is only *monotone increasing*; i.e., with the increase of a single variate, it does not decrease. Most of the means are homogeneous; but the exponential mean is not. The geometric, harmonic, contraharmonic, and power means—excepting the special case of the arithmetic mean—are not translative. Some means are not transitive or associative—e.g., the median and mode, and indeed, weighted means. For positive numbers, the geometric mean, the median, and the mode are potentive; but not even the arithmetic mean is potentive.

It is interesting to trace the evolution of the concept of the mean. Mathematicians at various times have characterized particular means of interest to them by specifying the properties of such means; that is, by laying down specific conditions or requirements for these means to satisfy. The fewer the conditions laid down, the more general has been the mean obtained.

With new forms of useful means emerging, it appears clear that very few conditions which we may feel inclined to impose will be satisfied by all kinds of means. In particular, one property to which mathematicians have clung rather tenaciously is that of homogeneity, closely associated with units of measure. Indeed it seems extremely natural to impose the condition that the mean of $km_1\ km_2, \cdots, km_n$ shall be k times the mean of $m_1, m_2, \cdots, m_n$. Yet the most important mean for the special problems of actuarial mathematics is an exponential mean, and this is not homogeneous. This exponential mean is the equated age for given different ages for joint annuities or insurance and is based upon the parameter c in the Makeham law of survival.

In the paper that follows I shall refer to the work of some of the many mathematicians whose joint efforts have enlarged our conception of means. In this progress, integration naturally generalizes summation. Indeed, it seems natural to go further; and to characterize means by the use of the language of point set theory.

August 5—THE CHIEF CHARACTERISTIC OF STATISTICAL MEANS,

EDWARD LEWIS DODD.

As preliminary to an inquiry as to what constitutes the chief property or characteristic of statistical means, a rapid sketch may well be made of the development of ideas about means.

In some of the early work on means, (Cauchy, *Analyse Algébrique,* 1821; Liouville, *J. de M.,* 1839), interest centered upon inequalities; and it was shown that the geometric mean does not exceed the arithmetic mean. Léon Marie (*Traité-Financiéres,* 1890) gives a proof that an exponential mean, with base less than unity (exact equated time for paying debts) is less than the corresponding arithmetic mean. O. Hölder (*Nachr. Göttinger,* 1889), and J. L. W. V. Jensen (*Acta M.,* 1906) dealt more generally with inequalities of means, using convex functions. This study of inequalities among means was continued through papers of J. F. Steffensen (*Skand. Aktuar.,* 1918, 1925; *J. Inst. Actuaries,* 1919), Birger Meidell (*Skand. Aktuar.,* 1921), Alf Guldberg (*Comptes R.,* 1922), and others; three recent papers are those of K. Toda (*J. Hiroshima,* 1934), B. I. Baidaff and J. Barral S. (*Bol. Mat.,* 1935), and Harald Bohr (*J. Lond. M. S.,* 1935). General means were formed from the elementary symmetric functions by André Durand (*Bul. Sci. M.,* 1902), Otto Dunkel (*An. of M.,* 1909), W. Sternberg (*Berich. Leipsiz,* 1919), and J. Mª Orts (*Rev. M. Hispano-Am.,* 1934). Dunham Jackson (*Bul. Amer. M. S.,* 1921) gave precision to the median.

Another set of papers dealt with means by setting up axioms or postulates. These were concerned almost exclusively with the arithmetic mean; but the individual postulates set up specify properties which serve to distinguish one kind of mean from another. Among this set are the papers of G. Schiaparelli (*Ist. Lombardo,* 1907), U. Broggi (*L' Enseign,* 1909), R. Schimmack (*Math. Ann.,* 1910), R. D. Beetle (*Math. Ann.,* 1915), O. Suto (*Tôhoku M. J.,* 1914), G. Bemporad (*Atti Lincei,* 1926, 1930), E. V. Huntington (*Trans. A.M.S.,* 1927), V. Furlan (*Metron,* 1928), S. Narumi (*Tôhoku M. J.,* 1929), L. Teodoriu (*Mathematica,* 1931), S. Matsumura (*Tôhoku M. J.,* 1933), I. Nakahara (*Tôhoku M. J.,* 1936). But Huntington deals not only with the arithmetic mean, but with the geometric, the harmonic, and the root-mean-square; Nakahara, with weighted means; and Furlan in his Type II. includes non-associative means, such as the median. Suto and others set up one postulate of extreme importance for studying the nature of means; viz., that the mean of n variates shall be a function $f(x_1, x_2, \cdots, x_n)$ such that $f(x, x, \cdots x) = x$.

Means especially adapted to finance were elaborated by C. E. Bonferroni (*Giorn. Mat. Financiara,* 1921, 1922, 1927; *Ist. Bari,* 1923-24, 1924-25; *Giorn. Ec. e Statistica,* 1926). Generalized means were used by E. L. Dodd (*Skand. Aktuar.,* 1922) and G. Darmois (*Statistique Mathématique,* 1928, p. 31).

A great advance in the study of the nature of means was made by O. Chisini (*Periodico di M.,* 1929) who contended that for statistical purposes a mean is primarily a *representative* number which may replace a set of numbers in a specific problem. With F denoting some functional relation under consideration, the mean of $m_1, m_2, \cdots, m_n$ is a number M which satisfies the equation:

$$F(m_1, m_2, \cdots, m_n) = F(M, M, \cdots, M) .$$

Chisini exhibited an *external* mean showing that *internality* or intermediacy is after all incidental in statistical problems. Of course, in *general analysis,* intermediacy has always been paramount; as, for example, in mean value theorems in calculus. Chisini's conception of the mean was taken up and reinforced by B. di Finetti (*Giorn. Ist. Attuari,* 1931) who also investigated the general properties of means, in particular the associative mean, and used integration freely to generalize summation. The papers of C. Gini and L. Galvani (*Metron,* 1929), M. Nagumo (*Jap. J. of M.,* 1930), A. Kolmogoroff (*Atti Lincei,* 1930), P. Martinotti (*Giorn. Ec. e Statistica,* 1931), Börge Jessen (*Acta Univ. Hungaricae,* 1931, *Mat. Tidsskr,* 1931), which appeared about this time, throw considerable light upon the nature of means. That of Gini and Galvani treats means for qualitative characters and means formed from frequencies placed upon cyclical series. Martinotti expressed the mean x of $x_1, x_2, \cdots, x_n$, in the implicit form:

$$\Sigma f(x) \cdot \varphi(y_i) \cdot x(z_i) \cdots = \Sigma f(x_i) \cdot \varphi(y_i) \cdot x(z_i) \cdots$$

where *auxiliary* functions $\varphi, x, \cdots$ are used.

The complete independence of certain properties of means is discussed in a paper by E. L. Dodd (*Ann. of M.,* 1934).

There may be some hesitancy in abandoning intermediacy as the primary characteristic of a mean (Latin, *medianus*) in statistics, on the ground that this diverges from general mathematical usage. However, for statistical purposes, the *representative* character of the number called the mean would seem to be of more importance than its *intermediate* character. It would not seem to be a bad guess, moreover, that in its *origin* an average or mean was fundamentally a *representative* number. Although external means do not often confront us, it is easy to construct such means with a function having a double-valued inverse.

The *implicit* definition of a mean given by Chisini is perhaps the most satisfying in its relation to actual statistics. But it is natural to try to replace implicit by *explicit* relations, or solutions. As almost equivalent to this implicit definition is a postulate used by Suto, Huntington, Narumi, and Teodoriu in their axiomatic approach to means; viz., that the mean $m = f(x_1, x_2, \cdots, x_n)$ is such a function that

$$f(x, x, \cdots, x) = x \ .$$

If we are interested in multiple-valued means—which do occur sometimes—we may require that at least one value of $f(x, x, \cdots, x)$ shall be x. Of course, this appears a very scant limitation to place upon a general function f. But it should be noted that none of the general properties—such as homogeneity, symmetry—found in certain means are common to all useful means.

If we wish the definition of a mean to appear in the language of point sets, the mean of Martinelli suggests the use of some auxiliary function. And, indeed, in the common generalization for calculus, a frequency function acts in this capacity.

DEFINITION. Let E and H be sets of numbers. Such a number t may be a real number or a vector number,

$$t = (t_1, t_2, \cdots, t_k) \ .$$

Let E_t be the result of replacing each number of E by a single number t. Then the mean m of numbers in E, relative to the set H, and to a function f, is given by $m = f(E, H)$; provided that the function f has been so constructed that for each t in E, $f(E_t, H) = t$, or at least one value of this f is t. It is to be understood here that when E is changed to E_t, the set H remains unaltered.

This definition does not specify integration. But an important special case is obtained by taking for E the set of real numbers x in some interval, (a,b), and for H the set of values of $\varphi(x)$, where φ denotes relative frequency. Then with a continuous increasing function ψ we may construct the well-known mean

$$m = \psi^{-1}\left(\int_a^b \varphi(x)\ \psi(x)\ dx\right) \ ,$$

where ψ^{-1} is the inverse of ψ. By the replacement E_t, $\psi(x)$ becomes $\psi(t)$; but H, which is $\varphi(x)$, remains unaltered, giving $m = t$.

August 5—A GENERAL INVARIANT CRITERION OF FIT FOR LINES, PLANES, AND FUNCTIONS EXPANDABLE IN SERIES, WHERE ALL VARIATES ARE SUBJECT TO ERROR,

CHARLES F. ROOS.

When it is desired to fit a straight line $y = ax + b$ to $m > 2$ observed points (x_i, y_i) the method usually employed is that of choosing a and b so that they minimize the sum of the squares of the residuals of the y_i, that is, $\Sigma(ax_i + b - y_i)^2$ is a minimum. Two regression lines can always be obtained, one by minimizing the residuals in the x_i, the other by minimizing the residuals in the y_i. The first assumes the observations in x to be perfect, and the second the observations in y. But, in the case of time series of economic data, both (or, more generally, all) variates are subject to error, and neither is the best line. Graphical curve-fitters have perhaps come closer to the best line, since, as the author has observed, they tend to minimize normal distance.*

While this gives a priori a more satisfactory line, there are obviously cases in which some intermediate lines are better, and, moreover, the method of minimizing normal deviates is not independent of the coordinate system chosen. In this lecture, the general problem was attacked and it was shown that, for the set of observations (x_i, y_i), the associated weights p, and the line, $ax + by + c = 0$, the most general function $\Sigma\, p_i\, f_i(x_i, y_i, a, b, c)$ which remains invariant under translation, homogeneous strain, and rotation is $\Sigma\, p_i(ax_i + by_i + c)^n$ where n is any number. For the quantity $ax_i + by_i + c$ to represent a distance in the direction $\tan\alpha = \delta y_i/\delta x$, where δy_i and δx_i are the errors in y_i and x_i respectively, we must also have $\cos\alpha\delta a + \sin\alpha\delta b = 0$. When $n = 2$ (the least squares' assumption), the conditions of solution are

*The literature on analytical methods for minimizing normal deviates is large. The earliest paper seems to be by R. J. Adcock who wrote in the *Analyst*, Vol. 4, 1877. Later fundamental papers on the same subject have been presented by Karl Pearson, "On lines and planes of closest fit," *Phil. Mag.* 6 Ser. Vol. 2, 1901, and L. J. Reed, "Fitting Straight Lines," *Metron*, Vol. 1, 1921.

$$\Sigma p_i x_i (ax_i + by_i + c) \sin \alpha = \Sigma p_i y_i (ax_i + by_i + c) \cos \alpha ,$$

and

$$\Sigma p_i (ax_i + by_i + c) = 0 ,$$

The method generalizes readily to planes, to hyperplanes, and to functions expandable in series involving the parameters linearly.

Suppose, as an application, that we wish to find a regression between wholesale prices of cotton goods and farm prices of cotton. We know that, over such short intervals of time as given by Table I, Y, the price of cotton goods, is equal

TABLE I

COST AND PRICE IN THE COTTON GOODS INDUSTRY

Date	Wholesale Prices of Cotton Goods Semi-Annual Av.	Farm Prices of Cotton Goods Semi-Annual Av.	Av. Hourly Earnings of Labor Semi-Annual Av.
1928—Jan.	153.7	100.1	42.5
July	151.0	100.5	43.2
1929—Jan.	149.7	99.3	42.8
July	140.0	97.7	42.2
1930—Jan.	119.3	90.7	41.9
July	84.2	78.8	41.2
1931—Jan.	74.2	71.0	39.1
July	51.3	61.0	38.0
1932—Jan.	44.5	64.7	36.5
July	48.3	53.6	33.3
1933—Jan.	53.3	54.2	32.9
July	64.7	87.4	43.1
1934—Jan.	81.2	87.3	45.9
July	96.5	86.0	47.0
1935—Jan.	94.8	83.1	46.8
July	86.7	84.1	46.1

to a weighted sum of X, the price of cotton at the farm, Z, the average hourly earnings of labor used in cotton manufacturing, and E which is composed of fixed charges and profits or erratic elements; thus

$$Y = a_0 X + a_1 Z + a_2 E \ .$$

For the data of Table I, the simple correlation between Y and Z is +.5195 and that between X and Z is +.7764. Both correlations are positive so that we may take

$$-\tan a = -k = \frac{\delta (Y/\sigma_y)}{\delta (X/\sigma_x)} = \frac{(.5195)^2}{(.7764)^2} = +.44$$

We, therefore, transform X and Y so that

$$x = \frac{X - (\text{mean of } X)}{\sigma_x} \ ,$$

$$y = \frac{Y - (\text{mean of } Y)}{\sigma_y} \ ,$$

and substitute directly in the equations of condition. If all observations are given the same weight, then $p_i = 1$ and consequently the line of best fit is

$$-(\Sigma y^2 + .44 \, \Sigma x_i y_i) \, x + (x_i y_i + .44 \, \Sigma x_i^2) \, y = 0 \ ,$$

or, more specifically,

$$-1.3953x + 1.3384y = 0 \ .$$

If one of the correlation coefficients, r_{yz} or r_{xz}, had been negative, the number k would have been positive. Similar considerations enable us to determine completely the signs of the direction cosines in the more general case of fitting a hyperplane.

(This paper will be published in full in an early issue of *Metron.*)

August 6—THE DEFINITION AND CONSISTENCY OF INDEX NUMBERS,

T. H. RAWLES, Dean of Freshmen, Colorado College.

Professor Fisher in *The Making of Index Numbers* defined index numbers as an average of price relatives and introduced a number of tests to indicate the properties which these numbers should possess. Divisia gave a definition of index numbers by means of differential expressions. Two methods are exhibited for the integration of these expressions and it is shown that one of them leads to Fisher's Ideal number.

The numbers obtained from Divisia's definition do not satisfy the circular test, a defect which is illustrated by the following example:

Year		0	1	2	3	4
Commodity "A"	p	10	15	10	5	10
	q	15	10	5	10	15
Commodity "B"	p	10	5	10	15	10
	q	5	10	15	10	5

If the price index is calculated for this system by means of the "link" method it is found that the prices have risen sixty per cent in the four intervals in spite of the obvious fact that both prices and quantities are identical in the 0th and the 4th intervals.

Starting again with Divisia's definition and introducing the additional assumption that the prices are functions of the quantities and, reciprocally, that the quantities are functions of the prices, it is possible to obtain index numbers which satisfy the circular test. This procedure leads to price indexes which are homogeneous functions of degree 1 in the prices of the commodities and quantity indexes which are homogeneous functions of degree 1 in their quantities. Economic systems which conform to these conditions might be designated conservative economic systems.

The amount by which the circular test fails was called by Fisher "the circular gap." If the logarithm of the circular gap for the price index is positive it indicates a market situation in which sellers are able to raise their prices in the face of a great demand. If it is negative it indicates a situation where the sellers increase their production when prices are high.

The quantity function $Q(q)$ may be regarded as an approximation to the utility function, the latter not being subject to the restriction that it is a homogeneous function of degree 1. Any particular indifference surface, $U(q) = k$, may be used to generate a quantity function, Q. If the equation $U(\frac{q}{Q}) = k$ is solved for Q the desired function is obtained. When Q is employed as a substitute for U an approximation is being made which resembles the approximation made in using Euclidean geometry as a local geometry even when the domain of its validity is limited.

The so-called "method of limits" does not appear to be very fruitful. Leontieff, who seems to have felt its limitations, introduced the device of referring all assortments of goods to a standard one in which the proportions of the quantities of the commodities are constant. Here again, it turns out that the indexes are independent of the choice of the standard assortment only if the utility function is homogeneous of degree 1.

Certain index numbers such as those used for calculating stock market averages where the quantities may be regarded as constant escape entirely from the criticism of this paper. They may be derived directly from Divisia's definition.

August 7—EVIDENCE OF STRUCTURE IN COMMON STOCK PRICES,

ALFRED COWLES 3RD, President of the Cowles Commission for Research in Economics.

If the stock market has advanced, for example, for one month, does this fact have any bearing on the probabilities of an advance for the ensuing month? In an attempt to answer questions of this type, sequences and reversals were counted, a sequence being when an advance follows an advance, or a decline a decline, and a reversal being when a rise follows a decline, or a decline a rise. Units of time considered were 20 minutes, 1 hour, 1 day, 1, 2, and 3 weeks, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, and 11 months, and 1 and 2 years. For any unit of time from 20 minutes to 6 months the excess of sequences over reversals was such that the probabilities were conclusive that the observed ratio could not have occurred in a random series such as one derived by penny tossing. For example, in the period from 1872 to 1935, using one month as the unit of time, there were 481 sequences and 278 reversals. The probability of such a ratio occurring in a penny tossing series is $< .000001$. Similarly, for the same period, using 6 months as the unit of time, 83 sequences and 42 reversals occurred, and the corresponding probability associated with this ratio was found to be .00036. Where larger units of time than 6 months were employed, non-randomness of the series could not be established on the basis of probabilities, although in every case where the unit of time was less than 2 years the sequences were found to outnumber the reversals.

Where indexes of stock prices for different industries such as motors, steel, oil, etc., were considered it was found that those which were stronger than the median in one month tended to be stronger in the succeeding month. For 61 industries during the 16 months from January, 1934 to April, 1935, when the general market movement was horizontal, of 915 observations, 570 were sequences and 345 reversals. That is, if the oil stocks were among the strongest 30 groups in January the probability would appear to be .623 that they would also be found among the stronger one-half in February. Considering individual stocks, for example, Standard Oil of New Jersey, instead of the oil stocks as a group, we found a distinct probability that a stock which was stronger than the median in one month would also be stronger in the succeeding month. Employing one year as the unit of time, instead of one month, the probability of a sequence was still greater.

Considering evidence as to periodicity in the stock market as a whole, a weekly cycle was shown to have existed during the last 15 years. Eliminating trend by the method of link relatives, the average close for Friday and Saturday was found to be higher than that for Monday in each of eight years analyzed. In three of these years the market was rising, in three declining, and in two horizontal. The average amplitude of the weekly cycle for all eight years was one-third of one per cent. The existence of a daily cycle was also indicated. Using a 20 minute series for eight periods of 25 days each, Saturdays having been omitted, it was found that for each of these eight periods the average of the closing quotations represented the low point in the daily cycle. Two of these periods represented rising markets, two declining, and four horizontal. The average daily cycle was observed to be one-quarter of one per cent.

August 7—SIGNIFICANCE OF ANALYSIS OF VARIANCE OF TIME SERIES,

HAROLD T. DAVIS, Professor of Mathematics, Indiana University, and Research Associate of the Cowles Commission for Research in Economics.

The statistical problem presented by time series in general, and economic time series in particular, is essentially that of untangling the effects of the element of secular trend which we shall designate by T, the quasi-harmonic elements which we shall designate by H and the erratic element designated by E.

The variance of a time series may thus be regarded as consisting of the following parts:

$$\sigma^2 = \sigma_T^2 + \sigma_H^2 + \sigma_E^2 \cdot$$

This paper is concerned essentially with the variance of the element of secular trend.

One school of economic thought has regarded economic time series as statistically equivalent to accumulated random series and hence essentially unpredictable. In order to analyze this premise a set of 1200 tosses of a coin was accumulated and compared with the 1200 monthly items in the series of rail stock prices from 1831-1930. The penny tossing series was accumulated by assuming a unit rise for each head that appeared and a unit fall for each tail. The essential difference between the two series was found in the fact that the standard deviation (with trend removed) for the penny tossing series was given by the formula

$$\sigma = .707\sqrt{n}$$

where n is the number of intervals from the origin, that is to say, it increased with the length of the series, whereas the standard deviation of the rail stock series quickly reached a maximum value and was essentially constant with respect to the length of the series.

A second problem studied was concerned with an investigation of the trend of 100 years of rail stock prices by months. A trend of 20 years, 1830-1850, was first fitted to the data and then extrapolated four years as a forecast. A period of four years was then deleted from the beginning of the series and four years of actual data from 1850-1854 were added to determine a new trend. This was continued through the whole 100 years of the series, twenty-one forecasts of secular trend thus being secured. It was found that the extrapolated trend showed a wide divergence from one 4-year period to another. The question thus proposed was to account for this wide divergence in these forecast lines. In order to do this the suggestive ideas set forth in a paper by Henry Schultz* were employed.

Using a straight line trend $y = a + bx$ the standard error of forecast x ranging from $-p$ to $+p$, is given by the formula

$$\sigma_f = \Sigma(A + A' x^2)^{\frac{1}{2}}$$

$$\Sigma = \sqrt{\frac{n}{n-2}}\,\sigma$$

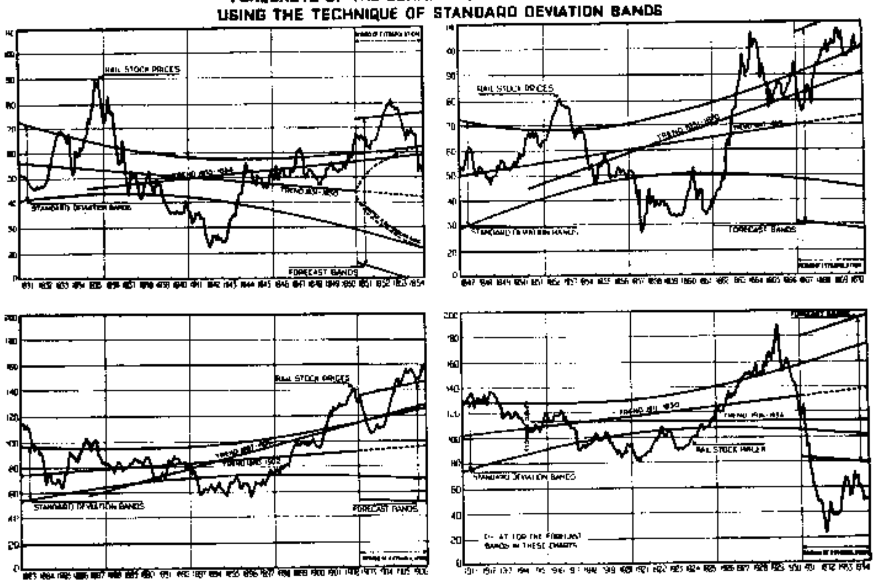
where $n = 2p + 1$ is the number of items in the series, A and A' are constants depending upon the forecast freedom of the series, and σ the standard deviation of the original series. If

*"The Standard Error of a Forecast from a Curve," *Journal of the American Statistical Association*, Vol. 25, 1930, pages 139-185.

a base period of N units is to be used to forecast the trend of a range of m units, the number of degrees of freedom to be assumed cannot be equal to N but must be assumed equal to N/m. We shall call this ratio, $k = N/m$, the *forecast freedom* of the series. Thus for a base series of 20 years and a forecast of 4 years, $k = 5$. Hence A and A' must be computed on the assumption that $p = 2$.

The hypothesis was subjected to statistical tests over the entire 100-year rail stock prices using a 20-year base and a 4-year forecast. Since σ_f yields a measure of the standard error of the trend, one expects that 68% of the extrapolated trends will lie within bands of width $2\sigma_f$ about the base trends. The exact count was 13 inside and 8 outside, a result which fully justified the assumptions since expected values were 14 and 7. In the accompanying chart four such typical bands are shown.

FORECASTS OF THE BEHAVIOR OF RAIL STOCK PRICES
USING THE TECHNIQUE OF STANDARD DEVIATION BANDS



If now on each side of the trend-forecast-bands, one constructs bands equal in width to the standard error, σ, of the original series, then it is to be expected that approximately 46% of the actual items of the time series in the 100 years of forecast, will lie within these outer bands. This expectation was justified by the experiment.

It is evident that this method gives a precise range for the forecasting of time series under the assumption that the total variance is compounded from the variance due to linear trend and the variance due to an erratic element.

August 8—SIGNIFICANCE TESTS FOR PERIODOGRAM ANALYSIS WITH APPLICATION TO PRICES OF COMMON STOCKS,

HAROLD T. DAVIS.

In the preceding paper the forecast freedom of a time series was discussed on the assumption that the variance was due entirely to a linear trend and an erratic element. The present discussion centers around the problem of how the recognition of one or more harmonic elements in a time series may be used to improve the forecast freedom of the series.

It should be recognized first that the probabilities involved in determining the structure of a time series must be regarded for the present as belonging to the category of inverse rather than direct probabilities, since the structure is discovered by observation and does not rest upon any secure a priori basis. Thus having taken numbers out of a hat, that is to say, having examined the historical series, one proceeds to guess the distribution of a second sample before it is drawn. If we consider a stock price index from 1897-1913 we see that it has a standard deviation of 15. Reduced by 3 harmonics, that is to say, on the assumption of 6 degrees of freedom, the series has a standard deviation of 8. Further reduced by the introduction of more degrees of freedom the standard deviation can be made as small as 3 or perhaps lower. Unfortunately, we have no way of estimating the number of degrees of freedom that should be used and hence cannot safely say precisely what the standard error of the erratic element should be.

As an example, let us consider the probability that there exists a 40-month cycle in industrial stock prices. This harmonic element is clearly revealed with a large significance for the period from 1897-1913. It again appears as the important harmonic in stock averages from 1914-1924 but is effaced in the following bull market. Relying on the probabilities involved here one would expect to find the 40-month cycle again appearing prior to 1897. This expectation is justified since the harmonic again appears in these averages from 1872 to 1897. Employing rail stock prices the 40-month harmonic appears in the data prior to the Civil War but it is again effaced in the turbulence of the Civil War period. The technique used in determining the significance of the period as revealed by the Schuster periodogram is due to R. A. Fisher.* The study concludes with the application of the Schultz technique as applied to the forecast band for harmonic analysis. The base period was from 1897 to 1914 and the period of forecast from 1914 to 1922. This period using the concept of the forecast freedom amply justified the method.

*See Schuster's original papers and R. A. Fisher, "Test for Significance in Harmonic Analysis," *Proc. Royal Soc. of London*, Vol. 125 (A), 1929, pp. 54-59.

ABSTRACTS OF PUBLIC LECTURES

July 10—THE DEPRESSION, ITS CAUSES AND CURES.

IRVING FISHER.

To me the most remarkable fact about the depression is that today, nearly seven years after it began, so few people understand it or even try to understand it. Even those very few who have done their best to understand it do not wholly agree, though they differ less and less as the evidence has grown. It is certainly high time that we should all try to read this evidence, especially as a fiery political campaign is now in sight when the evidence is likely to be distorted by both sides.

What I shall say is not designed to help or hurt either side in particular but merely to present the truth, as I see it, without fear or favor.

The depression, like all events in history, has many contributory causes. One of them, the capital gain tax, has been mentioned in my lectures on the income tax. Many others might be mentioned if there were time.

But one cause towers above all others, the collapse of our deposit currency. The depression was a money famine — a famine, not of pocket-book money but of check-book money, the money, or so-called money, recorded on the stubs of our check-books, our deposits subject to check.

In 1929, our check-book money amounted to 23 billion dollars. In 1933 before the "bank holiday" it was only 15 billions. That is, our chief circulating medium had shrunk by 8 billion dollars.

Moreover in 1929 all money circulated faster than in 1933 when people were hoarding. The money of 1929 did about 800 billion dollars' worth of business while the lesser money of 1933 circulating more slowly did only about 400 billion dollars worth. It is impossible to do business today without money. We did try to revive barter and we did try to improvise emergency money. One town in the State of Washington manufactured wooden money.

In almost all depressions and certainly in the big ones this factor of a money shortage has played the leading role. In many cases the clearing houses issued special "clearing house certificates" to eke out the scanty supply of circulating medium.

As long as a money shortage lasts business is hamstrung. With a shortage of 8 billions and with 15 billions of check book money left largely idle a business of 800 billions is simply impossible.

Some people put the cart before the horse and would have us believe that our check-book money shrank because business shrank—there wasn't enough business to use the money. These same people, almost in the same breath, tell us there was too much business to be done—"over-production"—which was not true either.

This over-production idea was that there were too many goods. The truth was simply there was too little money to buy them with.

Therefore, the destruction of goods, plowing under cotton, limiting production, was not the right remedy.

The right remedy was to increase the money supply and that is what has been getting us out of the depression. Of the shortage of eight billions about six billions have been restored.

This restoration was brought about not in the usual way by business men borrowing of banks but by the Government selling its bonds to the banks. Every billion dollars of bonds sold to banks created a billion dollars of new deposits subject to check. It is this new "money" or credit extended by the banks in return for Government bonds which has recreated six billion dollars out of the eight billions destroyed. This new money is now in general circulation on the stubs of your check-book and mine. It has started the wheels of industry going again.

This is not meant as justifying the big Government debt and expenditures. The restoration of our money supply could have been better done in another way.

We still need more money to restore business volume and the price level. But beyond a certain point not very far off any further additions to the circulating medium will be injurious. We want to overcome deflation but we do not want to cause inflation beyond normal.

And when we are "back to normalcy" we want thereafter to prevent either inflation or deflation.

This may be done by the new Federal Reserve Board under the new banking act of 1935. But that law may not be enough to prevent inflation. That law permits the Federal Reserve Board to double the reserve ratios. But that may not be enough.

It is the smallness of our reserve requirements which makes possible the contraction and expansion of the deposit currency. The average commercial bank is required now to keep a 10 per cent reserve. But all this reserve is kept as a deposit in a Federal Reserve bank which in turn is required to keep a 35 per cent reserve behind it. Thus every $100 of deposit subject to check which you and I have on the stub of our check book only needs to have $3.50 of actual money behind it in the Federal Reserve Banks. Such a small reserve requirement is simply ridiculous. No banking system can be run long on such a shoestring. You might as well try to carry a load of hay 100 feet wide on a truck 3½ feet wide. It is bound to upset.

We shall never have a really satisfactory and safe reserve until every reserve behind checking accounts is raised from 3½ per cent not simply to 7 per cent but to 100 per cent. The banks could then make service charges to carry the checking accounts. This would be going back to the original banking system. The Bank of Amsterdam, for instance, had a 100 per cent reserve.

Incidentally, the Government debt could largely be paid overnight by the Government re-buying bonds from the banks with new money to be used as reserve by the banks.

One New York Banker said to me that if the plan of 100 per cent reserve—the "100 per cent money" plan—can be enacted into law it will do more to stabilize banking than anything ever done since banking began.

It certainly would! But it would do even more. It would, by preventing such variations as from 23 billions of deposit currency down to 15 and up again, also prevent the depressions and booms which have cursed civilization ever since demand deposit banking began.

Another important source of instability is a fixed price of gold. The price of gold should vary as it does in England.

July 16—WHENCE THE FABULOUS WEALTH AND INCOME OF THE UNITED STATES,

CARL SNYDER.

It is a commonplace now to say that, in a space of a little over a century, the United States has become incomparably the richest nation that ever existed. Its wealth and income must be, normally, at least four or five times that of the next most highly developed countries, like Great Britain and Germany, and certainly equal to that of at least six other of the richest countries combined.

Whence came this astonishing primacy? It is often said: from our "great natural resources." But as to this, it may be observed:

(1) The especial "great natural resources" that have conspired to produce these riches are, first, our wonderful stores of coal and oil and gas; our iron and copper, and our huge pampas for the raising of corn, cotton, wheat, and meat.

Now, the interesting thing is that the United States had been growing at a remarkable and maximal rate for nearly two hundred years before these stores of "natural" wealth were extensively tapped. Both the growth of population and our industrial growth, so far as it can be measured, was rather greater *before* this exploitation of resources had gained headway, than after.

(2) The slender bands of settlers who crossed the Atlantic in these first two centuries were few in number and less in wealth. They did not find this a "rich" country. Quite the contrary. But they grew at an astonishing rate. Malthus, at the end of the Eighteenth Century, built his famous thesis as

to inevitable over-population on the estimates of this Colonial growth. But after a century and a half of this increase the country was so poor that, as Woodward in his remarkable *Life* points out, Washington was never able, even from a white population of three millions, to maintain more than about twenty-five thousand troops in the field at any time. At the final victory of Yorktown, the number of French troops present, who came in ships, exceeded that of all the "Americans."

Through all this time iron (chiefly from bogs) was smelted with wood charcoal. The general use of coal came long after. Our production of iron did not equal that of small Great Britain until near the close of the Nineteenth Century.

(3) After the incoming of coal and iron, and railroads, and the use of steam power, the rate of our national growth, alike of population and of industry, so far as the most careful measurements will disclose, seems rather to have declined than augmented; and this seems to have been especially true in our own generation. And all this was in the face of the immense trek of immigration to this country which began in the Fifties, the greatest the world has ever known. This is one of the strangest paradoxes in industrial history.

(4) We have no very serviceable estimates of national wealth prior to 1850 (and even the subsequent estimates are of no great value). But if we take the earliest estimates, of about 7 billions for 1850, and take the average indicated increase since (of around 4 1/2 per cent per annum compounded), the wealth of the United States when it began as a nation, in 1789, could scarcely have been much over half a billion dollars. And this was the accumulation of a century and a half. If it was more than that, then the subsequent rate of increase would have been less than 4 1/2 per cent.

Our vast accumulation of riches, therefore, has been a rather slow growth, from a long distant time, and apparently has been at an almost unchanging rate per capita, back, at least, to the days of the Revolution, and quite possibly long before that. How can we say this?

If the estimates of national "wealth" are of uncertain value, this is not true of the estimates of growth of industry and trade, at least as far back as a century ago. Some estimates which were made in my department revealed a most unexpected and almost unbelievable result.

It is well known that since our Civil War the rate of our population increase has been slowly declining, from around 3 per cent per annum to perhaps less than 1 per cent in the present day. Parallel to this declining rate has been a similar decline in the growth of industry and trade, so similar, indeed, that if the estimates of total production and trade be reduced to a *per capita* basis *the indicated rate of increase has not substantially changed throughout the last century and more;* not even, apparently, in any particular decennium of this century.

You will agree that this is a surprising finding, and I am quite sure that many economists, without ever having done a day's work on the subject, will be able to discover, from their inner consciousness, that it is palpably absurd. It will be especially trying to the gifted enthusiasts of the late "new era," and those who are able to discover wide variations or "cycles" in our rate of industrial increase. But it represents, I believe, the results of the most careful measurements which the available data will support.

Some enthusiasts, as I have already found, will find the rate too low; and others will be equally assured that the rate of increase shown is too high. They feel that such an even rate of human progress cannot possibly have continued so long. But these latter, I think, lose sight of the slowly changing character of our population with respect to age. A century ago, 60 per cent of our population was under twenty years of age. Today this proportion has fallen to around 40 per cent, owing largely to the parallel decline of the infantile death rate and of the birth rate as well.

This inevitably means, of course, of itself, a steady increase in the consumptive demand per capita; and correspondingly we must suppose a like increase in the per capita *product*, since a population 60 per cent adult ought to turn out more goods (and services) than if it were only 40 per cent adult.

Since, then, the imaginative or spontaneous conclusion is that the rate of growth I have given is *either* too high or too low, I conclude that, as is the usual thing, the judgment is purely emotional, and has little to do with the facts. Especially true of what we may call the emotional "sciences," of which economics seems a shining example.

I infer then we may reasonably conclude that the rate of technological improvement, the effect of discoveries, inventions, and improved methods, has apparently undergone little change in more than a hundred years and, so far as we may guess, back to the days of George Washington at least. Yet this estimated rate of increase of industrial growth per capita, of about 2 1/2 per cent per annum, perhaps a little more, is what has made it possible, for example, for this single nation now to own and run twenty to twenty-five millions of automobiles, at an annual cost exceeding *the total national income* of the people of France, or of any other country save Great Britain or Germany.

The greater part of this industrial increase has, of course, been due to the fabulous growth of mining and manufacturing. We have an estimate for the total *value* of manufactures of all kinds, factory, hand, and neighborhood industries, for 1810. This was around 145 millions of dollars. For Washington's first Administration, Tench Coxe estimates these values at from 20 to 40 millions. The total value of manufactures now, in normal times, and at somewhere near the same price levels, is around 50 or 60 billions. What made such an astonishing increase possible?

Clearly, for one thing, a somewhat parallel increase in plant investment. Actually, it would appear, a somewhat greater rate of increase. This is to say that more and more our industrial output is produced by machinery and less and less by human hands. This is mainly why the output per worker steadily rises. This requires a correspondingly increased investment in machinery and plant.

So we find from the earliest available estimates of capital invested in manufacturing (in 1825) a rate of increase, at least up to the World War, in actual dollars, of around *5 to 6 per cent* per annum, not varying very greatly from one census date to another. Whence came this capital?

Pretty clearly from industry itself, and largely from the manufacturing (and mining) industries; and certainly not from the imaginary savings of "the people." Certainly it did not come from farmers or from laborers, since, as we know, the savings of farmers and laborers and, indeed, of the vast bulk of the population, are very small, and for the most part negligible. This is disclosed by the fact that, generation after generation, from 90 to 95 per cent of all persons die without enough property to be worth recording in a will.

This is not saying that all accumulated property is handed down by means of a will, but it seems probable that the vast bulk of it is. And, in turn, by far the larger part of all the real or "reproductive" wealth of the country is nowadays represented by stocks and bonds, or ownerships of plant and machinery.

It is evident that if the rate of increase of the total industrial product, in all forms, in our generation, has been close to 4 per cent per annum, and the required increase of capital ("real" capital) has apparently been nearer 5 to 6 per cent, then the increase of workers' income could scarcely be equal to the increase in the total product, else there would be little or no saving. And this is precisely what we find.

A number of careful investigators, Faulkner, Mitchell, Douglas, Hansen, and many others, have in the last fifty years attempted to make estimates of "real wages," that is, actual wages relative to the average cost of living. These are, it is true, rather crude estimates, but inasmuch as there is substantial agreement between most of those who have investigated the subject, we must take their results as of "barometric" value. The various estimates of actual average wages, the "cost of living," and the relative or "real" wage, I have put together to trace the record for a century or more. These seem to indicate an irregular but very definite increase, for the entire period, for the average worker, of about *1 or 1 1/2 per cent per annum.* The periods of marked change are those of violent fluctuations of the price level—as in the usual inflations and subsequent deflations of war time. A long period of slow increase after the 90's to the World War.

But it is interesting that if we take the rate of increase shown for the first sixty or eighty years, to about 1900, and project it to the present day, this projected line comes very close to the actual estimates for the period '23-'29. Another blow, I fear, at the New Era myth, and for the uninformed enthusiasts who were able to discover a fabulous rate of increase in the period in, or after, the World War.

Here then seems the solution of our problem. If the entire product, the whole of the industrial output, per capita, were annually consumed, there would be, obviously, no savings whatever, and therefore no industrial progress. If the entire usufruct went to the owners of invested capital, then the workers, that is, the vast body of the population, would have no profit from the marvelous technical advance of the last hundred years or more. What actually appears to have happened then is a partition of the gain between the workers and the owners of capital.

That is, if the increase in total product, per capita, has been somewhere around 2 1/2 per cent and the net average gains, in real wages, to the total of workers around 1 1/4 per cent, then the balance must have gone to the owners of capital.

It is this rate of partition, so far as we may estimate it, which has made possible, and alone made possible, this amazing industrial growth. For clearly if "capital" had taken much more of the average annual gain, then the increase of capital, and therefore of the product, would have been much greater than it has been, and if, on the other hand, the worker had absorbed more of the increase, then the capital gain would have been inadequate to support such a rate of increase in the total product, and the producers and the consumers alike would have profited less.

It would seem as if there is a kind of "natural" balance in this distribution of the annual normal increment; and it seems probable that this partition of the "dividend" cannot seriously be disturbed without consequent results. Obviously if there were any unusual gains to capital, which would be the "excess savings" about which many economists talk much but prove nothing, there would result an undue plant expansion or "plant capacity," which would imply a corresponding diminution of the returns to capital.

Such excessive plant expansion would mean a high rate of construction activity, with a consequent high rate of employment, which always tends towards a rise of wages and probably, at times, excessively high wages, which prove a difficult factor in the subsequent reaction.

In other words, it seems as if we had here an admirable and beneficent mechanism, working for the highest attainable general good; in brief that our "capitalistic system" is far more highly adapted to existing economic conditions (i.e., human wants and disposition), and is altogether far more "natural" and far less "artificial" than the enormous number of moralists, journalists, amateur economists, and reformers who write so prodigiously upon these topics could imagine.

July 23—USE OF LAWS OF CHANCE IN INDUSTRIAL DEVELOPMENT,

W. A. SHEWHART.

There are many aspects of industrial development. The one here chosen for discussion is the economic control of quality under conditions of mass production.

An essential characteristic of Man, differentiating him from other animals, is his ability to predict and, within limits, to control the workings of nature. Among the earliest evidences of such control are the tools of our ancestors, tools which helped them to attain preconceived ends, tools which helped them to kill wild animals for food, to build their shelter from the elements, to obtain that with which to clothe themselves—in general, to satisfy their wants. Man is a creature of wants and the history of industry is largely a record of his struggle to produce physical things of such quality as to satisfy these wants. The object of this paper is to consider briefly some of the important steps in this conquest to control physical phenomena in the production of physical things to satisfy human wants, and to indicate the rôle played by laws of chance in this development.

Man started his sojourn on earth about one million years ago. For something like 700,000 years he apparently did not even dream of control. It was nearly 990,000 years before he began to fit piece-parts together. By about 1787, or only 149 years ago, the beginnings of mass production necessitated the making of interchangeable parts. Acting under the influence of the then prevalent concept of exact science, engineers attempted to make such parts "exactly" alike. They failed to attain this objective and introduced the "go" tolerance. Even yet they did not begin to think of chance causes systems as such but introduced the "go" tolerance in an effort to get around the effect of chance causes. Again they met with difficulties which led to the adoption about 1870 of the "go no-go" tolerance. Still they did not concern themselves with the study of chance causes as such.

At about this time, however, scientists began to realize the necessity of thinking in terms of the effects of chance causes, and industry had developed to a stage where the need for controlling chance variations was beginning to be appreciated on a national and international scale. It is but natural therefore that since 1900 we should find attempts at applying statistical and probability theory to engineering problems springing up on a very broad front and in many countries. Out of such studies one of the developments has been the theory of economic control of quality of manufactured product centered about the Quality Control Chart and its use.

This chart was first introduced in 1924. Whereas in 1870 or thereabouts, engineers introduced the "go no-go" tolerances or limits on the quality of pieces of product, the control chart in its simplest form introduces an aimed-at value somewhere between these tolerances and two so-called control limits spaced one on either side of the aimed-at value but (usually at least) within the tolerance ranges. The function of the control limits is to mark the range of variation in quality which *economically should be left to chance.* If variations fall outside these limits, action is to be taken to discover and remove, if possible, the causes of such variation even though the observed qualities lie within the tolerance limits. With the introduction of the quality control chart, engineers began a concerted study of the chance fluctuations of quality with a view to making use of laws of chance to reduce such variability to an economic minimum. This is in contrast with the objective of the introduction of the "go" and of the "go no-go" gauges which was simply to get around the difficulties caused by chance or unknown causes. This development has gone a long way toward providing an operational technique for (1) minimizing the cost of rejections, (2) minimizing the cost of inspection, (3) minimizing the tolerances that must be allowed and thus making more efficient use of materials with which an engineer must work, and (4) providing the maximum quality assurance at a given cost.

It is significant that industry through the study of laws of chance is well on the road to the development of a technique of making full use of such laws in the control of quality of product; that the full advantage of such efforts can come only through cooperation of industrial groups and that such cooperation is being developed on a national and even an international scale through engineering and scientific organizations. Progress in this direction should make for higher standards of quality—a fact of interest to each and every one of us.

July 30—PROBLEMS OF THE INTERNATIONAL DISTRIBUTION OF POPULATION AND RAW MATERIALS,

CORRADO GINI.

The problems of the international distribution of population and raw materials are only two particularly salient aspects of a larger problem, deriving from the disequilibrium between the political and the economic interdependence of the various states which matured during the last part of the nineteenth and the first part of this century. While, from the political standpoint, nations have always remained rigorously restricted to the national organizations, from the economic viewpoint they became bound together in increasing measure. But the acute political antagonism and the national egoisms which developed after the war prepared the ground for a breach of such a collaboration; and this was occasioned by the world crisis, when every nation, placed in a state of necessity, thought itself right in fully exploiting for its own advantage international economic relations.

The problem is, however, particularly serious with respect to the international distribution of raw materials and population, the marked inequalities in their territorial distribution, the particularly acute character which a crisis arising from a shortage of raw materials rapidly assumes, and the special difficulties of economic and extra-economic nature (such as racial antagonisms and considerations of prestige) which often arise from interfering with international distribution of population.

The problems of the distribution of raw materials and population cannot be solved if the fundamental problem is not faced. This is the adjustment between the political and the economic interdependence of nations, either by accentuating the first or by attenuating the second.

The radical and general solutions of a return to the economic isolation of national units or their fusion into one supernational political unit are out of the question. Indeed, the first solution would represent the ruin of practically all the states, and the second, far from any possibility of being realized in the immediate future, stands in contrast to the nationalistic tendencies which have been manifested in the postwar period.

It is then necessary to consider intermediate solutions.

One of these, often advocated, consists of limiting the sovereignty of states through the compulsory regulation, delegated to appropriate agencies or guaranteed by special dispositions, of the distribution of raw materials and labor forces. But when one passes to a complete program of attainment along such lines, the difficulties become evident. It is difficult to conceive a superstate agency confined to the regulation of only two fields, while the other fields remain entrusted to national or private initiative. On the other hand, experience has shown the inadequacy of the societary agencies, like the League of Nations; and reflection suggests that they are inescapable, as one cannot see why any state, as a member of a societary agency, ought to renounce its own interest for the interests of the others. So in societary agencies the viewpoints of the strongest members inevitably prevail, and we cannot expect from them solutions impressed with great equity and with greater regard toward the states which today feel themselves sacrificed. Also on the aims to be attained in the limitation of the sovereignty of states it is difficult to obtain the consensus of the interested nations. Some of these propose for international regulation the aim of assuring a complete freedom of international movements of raw materials and of populations, but such a freedom without regulation of international political relations would accentuate economic interdependence and aggravate its dangers. Others on the contrary would have the central agencies regulate the international distribution of the populations and of raw materials as the interallied executives regulated during the war the supplies of the allied countries. But the experience of these executives has shown the difficulty of establishing equitable and universally accepted principles for such distribution, as well as the slowness and rigidity of their practical application.

The alternative intermediate solution is that of accentuating the economic self-sufficiency of nations, either through a restriction of their needs, or through an integration of their territories.

In the field of population, the restrictive solution is proposed by the Anglo-Saxons and the French against the claims of the other Latin, the Slavic, and Oriental Nations. These ought to control their birth rate in order to avoid the necessity of expanding in the other territories. In the field of raw materials, a similar thesis is maintained by the Orientals against the Westerners in general and the Anglo-Saxons and French in particular, to the effect that they should reduce the excessive consumption of raw materials necessary to support the artificial development of their industries. But there is no prospect of such restrictions being accepted by the interested nations, unless they can be convinced that they are to their own advantage, or at least would further the progress of humanity. And such a demonstration appears extremely difficult, taking account of the high efficiency of the Western in comparison with the Oriental civilization and of the recognized danger threatening the Anglo-Saxon, Nordic, and French populations as a result of birth control, danger which would menace all white civilization were such to be adopted also by the other Latin and the Slavic populations.

The only solution which remains for our consideration is the integrative solution. It is clear that this cannot be a complete solution because it is not practically possible to add to a state territories which can guarantee it all the necessary raw materials and a complete outlet for its superabundant population, but only territories which suffice to lessen demographic pressure and the deficiency of raw materials or which constitute at least some reserve to which it can have recourse in case of necessity. It is equally clear that there cannot be a general solution, susceptible, that is to say, of adoption by all the states. But these are not decisive objections. Recognizing the impossibility of a complete solution, a partial one may well be considered satisfactory; and recognizing the impossibility of assuring a satisfactory solution to all nations, it may be well that such a solution be assured to those nations whose technical progress and whose demographic development are in keeping with the interest of humanity. On the other hand, the nations whose technical progress and whose demographic development are more accentuated have generally the tendency, and acquire sooner or later the power, to annex new territories, so that in the long run their particular interests fit with the general interest of mankind. The integrative solution is not a static one. History shows that the torch of progress was transmitted from one to another race and in the same race from one to another population, and that similarly the demographic expansion of a population is destined to give place to a stationary stage and finally to a numerical decline. Generally the leaders of an epoch do not pass from the stage of history before having adequately educated and elevated their successors. So that the objection leveled against the Western system of production—that it destroys itself by raising the standard of living of the populations which dominate and thus creating a dangerous competition—is really not an objection at all, but rather an indication that with such a system the Western nations carry out a civilizing mission.

It may be observed in this connection that the theory here presented definitely leads only to a justification of the crude reality. Certainly it leads us from theoretical abstractions and from Utopian programs to the actual behavior of the facts; and from those it goes on to explain—a result which truly, it may be well noted, constitutes the aim and the characteristic of every theory which is worthy of being called scientific. Because, in the field of social as in that of physical sciences, when a theory, although seemingly well constructed, does not correspond to the facts, one can say with assurance that its value is fictitious and that under satisfactory appearance, it conceals on the contrary some essential lacuna which makes it defective when compared with reality.

On the other hand, the deeper one goes into the study of society, the more one becomes convinced that in this field, as in that of biological phenomena, nature has provided much better than superficial observations may lead one to suppose; and that the scholar must go a long way in research and apprenticeship before attaining the power to apply effective correction to natural behavior.