COWLES COMMISSION
FOR RESEARCH IN ECONOMICS

REPORT OF SIXTH ANNUAL
RESEARCH CONFERENCE ON
ECONOMICS AND STATISTICS
AT COLORADO SPRINGS

July 1 to 26, 1940

THE UNIVERSITY OF CHICAGO

COWLES COMMISSION
FOR RESEARCH IN ECONOMICS

THE COWLES COMMISSION FOR RESEARCH IN ECONOMICS is a not-for-profit corporation, founded in 1932 for the purpose of conducting and encouraging investigations into economic problems. A function of the COMMISSION is to issue from time to time papers and monographs of an econometric or economic-statistical nature without, however, assuming responsibility for theories or opinions expressed therein. The COMMISSION is affiliated with the ECONOMETRIC SOCIETY, an international society for the advancement of economic theory in its relation to statistics and mathematics.

THE UNIVERSITY OF CHICAGO

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THE 1940 CONFERENCE

Although the offices of the Cowles Commission were moved from Colorado Springs to the University of Chicago in September, 1939, the Sixth Annual Research Conference on Economics and Statistics was held at Colorado College, Colorado Springs, as in former years. The Conference opened on Monday, July 1, and continued through Friday, July 26. Two lectures, followed by discussion periods, were scheduled each morning, the sessions beginning at 9:30 and closing about 12:30. In addition, two evening public lectures were given, by Carl Snyder and by Irving Fisher. Afternoons, Saturdays, Sundays, and July 4 were left free for recreation.

The scientific program was supplemented by a variety of entertainment. A picnic supper was held for those in attendance on the opening day. Teas for those attending the Conference were given by Professor and Mrs. Harold T. Davis at their home and by Headmaster and Mrs. Francis M. Froelicher at the Fountain Valley School of Colorado. Swimming, horseback riding, golf, tennis, and other sports were enjoyed by many, and a number of hikes and picnics were arranged. Points of interest visited on afternoon and week-end trips included the Garden of the Gods, the summits of Pikes Peak and Cheyenne Mountain, the Royal Gorge of the Arkansas River, Cripple Creek gold camp, Rocky Mountain National Park (Estes Park), and the presentation of *The Bartered Bride* in the opera house of the old mining town of Central City.

The number of people participating in part or all of the sessions of the Conference was 204, including 101 from out of town.

PARTICIPANTS FROM OUT OF TOWN

BARTKY, Professor WALTER, The University of Chicago, Chicago, Ill.

BURGESS, Dr. ROBERT W., Western Electric Company, 195 Broadway, New York, N. Y.

CAMP, Professor and Mrs. BURTON H., Wesleyan University, Middletown, Conn.

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COWLES, Mr. ALFRED, Cowles Commission, The University of Chicago, Chicago, Ill.

DAVIES, Dr. MAURICE B., The Denver and Rio Grande Western Railroad Company, Denver, Colo.

DAVIS, Professor and Mrs. HAROLD T., Northwestern University, Evanston, Ill.

DEAN, Professor JOEL, Cowles Commission, The University of Chicago, Chicago, Ill.

DEMING, DR. W. EDWARDS, Bureau of the Census, United States Department of Commerce, Washington, D.C.

DODD, Professor and Mrs. EDWARD L., The University of Texas, Austin, Texas.

DOUGLAS, Miss ELINOR, Training School for Southern Illinois, Carterville, Ill.

DRESCH, Dr. and Mrs. FRANCIS W., University of California, Berkeley, Calif.

FISHER, Dr. and Mrs. ERNEST M., American Bankers Association, Washington Building, Washington, D. C.

FISHER, Professor IRVING, Yale University, New Haven, Conn.

FLOOD, Dr. and Mrs. MERRILL M., Princeton University, Princeton, N. J.

GEIRINGER, Dr. HILDA, Bryn Mawr College, Bryn Mawr, Penn.

GOLDENWEISER, Dr. E. A., Board of Governors of the Federal Reserve System, Washington, D. C.

HAAVELMO, Mr. TRYGVE, University Institute of Economics, Oslo, Norway; present address, c/o Rockefeller Foundation, 49 West 49th St., New York, N.Y.

INGRAHAM, Professor MARK H., The University of Wisconsin, Madison, Wis.

JOHN, Miss ALICE, Commander Hotel, Cambridge, Mass.

JOY, Miss ARYNESS, Bureau of Labor Statistics, United States Department of Labor, Washington, D. C.

KREPS, Professor T. J., Temporary National Economic Committee, Washington, D.C., and Stanford University, Calif.

KRUPP, Miss SARAH, 3849 West Grand Avenue, Chicago, Ill.

LANGE, Professor OSKAR, Cowles Commission, The University of Chicago, Chicago, Ill.

LAURSEN, Mr. and Mrs. SVEND, Harvard University, Cambridge, Mass.

LEAVENS, Mr. and Mrs. DICKSON H., Cowles Commission, The University of Chicago, Chicago, Ill.

LEONTIEF, Professor and Mrs. WASSILY, Harvard University, Cambridge, Mass.

LEWIS, Mr. H. GREGG, Cowles Commission, The University of Chicago, Chicago, Ill.

LUSHER, Mr. and Mrs. DAVID W., Bowdoin College, Brunswick, Maine.

MCEWAN, Professor and Mrs. GEORGE F., P. O. Box 109, La Jolla, Calif.

MCFADDEN, Professor JOHN R., Kansas Wesleyan University, Salina, Kansas.

MCINTYRE, Professor and Mrs. FRANCIS, Indiana University, Bloomington, Ind.

MACNEILLE, Professor H. M., Kenyon College, Gambier, Ohio.

MATCHETT, Dr. GERALD J., Indiana University, Bloomington, Ind.

MAYER, Dr. WILLIAM E., 636 Church St., Evanston, Ill.

MENDERSHAUSEN, Dr. and Mrs. HORST, Colorado College, Colorado Springs, Colorado; present address, National Bureau of Economic Research, 1819 Broadway, New York, N. Y.

MITCHELL, Miss CLAUDIA, Calhan, Colo.

MOLINA, Mr. and Mrs. EDWARD C., Bell Telephone Laboratories, 463 West St., New York, N. Y.

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QUIRIN, Mr. and Mrs. E. L., Babson's Reports, Inc., Wellesley Hills, Mass.

RASHEVSKY, Professor and Mrs. N., The University of Chicago, Chicago, Ill.

REID, Professor and Mrs. WILLIAM T., The University of Chicago, Chicago, Ill.

ROLL, Professor and Mrs. ERICH, University College, Hull, England.

ROOS, Dr. CHARLES F., The Institute of Applied Econometrics, 405 Lexington Ave., New York, N. Y.

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SMITH, Mr. and Mrs. JOHN H., Cowles Commission, The University of Chicago, Chicago, Ill.

SNYDER, Mr. CARL, 33 Liberty St., New York, N.Y.

SPEEKER, Professor and Mrs. GUY G., Michigan State College, East Lansing, Mich.

SWED, Miss FRIEDA S., University Experiment Station, University of Wisconsin, Madison, Wis.

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WORKING, Mr. D. W., Denver, Colorado.

WORKING, Professor and Mrs. E. J., University of Illinois, Urbana, Ill.

WORKING, Professor and Mrs. HOLBROOK, Food Research Institute, Stanford University, Calif.

YNTEMA, Professor THEODORE O., Cowles Commission, The University of Chicago, Chicago, Ill.

HISTORY OF THE CONFERENCES

The Annual Research Conferences on Economics and Statistics, held at Colorado Springs under the auspices of the Cowles Commission for Research in Economics, originated in a series of informal meetings during the summer of 1935 after the meeting of the Econometric Society at Colorado Springs on June 22-24 of that year. At these gatherings various papers were presented and discussed by economists who remained in the vicinity. The meetings were so successful that it was decided to continue them in subsequent years. The following table gives a statistical record:

	1935	1936	1937	1938	1939	1940
Number of conference lectures	8	32	40	38	38	38
Number of lecturers	7	20	27	27	32	37
Average attendance per lecture	15	27	36	47	51	32
Participants from out of town	5	55	67	93	112	101
" local	20	24	41	99	122	103
" total	25	79	108	192	234	204

ABSTRACTS OF CONFERENCE LECTURES

Monday, July 1—Dynamics of the Business Cycle, HAROLD T. DAVIS, Professor of Mathematics, Northwestern University, and Research Associate, Cowles Commission.

The theory of monopoly in mathematical economics is based upon certain deductions which follow from the maximization of the profit function

$$\pi = yp - Q(u) \ ,$$

where y is demand, p is price, and $Q(u)$ is the cost of manufacturing u units. This concept of static economics has been enlarged in recent years by C. F. Roos and G. C. Evans through the introduction of the profit integral

$$\Pi = \int_{t_0}^{t_1} [yp - Q(u)] \, dt \, ,$$

where y and u are now regarded as linear functions of $p(t)$ and $p'(t)$, and where $Q(u)$ is assumed to be a quadratic function in u. From the postulate that the monopolist always strives to maximize his profits over fixed periods of time, let us say between annual meetings of his stockholders, it is possible to set up a dynamic theory which describes the movement of prices by means of the differential equation:

$$p''(t) + m\,p(t) = m\,p_0 \, .$$

But if Π is to be maximized, then a realistic determination of the parameters shows that m is essentially a negative number, from which we see that the solution of the differential equation is exponential in character. Hence, the possibility of explaining the existence of the business cycle by this analysis seems to be excluded. A realistic study of the demand for steel by R. H. Whitman (*Econometrica,* Vol. 4, 1936, pp. 138-152) shows clearly that demand, although it is influenced in part by price and its derivative, is more powerfully affected by a term measuring business activity.

The present paper is an attempt to give a new basis for the problem of dynamic prices, retaining, however, the attractive feature of the older theory that some extremal property is involved.

The main thesis of this paper asserts that an economy attempts to maximize the following integral:

$$J = \int_{t_0}^{t_1} (U - \kappa A - \mu B)\, dt ,$$

where κ and μ are constants and U, A, and B have the following meanings:

$U = U(x_1, x_2, \cdots, x_n)$ is the utility function of classical economics. We assume, at any point of time, the equilibrium conditions

$$\frac{\partial U}{\partial x_i} = \lambda p_i, \qquad (1)$$

where p_i are the prices of the goods x_i, and where λ is the marginal utility of money.

The quantity A is defined by

$$A = \sum x'_i p'_i ,$$

and measures the effect of shocks upon the economic system. Erratic shocks have long been recognized as important factors in economic variation.

The quantity B is defined to be

$$B = \sum (x_i - u_i)^2 ,$$

a function introduced to measure the effect of carry-overs and inventories upon the business cycle.

If we limit our analysis to one primary variable, perhaps a dominating commodity like steel, and if we take account of (1), then the first necessary condition (Euler's equation) that J be maximized leads to an equation of the form

$$p''(t) + m\, p(t) = I(t) ,$$

where m is positive and proportional to the marginal utility of money, and where $I(t)$, measuring the difference between production and consumption, may be assumed to correlate highly with the index of business.

This equation, under proper specialization of $I(t)$, accounts for the major changes in the price structure, as measured by the price of common stocks, since 1900. It agrees with the statistical observation of the existence of a 40-month cycle in stock prices, and with the fact that normal price movements do not show damping. By introducing the concept of mechanical resonance, the equation also accounts for the inflation of 1929 and the subsequent decrementary movement of the price series. On the somewhat doubtful assumption that m is directly proportional to the velocity of money (V), and inversely proportional to money supply (M), the period in prices is shown to be proportional to $\sqrt{(M/V)}$.

Monday, July 1—Some Observations on Business-Cycle Theory, CHARLES F. ROOS, Director of Research, The Institute of Applied Econometrics.

The choice of the starting point of a theory of business cycles is arbitrary, but there are advantages in choosing one at which the flow is simple and can be expressed in terms of an over-all slowly changing variable such as national income and some other such as price. The demand for semi-durable consumption-goods which satisfies these conditions can be shown to depend upon national income, consumers' stocks, and price changes. It can therefore be written in the form:

$$U(t) = A(t) I\alpha, \tag{1}$$

where α is the elasticity of income and $A(t)$ depends to some extent upon the distribution of income, but more upon recent purchases or consumers' stocks and price changes.

The retailer and wholesaler in turn will have a demand which is related to $U(t)$. It will depend upon their estimates of $U(t)$ and their opinions regarding price changes. It can be taken as:

$$(2) \qquad u(t) = (A_0 S^{\beta} p^{\gamma} B_0 \, dp/dt) \; U(t),$$

where A_0 and B_0 are empirical constants or functions of time which reflect physiological factors, β is the elasticity of stocks, and γ is the elasticity of price p. The quantity dp/dt is the derivative of price with respect to time, or rate of change of price, and $U(t)$ is given by equation (1) above.

This fundamental demand develops in the form of new orders on the books of manufacturers who are motivated by the urge to seek profits. These manufacturers would like to be able to maximize their profits over a period of time. But they actually are in control of only a few of the factors which determine profits.

The expected profit of each manufacturer may be represented by the integral

$$(3) \qquad \Pi = \int_{t_0}^{t_1} [\gamma p u - Q(u)] E(t_1, \; t) \, dt ,$$

where $E(t_1, \; t)$ is the force of interest, $Q(u)$ is the cost of production as an explicit function of production u and an implicit function of such factors as wages, technological developments, costs of materials, power costs, and the like. The factor γ is defined so that each entrepreneur deduces his expected price by multiplying present price p_0 by the psychological factor γ.

The economic process which actually takes place may be described as follows: Individual retailers and wholesalers, sometimes with and sometimes without the help of the sales representatives of the manufacturers, make estimates of the probable consumer demand, that is, of national income to which this demand is closely related, and of the effects of this expected demand upon supply and price. If their expectations are larger than their expectations of supply, i.e., manufacturers' expectations, they usually order in sufficient volume to induce a price rise, which in turn creates a speculative demand. While this speculative demand is developing, each buyer thinks he is among the few smart enough to see the "coming" additional price rise. Thus, in ignorance of the true state of the market, the wholesalers and retailers after a time have made sufficient commitments to cause stocks S to rise substantially.

Some then withdraw from the market and hope that others will be able to buy sufficient quantities to keep up the price while they correct their own speculative mistakes. But such orders do not develop, because demand cannot immediately rise fast enough, i.e., Say's law does not hold over the short term. The result is a downward turn in orders, followed by price weakness and then a further decline in orders. This downward spiral continues until prices are lowered sufficiently to attract purchases as a price speculation or until an inflationary outlook is presented. The upturn is likewise cumulative.

Examples of this process in automobile sales, residential building construction, and the demand for capital goods were described.

Tuesday, July 2—The Organization for National Defense, THEODORE O. YNTEMA, Professor of Statistics, The University of Chicago, and Director of Research, Cowles Commission, on temporary leave with the Advisory Commission of the Council for National Defense.

Mr. Yntema spoke informally, discussing the administrative and economic problems involved in industrial mobilization for national defense. He also outlined the economic difficulties facing the United States as a consequence of German domination of Europe.

Tuesday, July 2—The Demand for Steel, H. GREGG LEWIS, Instructor in Economics, The University of Chicago, and Research Associate, Cowles Commission.

The importance of steel in the national economy has for years invited questions concerning the effect of steel pricing policy on national economic welfare. In recent discussions of cyclical and long-run pricing policies, several economists have proposed that:

(1) Steel prices be made more flexible in the short run,

(2) Steel prices be considerably lower in the future than they have been in comparable situations in the past. Both proposals are directed at "stabilizing" and increasing steel output. Both are based on *assumptions* concerning the price elasticity of demand for steel.

These proposals have led to two general questions:

(i) What has been the range of values of the short-period price elasticity of demand for steel in recent years?

(ii) What has been the range of values of the long-run elasticity in recent years?

This is a preliminary report of an investigation to determine the answers to these questions. The price elasticity of demand corresponding to a price change at the beginning of a period of adjustment is defined as the ratio of:

(1) the percentage change (from the beginning to the end of the period of adjustment) in the time rate of steel sales

to (2) the given percentage change in price, other factors remaining constant.[1]

It is thus clear that elasticity of demand is a dynamic concept, in the sense that time is involved explicitly. "Short periods" are periods of adjustment approximately up to one year in length. The "long run" is a period of adjustment long enough for all the effects of the given price change to work themselves out in the above *ceteris paribus* sense.

In measuring these elasticities of demand three complementary approaches have been used:

(A) What Professor Tinbergen calls the "common sense" approach.[2]

(B) What has been rather unhappily called the "substitution" method.

(C) Multiple correlation of economic time series.

(*A*) *The Common Sense Approach.*

This "literary" method of analysis is that used in most

[1] Except to the extent that changes in other factors *are caused by the price change.*

[2] J. Tinbergen, "Econometric Business Cycle Research," *Review of Economic Studies*, Vol. 7, No. 2, February, 1940, p. 87.

current surveys of business conditions. It consists of reasoning from "a careful cataloguing of facts and of the month-to-month fluctuation in a number of economic times series."[3] Careful analysis by this method of the four years 1936-1939 led to the conclusion that in recent years the short-period elasticity of demand for steel has probably been very small—almost certainly less than unity, and that for very short periods there may actually be a positive relation between steel sales and steel prices.[4] This latter phenomenon arises in major part from price speculation in the short run.

(*B*) *The Substitution Method*

The ratio, R, of the cost of the steel used in the production of a unit of product-made-from-steel to the price per unit of the product was computed for each of several major steel products. Estimates were then made of the short-period elasticities of demand, E, of these products-made-from-steel. Assuming reasonably that in the short run the elasticities of substitution between steel and substitute materials are negligible, the quantities of $R \times E$ are fairly accurate measures of short-period elasticities of demand for steel by major consuming industries.

The values of R were found in most cases to be considerably less than 0.5, and in many cases less than 0.25. The corresponding values of E have probably in recent years not been greater than 2.0 or 3.0. This evidence helps substantiate the conclusion that the short-period elasticity of demand for steel $(R \times E)$ has been less than unity.

(*C*) *Multiple Correlation of Economic Time Series*

Linear demand functions were fitted to annual data, with steel bookings or steel shipments as the dependent variable, and the composite price of steel and one or more of the following as independent variables:

(i) Supernumerary national income,
(ii) Corporation profits,
(iii) Link relatives of the composite price of steel,
(iv) Link relatives of supernumerary income,
(v) Time-trend.

[3] Tinbergen, *op. cit.*

[4] See the author's *An Analysis of Changes in the Demand for Steel and in Steel Prices—1936-1939*, United States Steel Corporation, New York, 1939.

These multiple regressions yielded short-period price elasticities varying from —0.1 to —0.8. This evidence by itself was not accepted as conclusive.

Taking all the evidence, however, there is considerable reason to believe that over short periods the demand for steel is quite inelastic. Thus far attempts to measure the long-run elasticity have not been very satisfactory.

Before conclusive answers can be given to either of the questions studied, more adequate data must be found, and more satisfactory dynamic demand models developed for time-series analysis.

Wednesday, July 3—Tests of Sampling Error Applicable to Market Analysis and Control, MAURICE B. DAVIES, Statistician to the Trustees, The Denver and Rio Grande Western Railroad Company.

From the standpoint of business management, "market analysis" refers to the process of research by which the sales manager (or the railroad traffic manager) obtains the facts necessary for the intelligent mapping out of his sales strategy. It includes studies of customers and their preferences; of the geographic flow of present goods or services; and of the probable location of potential demand for future goods or services.

Since situations change rapidly, the management cannot be content with static facts which have become obsolete. Continuous observation is also needed, so that sales strategies may be shifted quickly at the first moment when the need for a shift can be discerned. This continuing statistical analysis of the current sales picture is a phase of "sales control."

The simplest test of significance usable by the market analyst is the application of Bernoulli variance to a frequency distribution where conditions of random sampling are known to have been fully met. This is illustrated by analysis of replies to a questionnaire, where the persons questioned were randomly selected.

Almost any static survey of market conditions may be adapted to "pure sampling" procedures—at least in part. The appropriate tests of significance are well known, so long as we deal with "pure frequencies," or "pure values," and so long as we analyze a static situation rather than a time series.

Following is an example of a more complex problem:

Consider the manufacturer of a single commodity which is sold at prices ranging from \$10 to \$30 per unit. Assume that this manufacturer sells through small retail dealers, on a franchise basis. If some expense is involved in the relation between manufacturer and dealer, both parties will be alert to discover if their relationship is ceasing to be fruitful. The performance during a few weeks or months is for this purpose taken as a sample of the long-time *rate* of performance.

Our problem arises from the chance fluctuations in number of sales from month to month. Where the number of sales expected in each month at any specified location is very small, we should expect the frequencies to fluctuate according to the Poisson distribution—the same distribution that is observed in biological cell counts with a haemocytometer. With larger frequencies, the chi-square distribution may be expected. That is, the variance of the number of sales in a month is equal to m, which is the expected number of sales per month.

By reference to a "confidence belt" chart, such as that presented by W. E. Ricker in 1937, the upper and lower limits may be estimated for the hypothetical universes from which the observed sample might have been drawn. These upper and lower "universe limits" are then respectively projected to an annual basis, by a factor which allows for the normal seasonality of the time interval at which the sample was taken. The result is an estimate of upper and lower limits of the annual rate of sales at this locality. If the sales are recorded only in dollar volume and not in frequencies, it becomes necessary to combine the variance of the frequency with the variance of the dollar value per unit sale; the two variances are combined by a first approximation for the variance of a product.

The foregoing discussion assumes that each unit sale is an event independent of every other unit sale. In certain lines of business, this assumption is almost perfectly correct.

In the railroad business, the techniques described above are adaptable wherever the assumptions can be maintained; as, for example, in studying the day-to-day fluctuations in number of passengers on a given train. A rough measure of variance can be made on the assumption that each passenger's presence is an independent event. A more accurate estimate could be made if it were known what percentage of the passengers traveling are in groups of two, three, four, or more.

In studying carloadings, the Bernoulli and Poisson concepts are useful where the analysis is extremely detailed so that very small numbers are used. With increasing size of sample, the Lexian aspects of the day-to-day distribution become relatively more important, and when the fluctuations of whole railroad systems are compared, the most helpful approach seems to be through an extensively adjusted analysis of variance and covariance.

Even where we are not yet able to identify the units which constitute truly independent events, a worth-while estimate may be computed from a constant "variance per thousand carloads" applicable concurrently to large and small samples in a comparative study.

Wednesday, July 3—Certain Aspects of the Theory of Genetic Equilibrium, MARK H. INGRAHAM, Professor of Mathematics, The University of Wisconsin.

At times certain characteristics of an individual plant or animal make it more probable that it will not mate or that it will mate to another individual of the same or different type. The probable number of offspring may also be affected. One among many mathematical approaches to the subject is given in this paper through the study of iteration of a single transformation of variables. This approach may prove fruitful, especially in certain applications to the social sciences. The discussion falls under three headings:

I. Genetic hypotheses and their mathematical formulation.

II. Mathematical theory and methods suggested by the hypotheses.

III. Possible extensions of the theory and their applications to social problems.

I. If a given characteristic such as color in rats having two aspects, for example, black and white, which are dominant and recessive, is determined by a single gene, subject in its behavior to the Mendelian law, an individual may be considered genetically as purely recessive, mixed, or purely dominant, both the latter two types appearing with the dominant characteristic. We assume that in preferential mating and reproduction the difference is determined by the apparent rather than the genetic type. Let the proportion of purely recessive to mixed be represented by x and the purely dominant to mixed by y and let x' and y' be the proportions in the next generation. Then x' and y' are given in terms of x and y where the parameters of the transformation are determined by the degree and character of the preferential mating and reproduction. There are essentially four cases. In the first the population becomes purely recessive; in the second, purely dominant; and in the third the population tends to a fixed proportion of the three types. In the fourth the end result depends upon the initial proportions. In this case, except for special initial proportions, the population becomes either purely dominant or purely recessive. There is, however, an invariant point and a curve through the point dividing the plane between values of x and y from which the population becomes purely recessive and points from which the population becomes purely dominant. Along this curve under iteration a given point approaches the above-defined invariant point. The equilibrium is unstable, however, and any accident which changes the proportions would make the end result tend to one pure type or the other. This process may be slow since for many initial proportions the tendency will be to approach the invariant point and stay near it through a long period before it becomes apparent that the proportion will approach one pure type or the other. If there is no preferential mating or reproduction after one generation, an invariant proportion is reached which depends upon the original proportions.

II. The above leads to a study of the convergence of the product of sequences of linear transformations. This study should be extended to the case of sequences of transformations whose parameters are determined only to within knowledge of their distribution functions.

III. The case of more than one gene acting independently in a probability sense is developed. The case of dependence should be studied. From the theory of this paper the incorrectness of a common assumption that a preferential birth rate for the groups of lower intelligence would necessarily lead to the degeneracy of the race can be shown. This assumption appears to be valid when the birth rate increases monotonically from genius to moron. It is probably true, however, that the highest reproductive rate, that is, reproduction of adults, is at a point somewhat above the lowest end of the mental scale. For such a case the assumption seems to be generally false.

Friday, July 5—A Comparison of Several Statistical Forecasting Methods, MERRILL M. FLOOD, Director, Statistical Section, State and Local Government Section, School of Public and International Affairs, Princeton University.

Satisfactory budgeting depends on the accurate estimation of anticipated revenues and expenditures. Where revenues are derived from business sources it is essential that forecasts of the general course of business be available for use. This is the situation in governmental budgeting, for example, when a fixed system of business taxes must be depended upon for revenue. A similar need exists in the case of certain governmental expenditures, particularly those for unemployment relief.

Whenever practicable, it is simpler to establish a relationship between each of the several revenue and expenditure series and a single "indicator" series, then to be forecasted, than to extrapolate for each budget item separately. If the individual budget series lag behind the indicator series the time interval for which $i(t)$ must be extrapolated is thereby reduced with consequent improvement in the forecasts. The yield of a personal net-income tax during 1940 may depend, for example, on business done in 1939. It is much easier, in general, to relate the separate budget items to a single indicator series than to forecast the indicator series $i(t)$ selected.

This paper examines certain of those methods which may be used to forecast values of $i(t)$ for $t = t_0 + 1$, $t_0 + 2$, $t_0 + 3, \cdots$, using only the previous observed values for $t = t_0$, $t_0 - 1$, $t_0 - 2, \cdots$ Three methods were applied to monthly sales data of a large mail-order company to obtain extrapolations one to four months in advance. The average percentage deviations for these various forecasts made for each of the months from January, 1937, through December, 1939, are shown in the table following.

AVERAGE PERCENTAGE DEVIATIONS

Method	Number of Months Extrapolated			
	1	2	3	4
A	2.82	3.03	3.24	3.27
B	2.87	3.26	3.62	4.72
C	4.42	5.83	6.61	7.97

Methods A and B were very much alike and gave very similar results. Each depends on the determination of a smoothed trend curve, fitted to the known observations, which is then extrapolated to yield forecasts of the trend curve. To these trend forecasts are applied the monthly seasonal correction factors to yield the desired monthly forecasts. The two methods differ principally in that the trend curve of Method A is simply the centered 12-month moving average extrapolated freehand.

The forecasts obtained by applying Method C were much less satisfactory. In this case the method of autocorrelation was applied, using data prior to 1936, to determine the parameters a_r and b_r in the estimating stochastical difference equations

$$m(t+r) = a_r m(t-1) + b_r m(t-2)$$

for $r = 0, 1, 2, 3,$ where $m(t) = i(t) \div i(t-12)$.

It seems a fair conclusion that the essential differences in the results of this experiment are due to the methods used in eliminating the monthly seasonal variation more than to any other cause. For forecasts of less than six months in advance the accuracy is good enough for the practical problems of budget control.

A somewhat different approach is suggested for further empirical analysis, based on the use of the linear difference equation, which might aid in extending forecasts beyond six months. It consists essentially in the examination of the changing amplitudes and phases of each component of the solution of the difference equation selected for use, as the initial points change with time. An application to a monthly capital-goods orders series was made and the result, shown graphically, suggests that the phase of the three-year cycle is essentially a step function but that the phase of the shorter cycle remains essentially constant.

The computation of the phase and amplitude functions is reduced, through a simple application of matrix theory, to the determination of the inverse of a matrix which is written explicitly in terms of the roots of the characteristic polynomial of the difference equation.

Friday, July 5—Alternative Statistical Methods for Estimating Marginal Cost Functions, JOEL DEAN, Assistant Professor of Statistics and Marketing, The University of Chicago, and Research Associate, Cowles Commission.

The growing interest in the economics of the individual firm, together with the crucial role of cost behavior in determining a variety of optimum adjustments to the firm's kaleidoscopic environment, have resulted in clearer recognition of the importance of empirical cost functions for business administration, for economics, and for public policy.

The cost of the individual firm is a complex function of many variables, the nature and relative importance of which is dependent upon the character and policies of the enterprise as well as upon the particular cost component concerned. Many of these determinants, moreover, have several significant aspects; output, for example, because of lack of homogeneity of product is rarely uni-dimensional. Isolation and measurement of the net relationship of each to cost is managerially useful not only for cost control but also for marginal calculations in determining optimum adjustments.

Marginal calculations involved in equating expected additional cost to expected additional revenue need not be confined to determination of traditional output and price optima, but properly apply also to the entire gamut of managerial decisions. The concept of marginal cost should, therefore, be expanded to embrace increments of expense with respect to all cost determinants and all policy decisions. The calculus of marginal equating of cost and revenue has universal applicability in business management.

There are two major alternative sources of data for estimating these various kinds of marginal cost by statistical methods: (1) successive observations of a single enterprise during a period characterized by variation in cost-determining operating conditions, (2) simultaneous observation of many enterprises variously situated with respect to the values of the cost determinants. Despite the difficulties encountered in statistical analysis of serially correlated successive observations, the first source is more promising than the second, because of the heterogeneity of firms with respect to product lines, technology, size, degree of integration, monopsony, managerial skill, and accounting methods.

Traditional theoretical models are not entirely satisfactory for statistical cost analysis. For example, the simple dichotomy of short run vs. long run must be displaced by a hierarchy of short runs, depending upon the number and importance of factors that are fixed, and the limits within which they are fixed. Moreover, cost observations may reflect shifts from one member of this hierarchy to another, rather than movement along the same short-run curve. Although firms and observation periods were selected with a view to obviating or minimizing this difficulty, the statistical cost curves obtained are not precise empirical counterparts of theoretical functions. Furthermore, comparability of cost observations with respect to input prices, which is usually assumed in theoretical cost curves, is more difficult to attain statistically if either monopsony or substitution among factors as a result of changes in their relative prices exists. When these problems could be avoided by selection of observations, rectification for changes in input prices was accomplished simply by deflation or by use of input price as an independent variable.

To determine marginal cost functions the rectified cost observations can be statistically manipulated in several alternative forms: average cost, total cost, differential cost, and first differences in cost. Moreover, an estimate of combined marginal cost can be obtained either from combined cost observations or by means of separate analysis of observations of its components. These alternative methods, when employed in several case studies of marginal cost behavior, yielded remarkably similar results. Samples of these findings were compared in a series of slides and the alternative procedures evaluated.

Monday, July 8—Sampling in Production Inspection, WALTER BARTKY, Associate Professor of Astronomy, The University of Chicago.

This report is limited to the sample inspection of large outputs or lots of pieces of product, the pieces falling into two classes: standard pieces and defective pieces. The simplest form of inspection procedure is to accept or reject the lot on the basis of the number of defective pieces found in a random sample. The errors encountered in this single-sample method of inspection, as well as other more complicated procedures, are:

A. The rejection of lots containing a low fraction (say p less than p_0) of defective pieces.

B. The acceptance of lots containing a high fraction (p greater than p_0) of defective pieces.

In an investigation of the cost assignable to a method of sample inspection, it is necessary to consider the cost of errors A and B as well as the cost of sampling. If it is assumed that for a given size of lot, costs A and B are functions, $A(p)$ and $B(p)$, of the fraction p of defective pieces, then the expected cost assignable to sampling is for a lot containing a fraction:

(a) p less than p_0: $I + A(p)[1 - P(p)]$;

(b) p greater than p_0: $I + B(p)\ P(p)$;

where $P(p)$ is the probability of accepting the lot and I is the cost of examining the sample.

If the frequency distribution of the fraction p of defects is known for a set of lots, it is possible to determine the sampling procedure so that the expected cost for the set is a minimum. In practice only a crude estimate of this distribution is obtainable on the basis of past performance, but a preliminary study of the effect of varying the distribution indicated that crude estimates were adequate for practical purpose. For this reason it was considered worth while to determine the sampling procedure that minimized the cost for the hypothetical case in which a set of lots could be classified into two groups: lots containing a fraction p_1 of defects and lots containing a fraction p_2, where p_1 and p_2 are fixed and $p_1 < p_0 < p_2$. If the cost of selecting and examining a sample is proportional to the number of pieces involved, then it can be demonstrated that for large lots the sampling procedure corresponding to minimum cost is one permitting the removal of more than one sample. For very large lots these additional samples become essentially equal in size, in fact, for a theoretical infinite lot the sampling plan is as follows:

Examine n_1 and accept lot if 0 defects are found in sample and reject lot if more than d defects. If number of defects is more than 0 and less than $d + 1$ continue sampling, taking additional samples of n until the total number of defects is either equal to the number of additional samples or greater than d plus the number of additional samples. In the former case the lot is accepted, in the latter case it is rejected. For such a sampling scheme it can be demonstrated that the probability of either passing or rejecting approaches unity as the number of additional samples increase. By an application of matrix theory, comparatively simple formulae were obtained for the probability of passing a lot containing a fraction p of defects as well as the expected number of additional samples.

Monday, July 8—Cost Functions in Merchandising, PHYLLIS VAN DYK, National Bureau of Economic Research and Cowles Commission.

Certain managerial decisions require analysis of accounting information by techniques which will yield useful bases for estimating effects upon cost of any given change in operating conditions. Statistical analysis constitutes one of the most effective of these techniques. In the case of merchandising problems, statistical cost functions are potentially useful in cost control, in sales-promotion policy, and in decisions of price policy.

There are several outstanding problems in the study of department-store costs which are either peculiar to, or especially accentuated in, merchandising cost analysis. Of prime importance among these is the instability of the unit of output. If sales volume is the measure of production, then the output unit, one dollar of sales service, may be a function of many factors, notably rising standards of service over time, competitive conditions, and the rate of output (particularly at the higher levels of sales volume, where service standards are apt to be lowered). It may thus occur that as the department approaches capacity, the volume unit changes and the cost function does not describe the same phenomena observed at the lower-volume levels.

The distinction between fixed and variable costs is particularly difficult to make in a department store. This is true not only because of allocation problems, but also because neither the plant nor the personnel of a given department is highly specialized. Both the equipment and the employees are easily interchanged, within limits, between various capacities.

It is also difficult to distinguish between production and promotion costs. There is, of course, not complete agreement concerning the correct theoretical criteria for making this distinction. But even if one of the conceptual schemes is accepted, it is almost impossible to make the division in practice.

A further problem is introduced, in any attempt to study a given department within a store, by the incomplete isolation of department units from each other. This interdependence, partly attributable to common services, is reflected in planning activities and in the accounting records, resulting in many problems of allocation and data rectification, some of which are insoluble.

The study described in this paper is a statistical analysis of the costs of a medium-priced dress department in a large metropolitan department store. It is one of a series of investigations of merchandising establishment expenses in which the central problem is the measurement of the relationship between various elements of cost and the factors affecting these costs.

The general procedure in all of these studies is elimination of certain influences by careful selection of department and time periods, and measurement of the effect of other important quantitative variables by means of data adjustments, graphic correlation analysis, and multiple least-squares correlation analysis.

The costs studied in this particular department were selling salaries, nonselling salaries, buying salaries, advertising expense, and the total of these four components. Separate analyses were made of first differences in advertising and in combined costs.

A rather large number of potential independent variables was considered. Some of them were found inoperative on the basis of a priori knowledge; others had to be discarded because of insufficient or nonquantitative information. The remaining variables were subjected to graphic analysis, and on the basis of this analysis, tempered by logical considerations, a limited number of series was retained for the least-squares solution.

There are, of course, many limitations on the interpretation and use of cost functions obtained by statistical methods. To mention only a few: accounting records are not designed for calculations of marginal cost, many important cost influences are immeasurable, and the use of statistical measurement for management decisions is posited on the stability of the determined relationships and certain fundamental conditions. Nevertheless, statistical analysis, despite its inadequacies, appears to have distinct advantage over alternative approaches and to be definitely superior to orthodox accounting procedures.

Tuesday, July 9—Elementary Analysis of Variance: The Theoretical Background, BURTON H. CAMP, Professor of Mathematics, Wesleyan University.

This is mainly an expository paper; the discussion pertains to the simple case of a rectangular table containing r columns and s rows. The notation is indicated in the diagram.

$j \backslash i$	I	II	r	Sums	(Sums)^2
I					
II					
			t_{ij}	R	R^2
s					
Sums			S	T	ΣR^2
(Sums)^2			S^2	ΣS^2	T^2

$$A = \Sigma\,\Sigma\, t_{ij}^2 - \frac{T^2}{r\,s} \qquad a = \frac{A}{rs-1}$$

$$B = \frac{1}{s}\Sigma\, S^2 - \frac{T^2}{r\,s} \qquad bs = \frac{B}{r-1}$$

$$C = \frac{1}{r}\Sigma\, R^2 - \frac{T^2}{r\,s} \qquad cr = \frac{C}{s-1}$$

$$D = A - B - C \qquad d = \frac{D}{(r-1)\,(s-1)}$$

$$E = A - B \qquad e = \frac{E}{r\,(s-1)}$$

$$F = A - C \qquad f = \frac{F}{s\,(r-1)}$$

$$T = \Sigma\, R = \Sigma\, S$$

The six quantities at the right are, in random samples from a normal universe, unbiased estimates of the variance σ^2 of the universe. Also, a is called the "variance of the whole table," b the "variance of the column means," c the "variance of the row means," d the "variance of the deviations," or "error," e the average within-column variance, f the average within-row variance. From these six quantities any pair may be selected and the ratio between the two members of this pair may be used to test the hypothesis that the table was obtained by chance from a single normal universe; for the probability distribution of each of these ratios may be found. This distribution is Fisher's z distribution only in those cases in which the two members of the pair are independent of each other.

Now there are fifteen such pairs, and the question naturally arises: Which of these fifteen should be chosen? In discussing this question we shall now consider only the special case in which the ultimate aim is to discover whether there is "significant" stratification by columns. One measure of such stratification is b, the variance of the column means. The question whether the observed b is large or small depends on the ratio bs/σ^2, and on its probability distribution. This is a Type III distribution, and the probability that bs/σ^2 would be as great as observed can therefore be computed easily whenever σ^2 is known. Unfortunately it is usually unknown, and so we commonly put up with the ratio between bs and some other one of the six estimates of σ^2.

We proceed therefore to consider next the relative merits of bs/a and bs/d as estimates of bs/σ^2. Consider first bs/a, and suppose one proceeds always on the theory that the normal hypothesis is to be rejected if the probability of obtaining a value for bs/a as large as the one observed is as small as ε (e.g., $\varepsilon = 0.01$). We may now show that the proportion of *these* cases in which $bs/\sigma^2 > k$ (e.g., $k = 2$) is independent of σ^2 and can be tabulated. Similar remarks apply to bs/d. The information thus obtained is valuable, but in the practical case, the choice between these two ratios will often depend on other considerations, external to the data. We may know, for example, or at least be willing to assume, that columnar stratification, if it occurs, will have been brought about by a redistribution of what would otherwise have been a random sample. In such a case the use of bs/a is indicated. Or we may know that such stratification may have been brought about by the addition of different constants to the different columns or rows or both. This is the common situation in agricultural studies. Then the ratio bs/d is to be preferred.

Tuesday, July 9—Mathematical Aspects of Some Sociological Problems, NICOLAS RASHEVSKY, Associate Professor of Mathematical Biophysics, The University of Chicago.

The behavior of an individual in a population is to some extent determined by the behavior of every other individual with whom he comes in contact. A given activity of an individual may either enhance or inhibit the same activity in another individual. When such influences are expressed by linear relations, the activity of a very large number of different individuals is described by integral equations, in which the kernel is determined by the distribution function of the individual personal characteristics.

For the case that an individual having a given personal characteristic is willing to associate only with such other individuals as do not differ too much from him with respect to that characteristic, it is shown that a sufficiently large group of different individuals will split into two or more "social classes." Expressions for the sizes of such classes are derived. For some simple hypothetical cases it is shown how some of the parameters which enter into the equations governing the structure of the social group can be determined by means of those equations from actually observable data. Furthermore some general properties of the variation with respect to time of the fundamental distribution function which enters into the equations are derived.

Further, an abstract mathematical theory is developed for the interaction of several social classes, of which one influences and controls the behavior of the others. In some cases such interaction results in the existence of two configurations of equilibrium for the social structure, characterized by different types of behavior. Each configuration corresponds to one definite type of behavior. The transition from one configuration to the other, in other words, the transition from one behavior to another, occurs rather rapidly. Equations governing these transitions are given. In other cases, namely when the efforts of the individuals to influence others into a given behavior lessen as the success of this influence increases, there is only one stable configuration characterized by a mixed behavior. Because of the dissimilarity of parents and progeny, the composition of each social class changes with time, as generations change. This results in the appearance of instabilities of the social structure and in relatively sudden social changes. Possibilites of quasi-periodic alterations of different social structures are discussed. Finally, the developed equations are applied to the case of physical conflicts between groups of individuals, such as riots, wars, and the like. Possible factors in addition to mere physical force which may determine the outcome of such conflicts are investigated.

Relations are also studied between two classes of population, of which one is characterized by a much greater ability to organize and supervise the productive activities of the other. Under some special and rather simple assumptions, it is shown that this kind of interaction results first in an increase of the ratio of the cumulative results of productive activities for the two classes in favor of the first. With time, however, this ratio reaches a maximum and declines. An expression for the "life span" of such organized classes is obtained.

Wednesday, July 10—A New Foundation of the Method of Maximum Likelihood in Statistical Theory, ABRAHAM WALD, Lecturer in Economics, Columbia University.

Let $f(x, \theta)$ be the probability density function of a random variable x involving an unknown parameter θ. Denote by $\hat{\theta}_n$ the maximum-likelihood estimate of θ calculated from a sample of n independent observations on x. It is known that, under certain restrictions on $f(x, \theta)$, $\hat{\theta}_n$ is an efficient estimate of θ, i.e., it satisfies the following two conditions: (1) $\sqrt{n}\,(\hat{\theta}_n-\theta)$ is in the limit normally distributed with zero mean and finite variance; (2) For any sequence $\{t_n\}$ $(n=1, 2, \cdots,$ *ad inf.*) of statistics which satisfies (1) we have

$$\lim_{n=\infty} \sigma_{\sqrt{n}(\hat{\theta}_n-\theta)} \leq \lim_{n=\infty} \sigma_{\sqrt{n}(t_n-\theta)} ,$$

where, for any random variable u, σ_u denotes the standard deviation of u. The justification of the use of the maximum-likelihood estimate has been mainly based on its efficiency. However, the restriction to statistics which are normally distributed in the limit is a serious one. It is conceivable that a statistic t_n exists which is not normally distributed in the limit and such that for any positive ε the probability that $|\sqrt{n}(t_n-\theta)| < \varepsilon$ converges with $n \rightarrow \infty$ to a value greater than the limit value of the probability that $\sqrt{n}(\hat{\theta}_n-\theta)| < \varepsilon$.

In this paper a property of $\hat{\theta}_n$ is given which is stronger than efficiency and which seems to give a satisfactory foundation for the use of the method of maximum likelihood in statistical inference based on large samples. We shall denote a region of the n-dimensional sample space by a capital letter with the subscript n. For any region U_n the symbol $P(U_n \mid \theta)$ denotes the probability that the sample point will fall in U_n under the assumption that θ is the true value of the parameter. For any region U_n we denote by $g(U_n)$ the greatest lower bound of $P(U_n \mid \theta)$. For any pair of regions U_n, T_n we denote by $L(U_n, T_n)$ the least upper bound of $P(U_n \mid \theta) - P(T_n \mid \theta)$.

Definition I. A sequence $\{W_n\}$ $(n=1, 2, \cdots,$ *ad inf.*) of regions is said to be an asymptotically most powerful test of the hypothesis $\theta=\theta_0$ on the level of significance α if $P(W_n \mid \theta_0) = \alpha$ and if for any sequence $\{Z_n\}$ of regions for which $P(Z_n \mid \theta_0) = \alpha$ we have

$$\limsup_{n\rightarrow\infty} L(Z_n, W_n) = 0 .$$

Definition II. A sequence $\{W_n\}$ $(n=1, 2, \cdots,$ *ad inf.*) of regions is said to be an asymptotically most powerful unbiased test of the hypothesis $\theta=\theta_0$ on the level of significance α if

$P(W_n \mid \theta_0) = \lim_{n=\infty} g(W_n) = \alpha$, and if for any sequence $\{Z_n\}$ of regions for which

$$P(Z_n \mid \theta_0) = \lim_{n=\infty} g(Z_n) = \alpha ,$$

we have

$$\limsup_{n\to\infty} L(Z_n , W_n) = 0 .$$

Let W'_n be the region defined by the inequality $\sqrt{n}(\hat{\theta}_n - \theta_0) \geq c'_n$, W''_n defined by $\sqrt{n}(\hat{\theta}_n - \theta) \leq c''_n$, and let W_n be defined by $|\sqrt{n}(\hat{\theta}_n - \theta_0)| \geq c_n$. The constants c'_n, c''_n, and c_n are chosen so that

$$P(W'_n \mid \theta_0) = P(W''_n \mid \theta_0) = P(W_n \mid \theta_0) = \alpha .$$

It has been shown that under certain restrictions on $f(x, \theta)$ the sequence $\{W'_n\}$ is an asymptotically most powerful test of the hypothesis $\theta = \theta_0$ if θ takes only values $\geq \theta_0$. Similarly $\{W''_n\}$ is an asymptotically most powerful test if θ takes only values $\leq \theta_0$. Finally $\{W_n\}$ is an asymptotically most powerful unbiased test if θ can take any real value.

On the basis of the above results it can also be shown that in large samples the confidence interval for estimating θ derived from the maximum-likelihood estimate will be "shortest" in the sense of Neyman's[1] theory of confidence intervals.

Wednesday, July 10—Telephone Trunking: A Problem in Economics, EDWARD C. MOLINA, Switching Theory Engineer, Bell Telephone Laboratories.

A telephone company is building two central-office exchanges in a certain city. How many trunk lines should the engineer specify to handle the anticipated traffic from one exchange to the other?

If this were merely a problem in physics two alternative

[1] J. Neyman, "Outline of a Theory of Statistical Estimation Based on the Classical Theory of Probability," *Phil. Trans. Roy. Soc. London*, Series A, Vol. 236, 1937, pp. 333–380.

solutions would suggest themselves; the minimum of one trunk or a maximum of, say, 2000 trunks which would make it possible for all members of the subscribers of one exchange to talk simultaneously with their friends in the other.

The obvious absurdity of these extreme solutions indicates that the engineer is interested in something more than pure physics. He must consider the *economics* of the situation confronting him. The real problem is: How many trunks should be installed in order that *adequate service be given at a price which will yield a fair return on the investment?*

A few comments on typical curves showing traffic fluctuations during the 24-hour day and during the so-called "busy hour" precede a short discussion of a simple "straight multiple" trunking problem. This discussion introduces the mathematical theory of probability and the solution of the simple problem is obtained in terms of the incomplete binomial summation

$$\sum_{r=c}^{r=n} \binom{n}{r} p^r(1-p)^{n-r} ,$$

where c is the number of trunks serving a group of $(n+1)$ subscribers and p is the proportion of time an individual subscriber converses during the busy hour. For certain ranges of values of the variables involved in the trunking problem recourse may be had to Poisson's exponential limit to the binomial, namely,

$$\sum_{r=c}^{r=\infty} \frac{(a^r e^{-a})}{(r!)} , \qquad a = np .$$

A Poisson "efficiency" table giving the average number of simultaneous conversations which can be cared for per trunk in terms of the number of trunks forming a group is presented. The bearing of such a table on the economic design of a telephone switching system is emphasized by some historical comments on the evolution of the dial system in use in our larger cities.

The earlier systems were such that the automatic switching apparatus functioned on the same numerical basis as that in accordance with which subscribers' numbers were designated. Today the automatic switches function on a nondecimal basis which capitalizes the greater efficiencies of large groups of trunks. But the subscribers are not deprived of the privilege of dialing decimally. A "translation" scheme takes care of the transition from one scale of enumeration to the other.

Without attempting to explain their forms, the more complex trunking arrangements confronting the engineer, such as the "graded," "random slip," "primary and secondary trunking," are mentioned, together with the mathematical tools to which recourse must be had: Laplacian generating functions, Erlang statistical equilibrium methods, complex function theory, and the like.

Thursday, July 11—The Probability Theory of Compatible Events, HILDA GEIRINGER, Lecturer in Statistics and Probability, Bryn Mawr College.

The classical Bernoulli-Poisson problem of *repeated trials* considers n independent trials with the probabilities of "success" equal $p_\nu (\nu = 1, \cdots, n)$. We ask for the probability $P(x)$ of exactly x successes and $n—x$ failures, or for obtaining x times the result "1" and $n—x$ times the result "0"; thus $P(x)$ is the probability *distribution of the sum* x.

Laplace was the first to consider the asymptotic behavior of $P(x)$. Introducing the *mean value* a_n, the *variance* s_n^2, and the *cumulative* distribution $V(y) = \sum_{x \leqq y} P(x)$, he found that

$$\text{(1)} \qquad \lim_{n\to\infty} V(a_n + y s_n \sqrt{2}) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{y} e^{-x^2}\, dx \equiv \phi(y).$$

But if, as n tends towards infinity, a_n tends toward a *finite* value a, we obtain the *Poisson* formula

$$\text{(2)} \qquad \lim_{n\to\infty} P(x) = \frac{a^x}{x!} e^{-a} \equiv \psi(x).$$

Far-reaching generalizations of these problems have been dealt with. Instead of *n alternatives* we consider n fairly general probability distributions $V_1(x), \cdots, V_n(x)$ where $V_\nu(x)$ indicates the probability of getting a result $\leqq x$, $(-\infty < x < +\infty)$. If, as before, $V(x)$ gives the probability that the sum of the n results is $\leqq x$, it has been shown, in a series of valuable contributions (Tchebycheff, Markoff, Liapounoff, P. Lévy, Feller, and others), that, under very general conditions, $V(x)$ converges toward a *normal distribution* ("Central Limit Theorem").

In another very important generalization of our problem rather arbitrary statistics, i.e., a certain general type of functions of the n results $F(x_1, \cdots, x_n)$, so-called *"statistical functions,"* have been introduced instead of the sum $(x_1 + x_2 + \cdots + x_n)$ of these results. If $V(x)$ denotes the probability that $F(x_1, \cdots, x_n) \leqq x$, then $V(x)$ converges (of course under certain restrictions) towards a normal distribution (von Mises).

In all these investigations the n trials are assumed as *mutually independent.* An important problem, *leaving this assumption out,* is given by the "Markoff chains." Here the probability distribution $V_\nu(x)\,(1 < \nu < n)$ depends—in the most simple case—upon the result of the $(\nu-1)$th trial.

In another conception of dependence ("arbitrarily linked events"), p_ν denotes the probability of success in the νth of n trials, $p_{\mu\nu}$ that of success both in the μth and νth trials, $\cdots p_{12\cdots n}$ that of success in all the n trials. Denoting by $P(x)$ the probability of x successes and by $V(x)$ the probability of $\leqq x$ successes within n trials, the author has investigated theorems, generalizing the results of Poisson and Laplace: If a_n and s_n^2 are the mean value and the variance of $P(x)$ we can indicate sufficient conditions for the validity of (1) and (2).

The conception of *arbitrarily linked events* is convenient in problems where the *order* of the n trials does not matter, and where the $p_\nu, p_{\nu\mu}, \cdots$ are the data given by the very nature of the problem. In other problems (like that of the "chains") the introduction of the $p_\nu, p_{\nu\mu}, \cdots$ would be artificial and other groups of probabilities have to be considered as the given data of the problem (cf. S. Bernstein, P. Lévy, and others.)

A very general problem, which comprises all those quoted above, is the following: Let $V(x_1, x_2, \cdots, x_n)$ $(-\infty < x_\nu < +\infty)$ be the (n-dimensional) probability of the first results being $\leqq x_1$, the second $\leqq x_2$, the nth $\leqq x_n$. We perform n trials and ask for the probability $V(x)$ that a "statistical function" $F(x_1, \cdots, x_n)$ of the n results might be $\leqq x$; the study of the asymptotic behaviour of $V(x)$ is a problem which is far from being accomplished. But a less difficult question which forms a particular case of this problem deals with the different forms of the so-called *law of large numbers*. This theorem, in its original form, asserts that (under general conditions) it is to be expected with a probability converging toward one, that the *observed* average $(x_1 + \cdots + x_n)/n$ differs from its *theoretical* value $(p_1 + \cdots + p_n)/n$ by as little as we please if n is sufficiently large. Thus the law of large numbers does not settle the exact asymptotic behaviour of $V(x)$; it states merely a particular feature of it, though the most fundamental one. Now this law of large numbers can be extended to the case indicated above, where the n distributions are *linked* in the most general manner and where we consider, instead of the sum, a *statistical function* of the n results. To use a modern term we can say that—under very general assumptions—a statistical function $F(x_1, \cdots, x_n)$ "converges stochastically" toward its "theoretical value." Some of the above-quoted new results may be of a certain importance for current problems of statistics.

Thursday, July 11—Sampling Problems of the 1940 Census, W. EDWARDS DEMING, Mathematical Adviser, Bureau of the Census, United States Department of Commerce.

1. Introduction. The preliminary planning for taking a part of the census by sampling methods, involving decisions concerning the information to be asked on the supplementary schedule, the sampling ratio, and the style of schedule to be used, were carefully thought out over a period of many months by Drs. Truesdell and Hauser in conferences with other members of the census staff and with commitees from other interested government and private organizations. The solution of the relatively simpler problem of choosing the "sampling line numbers" on the population schedule is to be discussed in this paper, and the solution that I shall present is the result of a joint study carried on mainly by Messrs. F. F. Stephan and Morris Hansen, assisted by the author. Since for administrative reasons it is not feasible to attempt a random selection of sampling line numbers in the field, they must be designated in advance.

As a background it is desirable to have in mind the population schedule used by the enumerators. This is a so-called "line schedule"; each line contains a name and the information associated with it, and there are 40 lines on each side of a sheet. Out of every 40 lines, 2 are designated "supplementary," which means that ordinarily, out of 40 people enumerated consecutively, 2 are included in the sample. For them, information is obtained not only for the main schedule, but also for certain additional supplementary questions. It will be observed that the sample is highly stratified.

For tabulation we may think of two sets of cards, A and B. There is an A card for every person enumerated, and it contains the information on the main schedule. There is a B card for every person in the sample, and it contains not only most of the information from the main schedule, but also the supplementary information for that person. The sample serves three purposes: (i) the B cards, being only 1/20th as numerous as the A cards, can be cross-tabulated at less expense, thus permitting more studies to be carried out with the same funds than would be possible with the A cards; (ii) they can be tabulated quicker, permitting early estimates, months ahead of the A cards; (iii) they supply the only source of tabulation for the information on the supplementary schedule. Our job is to get a good sample, one that when tabulated will show agreement with the subsequent tabulation of the A cards. In the Shewhart sense of the word, we have agreement when the results of the sample will not lead to action different from that which would be taken on the basis of complete tabulation.

2. Two Kinds of Sampling Errors. There are two kinds of errors in sampling, i.e., there are two main sources of disagreement between the sample and a complete count, and these can be classed as follows—

I. Process biases.
  (a) Order of enumeration in blocks and within households.
  (b) Varying sizes of the enumeration districts.
  (c) Miscellaneous variation in the distribution curves of characters plotted against line number.

II. Statistical fluctuations.

The latter are easy to control—we need only refuse to cross-tabulate the B cards to a point where they get too thin. On the basis of experimental runs with the 1940 sampling scheme applied to the census of 1930 and the Special Census of Indiana in 1939, it is clear that for studying patterns in cross classifications we may deal with as low as 250 persons per cell, but that for estimates of proportions depending on a single cell alone, we require about 1000.

3. Bias Arising from the Order of Enumeration. Studies conducted with the object of learning how such household characteristics as rent, value of home, number of children, number of lodgers, etc., vary from one household to another in city blocks, have given quantitative measures of corner influence and the bias that would result from an overrepresentation of corner houses in the sample. Within any one household there is, moreover, a variation in such personal characteristics as age, sex, education, etc., from one member to another. With the sampling line numbers fixed in advance, a specified order of enumeration is thus apt to introduce a bias. But sample or no sample, an enumerator must be systematic; he must commence at a corner house and work around the block; and within a household the members must be listed systematically — head, wife, oldest child, etc. At a corner, the corner-influence and order-of-enumeration biases therefore operate in phase: line 1 of an enumerator's first schedule always lists a head—more than that, the head of a corner house if the corner is occupied; line 2 lists the wife (except in the 8 per cent of households occupied by one person), etc. Now heads of corner houses deserve to be represented in the sample, but they do not represent the population; and similar remarks hold for the persons on lines 2, 3, 4, etc.

If it had been feasible to use 20 styles of the schedule, the problem of giving each line its 1-in-20 representation would have been simple; but to keep the number of styles at a minimum and still get a representative sample is another matter. A satisfactory solution was worked out with five styles (see table). Lines 1-6 (the lines of greatest variability) get their 1-in-20 representation through the W, X, Y, and Z styles, each of which is used in 1 out of 20 enumeration districts. The other lines in these styles take care of secondary effects at the middle and end of a sheet, arising from the possibility (it turned out to be a reality) that some enumerators would misunderstand the instructions and fail to use the bottom lines, preferring to commence a new family or new block at the top a sheet (lines 1 or 41).

Calculations based on corner influence, and on tables prepared to show what proportions of the heads, wives, etc., of the 1st, 2d, 3d, etc., family will fall on various lines of the population schedule, showed that line 14, if used in 16 out of 20 enumeration districts, would balance the characteristics obtained on lines 1-6 in the other 4 districts, and this accounts for the presence of line 14.

4. Bias Arising from Different Sizes of Enumeration Districts and from Miscellaneous Variation. When the enumerators' records for an entire city or state are pooled, and the total frequency of some characteristic, or ratio, or other function of frequencies is plotted against line number x, it is found that after the line numbers of styles W, X, Y, Z have been deleted, a cubic in x will give a good fit. By a scheme that I choose to call the Stephan method of unbiased abscissas, it is possible to find three companion lines to line 14 in the V style, such that no matter what be the shape of this cubic, the four lines in style V will give the average ordinate under the cubic. These companion lines turned out to be 29, 55, and 68, as seen below. The line numbers of the five styles, combined with a distribution system that preserved the 16:1:1:1:1 ratios within counties, cities, and states, eliminate the process biases and should give a sample representative of the population in the characteristics that are to be studied.

TABLE OF LINE NUMBERS FOR SAMPLE

Style	V	W	X	Y	Z
Proportion	16	1	1	1	1
Line numbers	14	1	2	3	4
	29	5	6	39	40
	55	41	42	44	46
	68	75	77	79	80

Friday, July 12—Some Results of an Empirical Study of the General Equilibrium, WASSILY LEONTIEF, Associate Professor of Economics, Harvard University.

This paper presented the results of the author's study of interrelationships of American industries in the years 1919 and 1929. The analysis is based on a simplified theory of general equilibrium (interdependence). Factual data are obtained from a detailed statistical table of interindustrial and household payments. Price changes and output variations of individual industries are computed on the basis of various "primary changes."[1]

Friday, July 12—The Stability of Equilibrium, PAUL A. SAMUELSON, Society of Fellows, Harvard University.

It is the task of *comparative statics* to determine the equilibrium values of given variables (unknowns) under postulated conditions (functional relationships) with various data (parameters) being specified. Changes in these parameters result in changes in the equilibrium position. The

[1] A complete analysis of the subject will be presented in the author's forthcoming book, *Structure of American Industries, 1919–1929*, to be published by the Harvard University Press.

fruitfulness of the method of comparative statics depends upon the knowledge it can give of the direction of change of the equilibrium quantities, even though it is *not* concerned with the period of transition or the path from one equilibrium state to another.

Too often the older economists neglected to pluck the fruit before them and contented themselves with verifying the determinateness of equilibrium by counting variables and independent, consistent equations. While no doubt a necessary preliminary step, such a procedure stops short of the derivation of *operationally meaningful* theorems. Only by an examination of the quantitative and qualitative properties of the equilibrium equations can useful information concerning the direction of change of the equilibrium position be derived.

In the absence of precise, detailed, empirical quantitative information—such as we do not have nor are likely soon to have—our knowledge of the structural properties of the economic equilibrium functions is usually derived from two somewhat distinct sets of conditions: (1) maximum conditions within the firm or household; (2) stability conditions relating to the interaction between economic units. Although inadequately explored until comparatively recently, the first set of conditions are best known and are not separately discussed here; from certain points of view, however, they can be fitted in as special cases of the second set.

Upon examination it will be found that the definition of stability of a comparative statical system rests implicitly upon the introduction of dynamical considerations, however rudimentary they may be. We are faced with this paradox: *In order for the analysis of comparative statics to be useful, it must be generalized to include the determination of the dynamical path of the system in and out of equilibrium.*

As an example one may consider the determination of price (P) and quantity (Q) in a particular-equilibrium commodity or service market. The two unknowns are determined by the interaction of supply and demand. By itself this does not tell us how the system will behave if (say) demand increases. If any of these well-known alternative dynamical systems are assumed, however, the following nonequivalent propositions become true (at least in the small):

(1) Price must always rise; quantity may fall or may rise by any amount.

(2) Quantity must always increase; price may fall or may rise by any amount.

(3) Price must always rise; quantity may fall or rise although it cannot rise by as much as it would from an "equivalent" increase in supply.

The first conditions arise from the Walrasian stability conditions in contrast (*sic*) to the second or so-called Marshallian stability conditions. The third represent the cobweb phenomenon. Each is derived respectively from the following assumed dynamical systems:

$$(1) \qquad \frac{dP}{dt} = (a - b)\, P + \cdots ,$$

$$(2) \qquad \frac{dQ}{dt} = \left(\frac{1}{a} - \frac{1}{b}\right) Q + \cdots ,$$

$$(3) \qquad Q_t = \frac{b}{a}\, Q_{t-1} + \cdots ,$$

where a = slope of demand curve at equilibrium point referred to the price axis, and b = slope of supply curve at equilibrium point referred to the price axis.

One could easily multiply *ad infinitum* analogous alternative dynamical assumptions and corresponding stability conditions. More generally, the approach outlined above admits of generalization to systems of the wide type

$$\frac{dx_i}{dt} = f^i(x_1, x_2, \cdots, x_n),$$

or

$$x_i(t+1) = g^i[x_1(t), \cdots, x_n(t)] \qquad (i = 1, \cdots, n),$$

and to still more general systems.

Among the results suggested are: (1) The attempt to unify diverse stability conditions for a single market necessarily must fail. (2) The Marshallian supply curve, forward-falling because of external economies, is not a supply curve in the sense that it defines the amount forthcoming at a given price at which all can sell without limit; it is, however, a supply curve in the sense that it is traced out by shifts in (sufficiently steep) demand schedules. (3) The stability conditions of Hicks (*Value and Capital*, p. 315) do not seem to be either necessary or sufficient under the envisaged dynamical setup. (4) Stability in the dynamical sense guarantees uniqueness in the statical sense, at least in the small. (5) Where dynamical systems are related to ordinary maximum and minimum problems, *symmetry* and *definiteness* in certain quadratic forms are involved, so that stability is guaranteed under broad conditions. (6) Differential equations derived from variation problems display interesting stability properties. (7) The techniques outlined here are of even greater interest when applied to problems of business cycles (e.g., when applied to the Keynes-Lange model).

Monday, July 15—Measuring the Social Performance of Business, THEODORE J. KREPS, Professor of Business Economics, Graduate School of Business, Stanford University, and Economic Adviser, Temporary National Economic Committee.

The *raison d'être* of private business is its ability to achieve certain social objectives. Under competition these were traditionally thought to be achieved automatically. They are uniformly recognized by business men to be valid. The United States Chamber of Commerce and the National Association of Manufacturers have publicly gone on record repeatedly even defining business in social terms. Thus the National Industrial Conference Board in its volume *Studies in Enterprise and Social Progress* states:

> Enterprise may be broadly described as a way of collective life in which the arrangements and processes of making a living are based upon unconscious, voluntary cooperation of individuals in producing, exchanging and consuming the greatest possible amount of the goods and services they want with the least aggregate loss or sacrifice to themselves, through their experimental, competitive efforts to utilize available natural resources and develop potential human capacities.

Six tests have been applied, namely, employment, production, pay rolls, dividends and interest, consumer funds absorbed, and consumer effort commanded. These six tests have been utilized to measure the social performance of 22 individual industries. The results are shown in the accompanying table.

The table gives two sets of ratings, one on the narrow but important basis of three criteria: production, employment, and pay rolls; the other on the broader basis of seven criteria.[1] In terms of the first, chemicals and canning and preserving stand clearly at the top of the list. Petroleum refining, knit goods, agricultural implements, baking and confectionery, and electric light and power belong in the next highest group. Paper and pulp, slaughtering and meat packing, oil and gas producing, boots and shoes, woolen and worsted manufacturing, iron and steel, and automobiles belong in the average-to-medium-lower class. Tobacco manufacturing, cotton goods,

Industry	Composite Rating Based on Three Criteria			
	Cumulative Increment	Rating	Cumulative Rank	Rating
Baking and Confectionery	+ 3.15	6	21	6
Knit Goods	+ 4.02	4	18	5
Canning and Preserving	+10.19	1	5.5	2
Agricultural Implements	+ 3.56	5	17.5	4
Petroleum Refining	+ 5.40	3	15	3
Automobile	— 1.95	14	38	14
Boot and Shoe	+ 0.47	11	32	11
Slaughtering and Meat Packing	+ 1.36	9	25	7
Cotton Goods	— 3.26	16	44	15
Woolen and Worsted Goods	— 1.30	12	35	13
Chemical	+10.03	2	4	1
Electric Light and Power	+ 2.83	7	26	8,9
Iron and Steel	— 1.52	13	33	12
Paper and Pulp	+ 1.64	8	26	8,9
Flour and Other Grain Mill Products	— 4.43	17	47	17
Oil and Gas Producing	+ 1.18	10	31	10
Furniture	— 9.99	20	61	20
Metal Mining	— 5.93	18	52	18
Tobacco	— 2.47	15	45	16
Coal Mining	— 7.00	19	56	19
Lumber and Timber Products	—11.36	21	64	22
Railroad	—11.55	22	63	21

[1] Because of the limitations of space, only the ratings based on three criteria are shown in the table herewith.

flour and grain milling come in the definitely-below-average category while metal mining, corn milling, furniture making, manufacture of lumber and timber products, and steam railroads are conspicuously low in performance.

On the basis of seven criteria there are one or two notable shifts, chemicals in particular going down from first to tenth place. But the five industries at the bottom of the list are identical.

Concerning these measurements the fact should be noted that social performance shows no correlation with the amount of government regulation or the durability of product or the nature of the market. Nor does such a general factor as confidence afford an explanation for such extraordinary differences in behavior among the individual industries.

The usefulness of such measures is seriously limited at present by the lack of comparable data. It is also limited by the fact that in the main only general figures are available covering the nation as a whole. Obviously detailed figures are necessary giving performance by type of product, by geographical area, by company, and so forth.

The importance of such measurements is their potential usefulness to a Bureau of Industrial Economics. If suitably developed, they might furnish economic tests of superior utility in the enforcement of antitrust legislation to the present excessive reliance on the legal doctrine of conspiracy. If industries are steadily creating more employment, producing more product at lower prices, and increasing their disbursements in the form of pay rolls, such facts as the number of producers, whether one, few, or many, or the meeting of minds to determine sales, prices, and production, become of minor significance, if not irrelevant.

Monday, July 15—"Wage Stop," "Deferred Pay," and the Traditional Methods of War Finance, HORST MENDERSHAUSEN, Instructor in Economics, Colorado College (now Research Associate, National Bureau of Economic Research).

Money is not the sinews of war. Contemplating the treasures of King Croesus of Lydia, the Athenian statesman Solon remarked poignantly that he could not consider Croesus powerful on that ground; somebody having more iron might come and take his gold away. The real sinews of war are man power, raw materials, industrial equipment, and an efficient organization. War finance is instrumental for the most effective mobilization of these resources.

War finance has two tasks: (1) to divert a considerable portion of purchasing power from individual consumption and private investment to the acquisition of war materials by the government; and (2) to distribute the economic burden of war over the various social classes in some justifiable manner.

The traditional methods of war finance are taxation, borrowing, and inflation. Direct taxes, such as our income taxes, make a rational distribution of the burden possible. Each individual is hit by them according to his capacity to pay. Indirect taxes (customs and excise duties) provide a less transparent distribution of the burden. Financing the war from taxes helps to acquaint people with the fundamental truth that the bulk of the war burden is borne by the generation waging the war. This generation has to make most of the sacrifices. People who like to deceive themselves over the meaning of war finance may oppose taxation on that ground. There are, however, also reasonable objections. At the beginning of the war taxation may be too slow in bringing in sufficient revenue; it is difficult by means of direct taxation to reduce the consumption of the lower income classes sufficiently.

In the past, loan finance has played a larger role than taxation, mainly because governments were afraid to put a definite burden on the people. In general, borrowing does not make it possible for the generation waging the war to shift the burden to future generations; but it renders the distribution of the burden less evident. War loans open up a period of protracted haggling about how to levy the taxes necessary to service the war bonds. Whether the original subscribers to the bonds will carry the burden or whether they will be able to shift it to other persons of their generation depends on how these taxes are levied and on whether the government does not decide to default on its debt. However, loans are a way of raising revenue quickly, particularly from the wealthier classes.

Currency inflation is the least equitable way of war finance. In wartime the nation's resources are likely to be fully employed after a short period of transition. If then more money is put into circulation prices will rise. Hence, all people living on fixed incomes such as salaries, annuities, and interest from capital will suffer. Inflation may take place by issuing additional paper money or by allowing an expansion of bank credit. In the World War loans and inflation were the predominant instruments of finance.

In the present war a few new techniques have been used or proposed. They aim mainly at the avoidance of inflation and a stronger reduction of the purchasing power of the lower income classes.

In Germany a general wage stop has been decreed. While workers are compelled to produce more, their incomes are kept constant. In addition, consumers' goods have been rationed and their prices fixed. Hence many people find themselves with money on hand which they cannot spend on consumption goods. An increase of "voluntary" saving ensues. Simultaneously, employers' profits are taxed away or invested under government supervision. Thus the purchasing power of the lower income classes is kept down, their income is partly pumped upward to corporations and wealthier individuals and then taken care of by the government.

In England, Professor Keynes proposed an extension of the income tax to the lower income brackets by means of a system of "deferred pay." A proportion of income rising (with income) from 3.5 to 85 per cent is proposed to be levied. For the lower income classes all of the levy is considered as compulsory saving, not as a tax. Their payments are placed in blocked savings accounts. Except in cases of emergency people cannot withdraw these savings; they are at the disposition of the government. As income rises the percentage of deferred pay declines and that of the income tax proper rises. This system makes it politically easier to restrict the purchasing power of the lower income classes. It is particularly suitable for a democratic society. Furthermore, it can be used for bringing about a more equal distribution of incomes after the war. Finally the freeing of the blocked accounts after the war may be timed in such a way as to help combat an eventual postwar slump.

A difficulty in the operation of such a system as Keynes proposes is to prevent people from compensating for their compulsory savings by decreasing their voluntary savings, or by drawing down their savings accounts. So far the system has not found application in England.

Tuesday, July 16—Economic Developments in the Northern Countries since 1930, KAARE PETERSEN, Research Associate, Central Statistical Bureau of Norway.

The Northern Countries, Norway, Sweden, and Finland, are all small countries. They are dependent on an extensive foreign trade, and their exports are of a highly specialized nature. In Finland more than 80 per cent of the exports consist of wood products; in Sweden 40-50 per cent of the exports are wood products; and in Norway, where the export trade is of a less specialized nature, 25-30 per cent are wood products.

As the Northern Countries are all so dependent on their foreign trade, naturally economic developments in those countries are closely bound to the developments in their customer countries. The main features of the depression and business revival following the crisis of 1929 are well known to all students of economic theory. The great depression was followed by an almost complete change in the economic policy in the leading European countries. Self-sufficiency has been one of the aims of the new commercial policy. Consequently the nature of international trade has undergone a rapid change under pressure of restrictions on trade. The tendency to build up key industries in a protected home market has been one of the outstanding features in several of the more important countries of Europe. This policy of course greatly affected world trade, and the final result was a steep decline in imports and exports. The Northern Countries were most vulnerable to this commercial policy, on account of their extensive foreign trade, and it should therefore be expected that the foreign trade of Norway, Sweden, and Finland would be more severely affected by this restrictive trade policy than the other countries in Europe. The depressing effect of modern commercial policy should easily be traceable in the economic system of the Northern Countries.

The fact is however, that the economic development in the Northern Countries has been far more favorable than in any other country in Europe or in America. In Norway the index of industrial production went up from 94 in 1930 to 120 in 1937. In Sweden the figures were 83 and 121, and in Finland 80 (in 1929) and 124. In England on the other hand production went up from 94 (in 1929) to 116 in 1937. In France there was an 18-per-cent decline in the same period, and in United States the index fell from 132 (in 1929) to 122 in 1937. The same main features can be found in the foreign-trade figures. While the volume of the world trade fell 3 per cent, the volume of the foreign trade of Norway rose 35 per cent from 1929 to 1937, the exports and imports of Sweden rose about 40 per cent, and in Finland there has been a 50-per-cent rise. In view of the restrictive trade policy followed in Europe during the period, these figures are most astonishing.

The foreign-exchange policy in Norway, Sweden, and Finland has been of great importance as a stimulating factor. In 1931 when the English pound went off gold, the situation in the Northern Countries became one of extreme gravity. England was one of the main export markets of the Northern Countries, and maintaining the gold standard after England had adopted a paper standard could not have been done without adjusting prices and wages to a much lower level. Such an adjustment would no doubt have been followed by unemployment and labor unrest. The only thing for the Northern

Countries to do, therefore, was to follow the pound sterling. This later proved to be a very fortunate step. While the prices in the gold countries fell during 1932 and 1933, the prices in the sterling-bloc countries were firm or moved upward. The undervaluation of the currency in the Northern Countries after 1933 helped them to maintain their foreign trade, and even develop new markets.

Beside the monetary policy we may regard the international economic trend and its effect on exports as being the most important factor in the economic revival of the Northern Countries. The financial policy of Norway, but especially that of Sweden, has been credited with turning the economic tide in the years after 1931. The financial policy of Sweden has no doubt been of great significance. We must remember, however, that the upswing started in 1932, while the new financial program was put into action as late as 1934. An inquiry leads up to the conclusion that the Swedish unemployment policy and new financial policy have played a secondary role only, in bringing about the upturn in 1932-33. In Norway, a new financial policy was put into action by the government in 1935-1936. At that time however, Norway was well out of the economic depression.

Tuesday, July 16—The Use of Weighted Regressions in the Analysis of Economic Series, JOHN H. SMITH, Research Associate, Cowles Commission.

When the relationships among a set of economic variables are to be determined statistically, one approach to the problem of estimation is to apply the method of maximum likelihood on the basis of a simple statistical specification. In all cases considered the k observed series constitute a closed set, except for an independent normally distributed random disturbance or error component in each variable, and all relations are linear. Variables are expressed in deviations from the mean in units proportional to the standard deviations of their respective disturbances. Coefficients of the homogeneous linear equations which describe the relationships among the systematic parts are then estimated by the method of maximum likelihood. Correlated errors are considered as effects of neglected variables and it is shown that in such cases it is impossible to set limits to the value of any regression coefficient which would exist in the completed set of variables.

The case in which the systematic parts of the k observed series satisfy k—1 linear relationships is an extreme case of multicollinearity in which these relationships determine a line in k-space. Maximum-likelihood estimates of the direction cosines of this line are proportional to the elements of any row or column of the adjoint of the characteristic moment matrix based on the largest characteristic root. These can be obtained most easily by applying Hotelling's iteration method to the moment matrix, with units chosen as above, instead of to the correlation matrix as in Hotelling's method of principal components.

When the systematic parts satisfy only one linear relationship, which is called the weighted regression, maximum-likelihood estimates of the coefficients are proportional to the elements of any row or column of the adjoint of the characteristic moment matrix based on the smallest characteristic root. Numbers proportional to these coefficients are most easily obtained by applying Hotelling's iteration method to the moment matrix after first subtracting from each diagonal entry a number greater than half the sum of the largest and the smallest characteristic roots.

In order to be able to make an intelligent choice among available statistical techniques, it is necessary to have solutions for problems of estimation in connection with simple specifications. The mere presence of "errors" in all variables is not sufficient justification for the use of weighted regressions in preference to ordinary elementary regressions. The nature of economic problems which may profitably be identified with the estimation of parameters in weighted regressions should be carefully studied before making such a choice. In general it would seem that the relevant relationship is that which expresses the variable in which changes are to be predicted as a function of controllable or observable components of related series. This is true even when the relationship is better described as a probability distribution than as a single-valued function.

Sampling variance of weighted regression coefficients are infinite when the systematic parts satisfy two or more linear relationships. This fact has often been cited by those who argue that the t-test is invalid in such cases. Such an argument is not good evidence, however, because it can be shown that multicollinearity among the systematic parts is not sufficient in itself to change the sampling distributions of the usual test criteria. Thus the techniques of confluence analysis which serve a useful purpose in the study of multiple relationships should be sharply distinguished from tests of significance.

Like all other scientific constructs, statistical specifications must always be more or less unrealistic. In the case of economic time series, attempts to make more realistic specifications have often resulted in specifying important problems out of existence. Statistical tests are designed to be useful in the problem of making a choice among a certain set of alternative hypotheses. To the extent that the set of hypotheses among which the test is an effective aid in making a choice does not coincide with the set of hypotheses which are plausible alternatives to the one which is tested, the statistical test is ineffective and often misleading. In the attempt to devise specifications consistent with serial correlation, some statisticians specify that only the deviations from regression are random. To an economist whose alternative hypotheses are chiefly concerned with changes in relationships among components identified statistically as systematic parts, the exact test of significance based on such a specification is practically useless.

Wednesday, July 17—The Velocity of Circulation of Money, IRVING FISHER, Professor Emeritus of Economics, Yale University.

The question of whether the velocity of circulation of money is a constant or a variable is of great importance both for monetary theory and monetary policy.

As to theory, if velocity is substantially constant, the so-called quantity theory of money has very definite validity; for then, according to what I have called the "equation of exchange," an increase in the quantity of the circulating medium can not express itself by a change in the velocity of the circulation of money, as has sometimes been claimed, but must affect the price level or the volume of trade, or both.

So far as monetary policy is concerned, if the velocity of circulation of money is simply a cushion for changes of quantity, any attempt to control the price level or volume of trade by controlling money would be futile.

R. H. Hemphill, formerly credit manager of the Federal Reserve Bank of Atlanta, has criticized the present Board of Governors of the Federal Reserve System and especially its chairman, Mr. Eccles, for maintaining that monetary control is almost futile because of velocity variations, although the board took office with the idea of monetary control uppermost. Mr. Hemphill claims that the statistics of velocity are so inaccurate that the conclusions of Mr. Eccles are incorrect.

Apparently the first figure offered for the velocity of circulation of money—which was for "pocket-book money," was one based on turnover of cash by Yale students in my classes. The figure found was 45 times a year which undoubtedly is much too high owing to the fact that the students were not representative of the country as a whole and that the larger the purse of an individual the more rapid his turnover. Moreover the figures were too meager to be very reliable. Afterwards, Professor Kemmerer obtained similar results; likewise now known to be too high. Still later, with the aid of certain statistics for one day in 1896 and another in 1909 which had been worked out at the office of the Comptroller of the Currency and a special study of the Aldrich Monetary Commission and by means of a new formula, a series of figures for velocity between 1896 and 1909 was worked out by me and published in the *Purchasing Power of Money* showing the velocity to be between 18 and 22. These are believed to be at least approximately correct.

Apparently no other statistics for velocity of pocket-book money have been offered up to date.

On the other hand, for "check-book money" that is, demand deposits, figures for velocity have been increasingly abundant.

The first known figures are those of Pierre des Essars who in 1895 published the velocity of circulation of the demand deposits of special banks namely, the state banks of France, Germany, Belgium, Italy, Spain, Portugal, and Greece. He found enormous differences between the various banks but less important differences from time to time for any one bank. The figures were for the years 1884-1894 inclusive. He found that in "crisis" years the velocity at the Bank of France was at a maximum and in "liquidation" years, at a minimum; these years being selected according to those of Juglar. The results are similar to those found later in this country.

The next studies in this line were those of Professor Kemmerer in his book *Money and Prices* in 1907 and the next in my *Purchasing Power of Money* in 1911.

The next, and far more comprehensive studies, were those made by the Federal Reserve Bank of New York, and showed a velocity in the neighborhood of 30 times a year for the country as a whole. W. R. Burgess reported these findings in the *Quarterly Publications of the American Statistical Association*, 1922. He found great differences between different localities, some seasonality, and other interesting relations.

Carl Snyder, also of the Federal Reserve Bank of New York, made important contributions and showed that there was apparently no trend in velocity. He also showed that velocity varies directly with trade and that, consequently, the price level corresponds to the money supply even more than if velocity were perfectly constant. James W. Angell of Columbia University and Harold T. Davis of Northwestern University also made important contributions.

Careful study shows that comparison between predepression and the postdepression velocities exaggerates the apparent drop in velocity because of the differences in the methods of collecting statistics. Up to the stock-market crash in 1929, many of our demand deposits were misclassified as time deposits, to take advantage of the lower requirement for reserve, while after the "bank holiday" in 1933 such misclassification was no longer tolerated. The sudden drop between these two dates, October, 1929, and March, 1933, is largely due to the fact that banks were mending their ways on this subject.

There was also a similar change in practice in regard to "deferred credit." Prior to the stock-market crash, it was common to give immediate credit on all checks deposited despite the good intentions of the banks to require a waiting period during which the check travelled from the bank in which it was deposited to the one on which it was drawn and word of its receipt was sent back. This rule of deferred credit is now much more generally enforced.

There are other reasons of a somewhat similar nature to explain the drop between 1929 and 1933 which was probably much more apparent than real. If we had comparable statistics and accurate statistics, there are strong indications that we would find a greater degree of stability in velocity particularly if speculative transactions were eliminated. In speculative transactions whether in securities, wheat, cotton, or other commodities, the turnover is many times as fast as in any ordinary checking account, and for this reason very little money is required to finance these speculative accounts.

The final conclusion is that velocity of circulation differs between individuals and banks enormously, but is fairly constant for nonspeculative accounts. Consequently a control of the quantity of money would not be futile through any inverse change in velocity.

Wednesday, July 17—The Problem of Testing Economic Theories by Means of Passive Observations, TRYGVE HAAVELMO, Research Associate, The University Institute of Economics, Oslo, Norway.

Suppose that, in order to "explain" a certain group of economic variables, we construct a system of hypothetical equations between these variables, the equations being of specified *form* but containing a certain set of unspecified *parameters,* say $\alpha_1, \alpha_2, \cdots, \alpha_k$, or for shortness α, where α denotes a point in the parameter space $\alpha_1, \alpha_2, \cdots, \alpha_k$. For any particular parameter point α let $S(\alpha)$ be the set of all value sets of the variables satisfying *all* equations simultaneously, and for each single equation (say No. i) let $S_i(\alpha)$ be the set of all value sets of the variables satisfying *this* equation. $S(\alpha)$ is a subset of any $S_i(\alpha)$. Now suppose our "observation material" consists of the set $S(\alpha^o)$, where α^o is a particular parameter point. The problem is: Can we find from $S(\alpha^o)$ the parameter point α^o ? This problem is equivalent to the following: Is $S(\alpha) \neq S(\alpha^o)$ for any $\alpha \neq \alpha^o$? This depends upon the form of the equation system. In broad economic terms the meaning of the problem is this: Can we measure economic structure relations (e.g., individual indifference surfaces or other "behavioristic" relations) by means of data which satisfy simultaneously a whole network of such relations, i.e., data obtained by a "passive watching of the game" and not by planned experiments? To bring out the main ideas we neglect here the statistical aspect of the problem and deal with exact equation systems.

Let

$$(1) \qquad f_i(x_1, x_2, \cdots, x_m, x_{m+1}, \cdots, x_n; \alpha_1, \alpha_2, \cdots, \alpha_k) = 0, \qquad i = 1, 2, \cdots, m,$$

be a system of m equations between $n \geq m$ variables, x_1, x_2, $\cdots, x_n$, involving k parameters, α_1, $\alpha_2, \cdots, \alpha_k$. And let $S(\alpha^o)$ be the "observation material." Let it be known that, for *all* values of the α's, the system (1) can be solved uniquely with respect to m variables, say the first m ones:

$$(2) \qquad x_i = \phi_i(x_{m+1}, x_{m+2}, \cdots, x_n; \alpha_1, \alpha_2, \cdots, \alpha_k), \qquad i = 1, 2, \cdots, m,$$

and assume that $\frac{\partial \phi_i}{\partial \alpha_j}$ $\quad (i = 1, 2, \cdots, m; \quad j = 1, 2 \cdots, k)$ are continuous functions of the x's and the α's. "All" α's may in practice mean: all a priori possible α's. Still the question of nontrivial necessary *and* sufficient conditions for the uniqueness of the α's with respect to the set $S(\alpha^o)$ is of a delicate nature. However, it is possible to give certain sufficient conditions for a unique solution—at least locally—and certain sufficient conditions for nonexistence of a unique solution, which are of practical value. We shall mention two such ones:

I. If the k sets of m functions each,

$$(3) \qquad \frac{\partial\phi_i}{\partial\alpha_1}, \quad \frac{\partial\phi_i}{\partial\alpha_2}, \quad \cdots, \quad \frac{\partial\phi_i}{\partial\alpha_k}, \qquad i = 1, 2, \cdots, m,$$

are *linearly independent* for *all* values of the α's, i.e., if, whatever be the parameter point α, it is *not* possible to find k numbers $c_1, \cdots, c_k$ independent of the x's and not all zero such that

$$(4) \qquad c_1\frac{\partial\phi_i}{\partial\alpha_1} + c_2\frac{\partial\phi_i}{\partial\alpha_2} + \cdots + c_k\frac{\partial\phi_i}{\partial\alpha_k} \equiv 0, \qquad i = 1, 2, \cdots m,$$

for all values of $x_{m+1}, \cdots, x_n$, *then* there exist at most a certain number of different *isolated* sets of α's, which produce the same set $S(\alpha^o)$.

II. If there exists a positive integer, h, such that, whatever be the α's, a relation of type (4) *holds* among any r of the k sets of functions in (3) when $h > r \geq k$, while there are h sets of functions in (3) for which *no* relation of the type (4) exists, then there are, even in a small neighborhood of α^o, infinitely many sets of α's for which $S(\alpha) \equiv S(\alpha^o)$.

The question of linear dependence of functions may be studied by means of Gramian determinants. This criterion may be extended to functions of several variables.

Analogous rules for systems of functional equations (dynamic systems) were given in the lecture, and also examples.

Thursday, July 18—Britain's International Accounts, 1924-1939, ERICH ROLL, Professor of Economics and Commerce, University College, Hull.

An empirical study of Britain's international economic position during the fifteen years of armistice which preceded the outbreak of the second World War offers many attractions. It should supply material for testing certain current theorems; it should illuminate the final phase in a long trend of development of the metropolis of the world's greatest empire; and it might also produce useful evidence for an analysis of the present position of economic imperialism. This paper provides merely the beginning of such a study.

The period under review is not entirely suitable for the application of the methods used by Taussig, Viner, Graham, Williams, and White. A test of certain aspects of the classical international trade mechanism is not easy to undertake when the number of significant variables is greatly increased and when none of them can legitimately be assumed at rest for the purpose of simplifying the analysis. A mere enumeration of the respects in which this period differed from that examined by Taussig shows the difficulty of a rigorous quantitative and realistic treatment. Moreover, the greater complexity of the period makes adequate statistical material more than ever indispensable. But, as is well known, much information is entirely lacking and much is seriously deficient. This is all the more regrettable, since the theory of international trade has been greatly refined in recent years and since, on a number of points, controversy is still acute. One has only to think of the different views on the effects of capital movements on the terms of trade to realize how helpful the evidence of figures would be.

In this paper, I have, therefore, attempted to survey briefly some of the outstanding features of the period to indicate some of the possible theoretical conclusions that seem to suggest themselves, and to show the directions, statistical and analytical, in which I hope to continue this study.

The paper begins with a survey of the general characteristics of the period and an analysis of the trend which these characteristics reveal. Against this background of uneven economic development, of the increase of monopolistic devices, of the intensification of inter-imperial rivalries, the changes in Britain's international economic position are sketched. A brief summary is given of the developments of merchandise trade in its relation to business fluctuations, intensification of inter-Empire trade, and rearmament. Similarly, the movements of several significant series are described, including capital and gold movements, gold holdings, exchange fluctuations, and others. The trend of Britain's balance of payments

— and with it, the changes in the position of sterling — becomes the most important single symptom of the profound structural changes in Britain's international position. This trend is described and analyzed also in relation to certain disputed points in the theory of international trade. Special attention is also given to changes in the terms of trade during the period. A concluding part touches briefly upon some wartime problems, particularly in the field of exchange control.

Thursday, July 18—Perpetual Turbulence versus Equilibrium, ROBERT W. BURGESS, Statistician, Western Electric Company.

In many economic discussions there is a concentration of interest and argument on what will ultimately be true when abrupt changes no longer disturb the even flow of events and when all the tendencies actually observed, or believed for theoretical reasons to exist, will have reached an equilibrium. The fundamental thesis of this paper is that the mode of action of abrupt changes, and the rate at which each tendency acts before the ultimate state of equilibrium is reached, are also important questions requiring careful analysis and, in fact, in many practical problems, are more important than the ultimate state of equilibrium from the point of view of understanding the real cause-and-effect relationships and the probable effect of proposed changes in laws or customs.

The American economy is characterized by frequent introduction of new inventions and new organizations, the creation of new industries and new companies, and the disappearance of old units. Changes frequently occur in the relative importance of different industries and in the relations between Government and industry. New products and new designs of old products and new methods of manufacture have come in an irregular stream. All of this process of change is properly described as continual rather than continuous; that is to say, it proceeds in hop, skip, and jump fashion rather than just rolling along.

This characteristic of American industry is not fully described by using the word "dynamic" in contrast with "static" because of the fact that many of the changes are discontinuous and abrupt rather than gradual and subject to definite laws expressible by simple mathematical formulas. In the terminology of mathematics, the first and second derivatives are discontinuous and the ordinary procedures of infinitesimal calculus do not apply.

If this description of the economy is accepted as substantially correct, many orthodox economic concepts are unrealistic or inadequate at a number of points. For instance:

1. The idea that in any industry there will be certain concerns which can be recognized to be marginal, that is to say, to earn on the average at the same rate as earned by industry as a whole. Conditions often change sharply before they have been stable long enough to produce an adequate record for selecting marginal companies.

2. The idea that the rate of profit earned in different industries will tend to be the same if conditions are stabilized for a considerable time. This idea is inadequate without some analysis of the strength of this tendency towards uniformity and of the rapidity with which it works.

3. The idea that a manufacturer or dealer can usually forecast pretty accurately what the demand for a particular product will be, and even that he can forecast accurately how the demand will vary as price to the consumer is changed.

4. The idea that the demand for capital goods results largely from growth in population or purchasing power, whereas in the main it actually results from changes, that is to say, the creation of new industries or new or improved products, the economic rise of certain geographic regions at the expense of others, and new inventions and new industrial processes in old industries.

5. The idea that maximization of the excess of income over cost can be used as the fundamental economic principle, ignoring the fact that this principle applies only in the long run and after due allowance for the elements of uncertainty in the business man's forecasts of probable income and costs. Such forecasts are known to be uncertain because of the prevalence of turbulence.

Friday, July 19—Some Uses of Iso-Outlay Curves in Economic Analysis,* FRANCIS MCINTYRE, Assistant Professor of Economics, Stanford University, and Research Associate, Cowles Commission (now Assistant Professor of Economics, Indiana University).

This paper undertakes a summary and appreciation of recent developments in the pure theory of production. Indebtedness to the works of Schneider, Carlson, Hicks, Allen, and Frisch will be noted by all who have read in this field. Considering the production of a single commodity by a single unit of control employing varying quantities of several factors of production, we may suppose the product demand horizontal or negatively inclined, the factor supply horizontal or positively inclined.

Since the generalization for N factors appears merely a simple extension of the two-factor case, diagrammatic simplicity may be had without real loss. If product X is obtained employing variable factors A and B, the total-product surface will yield contour lines whose projections upon the two-factor plane constitute loci of factor-quantity combinations providing equal output. These projections are called isoquants, are always of negative slope, and generally convex toward the origin, though they may be linear (infinite elasticity of substitution) or concave toward the origin ($\sigma < 0$). The total-outlay surface contour-line projections will be loci of factor-quantity combinations providing equal total cost. These loci may be called isocosts or iso-outlay curves. The optimum combination of variable factors is obtained for the production of any given output at the point of tangency of the isoquant for that output with the lowest possible (leftmost) isocost. A line connecting such points of tangency on the several isoquants is called an expansion path, since increases in output will be obtained by increasing the employment of factors A and B along that path.

* I owe much to Mr. R. W. Jastram of Stanford University (now at the National Bureau of Economic Research), with whom the outline of this paper was drawn up and plans for its joint presentation made. Since the pressure of other duties prevented Mr. Jastram's participating in the actual preparation of the manuscript, I must assume full responsibility for the paper and its abstract.

A concave isoquant is incompatible with fixed factor prices unless the employment of one factor be abandoned, but it is compatible with factor supply schedules sufficiently steeply positively inclined to insure greater curvature of the iso-outlay curve than of the isoquant. Negatively inclined supply schedules of the factors are even possible. If the negative inclination is moderate, the iso-outlay curves may still be concave to the origin, which presents no new problem. If the iso-outlay curves are convex to the origin, they must be less so than the isoquants.

If some factor exists in such lumps as to make plant capacity discontinuous, the long-run average-cost curve may still be continuous, but will not have a continuous first derivative. The isoquant in this case may be represented by dots, connected by dashed straight lines. The dots will arrange themselves in vertical lines (if the discrete variable be measured along the base axis) above the quantities of factor A (say) which it is possible to employ. The isocosts will similarly consist of relevant dots and connecting dashes. The dot in any isoquant which is coincident with the dot of the lowest possible isocost will indicate the appropriate factor quantities for the production of the given output.

If an isoquant is drawn for each possible unit of output, and the appropriate isocost for each isoquant is extended to the B-axis, the intercepts will mark off intervals which are simply MC/p_b (if the price of B is assumed constant). Thus these intervals are proportionate to marginal cost. The addition to plant of a lump of factor A will generally make MC/p_b quite small at first, but the numerator will increase as this lump is used more intensively with each increase in output. In the discontinuous case above considered this will give rise to a recurring, telescoping pattern of isocost intercepts on the B-axis.

Assuming only that a given output will bring one and only one price, each isoquant is also an isoincome curve. Successive isoincome curves may be drawn for each dollar of possible gross revenue. If the relevant isocost is drawn for each isoincome curve, the intervals between intercepts on the axis of the factor fixed in price would be proportionate in length to the (marginal) cost of securing an additional dollar of gross additional unit of output. Should the entrepreneur face perfect competition, only the factor of proportionality would revenue, rather than to the (marginal) cost of securing an differ.

Subject to the limitations of partial-equilibrium analysis, it may be possible to investigate the effect of pay-roll taxes or other factor-cost-increasing devices upon employment, using these tools. Expansion paths might be drawn before and after the imposition of a tax, which would show the burden each factor would sustain in any reduction in output.

Friday, July 19—The Study of Business Fluctuations by Means of Economic Models, FRANCIS W. DRESCH, Instructor in Mathematics, University of California.

The mathematical models of the present paper constitute a second step in the program of model building directed towards the construction of a sequence of such models each more complex and more realistic than its predecessor yet simple enough to serve as a basis for theoretical and empirical investigations into macrodynamic economics. As in a previous paper,[1] we are concerned with the general economy and with variables of aggregative type such as national income, price indices for capital and consumption goods, corresponding indices of production volume, etc. The assumption of equality of supply and demand, however, has been abandoned and new relations purporting to describe equilibrating mechanisms have been introduced. This generalization permits fluctuation in inventories and in unemployment somewhat after the fashion of the cobweb pattern and thus brings the model into closer conformity with the observed movement of relevant economic variables. Of much greater significance, however, is the fact that the present model nets one additional equation. Whereas one price had to be left indeterminate as *numeraire* in the

[1] Francis W. Dresch, "A Simplified Economic System with Dynamic Elements," in Cowles Commission, *Report of Fifth Annual Research Conference . . . 1939*, pp. 18-21.

previous model, a determinate solution is now obtained for each variable. This result is due to the fact that from the possibility of variation in commodity stocks there follows the possibility of variation in hoarded credit balances (investments in liquidity) and further of a disequilibrium between the supply function for savings and the demand function for investment funds. An extra relation (involving the rate of interest) is thus desirable to govern the above divergence and this relation does not depend on the other relations of the system. In the earlier model the absence of inventory variations required that income be either invested or consumed so that the supply function for investment funds was determined by the demand function for consumption goods and consequently an equation of demand and supply for consumption goods implied an equation of demand and supply for savings. In the new model savings enters as an independent variable but a demand function for liquid funds (or alternately a supply function for savings) enters as a new relation to compensate.

In its present form the system consists essentially of 15 equations (some involving time derivatives) on 15 independent variables. To facilitate comparison with data, the necessary empirical functions have been taken in certain convenient forms involving altogether 23 empirical constants. Given a set of values for these 23 parameters, it would be possible, save for exceptional cases, to solve for each of the 15 variables as a function of time. On the other hand, being given the appropriate time series for the variables, it would be possible to estimate the parameters and to test the model for consistency. Unfortunately it is very difficult to obtain data for the complete set of 15 series. Consequently an indirect approach has been adopted. Certain parameters have been taken arbritrarily as zero yielding a system for which it happens supply is constantly equal to demand, corresponding roughly to an infinitesimal period of adjustment. This system may be regarded as a description of a long-run situation and the parameters involved in it may be determined from available time series. The general system is then treated as a perturbation of the long-run system and may be transformed into a set of equations on ratios of deviations from trend to trend.

A second application of available data permits estimation of the remaining parameters. This procedure, however, depends on a tedious process of successive approximation and the empirical results are as yet very incomplete.

Monday, July 22—The Theory of Technological Unemployment, OSKAR LANGE, Associate Professor of Economics, The University of Chicago, and Research Associate, Cowles Commission.

The purpose of this paper is to analyze the effect of technical progress on employment in relation to monetary and to wage policy. The analysis is based on the following model of a simplified economic system. All production is divided between production of consumption goods x_1 and production of investment goods x_2. Consumption goods and investment goods are regarded each as one homogeneous commodity. Only two factors of production are used, i.e., labor (a) and capital services (b), which are measured in physical terms.

The production functions of consumption goods and of investment goods are:

$$x_1 = f(a_1, b_1, T_1), \tag{1}$$

$$x_2 = \phi(a_2, b_2, T_2), \tag{2}$$

where a_1 is the labor employed in producing consumption goods, a_2 the labor employed in producing investment goods; b_1 and b_2 have a similar meaning with regard to capital services. All variables are flows per unit of time. T_1 and T_2 are parameters.

Total employment is

$$a = a_1 + a_2, \tag{3}$$

and the total supply of available capital services is assumed to be given. Thus

$$b_1 + b_2 = \text{constant}. \tag{4}$$

The value of the marginal product of labor is equal to the wage rate. Denoting the wage rate in the consumption-goods industry by w_1 and in the investment-goods industry by w_2 we have:

$$w_1 = f_a p_1 , \tag{5}$$

$$w_2 = \phi_a p_2 , \tag{6}$$

where p_1 and p_2 are the prices of consumption goods and of investment goods respectively.

Similarly the value of the marginal product of capital services is equal to their price. Denoting the price of capital services by π we obtain

$$\pi = f_b p_1 \ ,$$

$$\pi = \varphi_b p_2 \, .$$

Since, however, the price of an investment good is the discounted value of all its future services, we have:

$$\pi \int_0^{\tau} e^{-it}\, dt = p_2 \, ,$$

where i is the rate of interest and τ is the average durability of investment goods. Denoting the integral above by k we obtain

$$p_2 = k f_b p_1 \, , \tag{7}$$

$$p_2 = k \phi_b p_2 \, . \tag{8}$$

The total demand for consumption goods (propensity to consume) is a function of the real income:

$$x_1 = F(x_1 + x_2 \frac{p_2}{p_1}). \tag{9}$$

The argument of this function is the real income measured in terms of consumption-goods prices.

These 9 equations determine our system which consists of the 9 variables: $x_1 , x_2 , a_1 , a_2 , a , b_1 , b_2 , p_1 , p_2$.

Equations (5) — (8) can be reduced to the following two:

$$\lambda \frac{f_a}{\phi_a} = k f_b \ , \tag{5a}$$

(6a) $$1 = k\phi_b ,$$

and (9) consequently can be written:

(7a) $$x_1 = F(x_1 + x_2 k f_b),$$

where $\lambda = w_2/w_1$. Thus all the physical variables in the system are found to depend only on the ratio of wage rates and not on their absolute level. The number of equations and the number of variables are reduced to 7. The system contains T_1, T_2, λ, and k (which depends on i and on t) as parameters.

Technical progress is treated as a change of the parameters T_1 and T_2. Its effects, as well as those of wage policies and of monetary policies, are obtained by differentiating the system totally and by expressing the differentials of the variables in terms of the differentials of the parameters. Using logarithmic differentiation we obtain from (1) and (2):

$$d \log x_1 = \frac{Ex_1}{Ea_1} d \log a_1 + \frac{Ex_1}{Eb_1} d \log b_1 + \frac{Ex_1}{ET_1} d \log T_1 ,$$

$$d \log x_2 = \frac{Ex_1}{Ea_2} d \log a_2 + \frac{Ex_2}{Eb_2} d \log b_2 + \frac{Ex_2}{ET_2} d \log T_2 ,$$

where Ex_1/Ea_1, etc., are partial elasticities. Taking into account these two relations we obtain from (4) and (5a)–(7a) three linear equations of the type:

$$A_1\, d \log a_1 + B_1\, d \log a_2 + C_1\, d \log b_1 = P_1 ,$$

$$B_2\, d \log a_2 + C_2\, d \log b_1 = P_2 ,$$

$$A_3\, d \log a_1 + B_3\, d \log a_2 + C_3\, d \log b_1 = P_3 .$$

The coefficients of these equations are:

$$A_1 = \frac{Ef_a}{Ea_1} - \frac{Ef_b}{Ea_1} ,$$

$$A_3 = X_1(1 - F')\frac{EX_1}{Ea_1} - F'q\,\frac{Ef_b}{Ea_1} ,$$

$$B_1 = -\frac{E\phi_a}{Ea_2} , \quad B_2 = -\frac{E\phi_b}{Ea_2} , \quad B_3 = F'q\,\frac{EX_2}{Ea_2} ,$$

$$C_1 = \frac{Ef_a}{Eb_1} - \frac{Ef_b}{Eb_1} + \frac{b_1}{b_2}\frac{E\phi_a}{Eb_2} , \quad C_2 = \frac{b_1}{b_2}\frac{E\phi_b}{Eb_2} ,$$

$$C_3 = x_1(1 - F')\frac{EX_1}{Eb_1} - F'q\,\frac{Ef_b}{Eb_1} + F'q\,\frac{b_1}{b_2}\,\frac{EX_2}{Eb_2}\ ;$$

$$P_1 = d\log K - d\log\lambda - \left(\frac{Ef_a}{ET_1} - \frac{Ef_b}{ET_1}\right)\ d\log T_1$$

$$+\ \frac{E\phi_a}{ET_2}\,d\log T_2\ ,$$

$$P_2 = -\,d\log K - \frac{E\phi_b}{ET_2}\,d\log T_2\ ,$$

$$P_3 = [F'q\ d\log K - X_1(1 - F')\frac{EX_1}{ET_1} - F'\ q\,\frac{Ef_b}{ET_2}]\ d\log t_1$$

$$+\ F'q\,\frac{EX_2}{ET_2}\,d\log T_2\ .$$

All the terms in the coefficients have an economic interpretation. Thus

$$\frac{Ef_a}{Ea_1} - \frac{Ef_b}{Ea_1} = \frac{E\,(f_a/f_b)}{Ea_1}$$

is the elasticity of the marginal rate of substitution of a for b, F' is the marginal propensity to consume and $1 - F'$ is the marginal propensity to save, q stands for x_2p_2/p_1 which is the value of gross investment expressed in terms of consumption-goods prices. The technical progress in consumption-goods production is defined as labor saving, neutral, or capital saving, according as to whether

$$\frac{Ef_a}{ET_1} - \frac{Ef_b}{ET_1} = \frac{E\,(f_a/f_b)}{ET_1} \lesseqgtr 0\ .$$

Solving the three equations we obtain the effects of technical progress and monetary policy (monetary policy acts via the rate of interest) and wage policy on employment of labor in consumption-goods and in investment-goods production. The effect on total employment can be obtained from (2). Putting $d\log K = 0$ and $d\log\lambda = 0$ we obtain the effect of technical progress when no compensating monetary and wage policy is present. Putting $d\log a_1 = 0$ and $d\log a_2 = 0$ we can find the monetary and wage policy, or any combination of them (i.e., $d \log K$ and $d \log \lambda$) which will exactly compensate the effects of technical progress on employment.

The system presented is intended to serve as a skeleton model for an econometric analysis of the problem of technological unemployment.

Monday, July 22—The Foreign-Trade Multiplier, SVEND LAURSEN, University of Copenhagen.

The "multiplier" analysis is usually undertaken under the assumption of a closed economy. In recent years there has, however, been a tendency also to introduce foreign trade in this analysis. Apart from minor differences in formulation there are two fundamental ideas underlying the approach. In the first place, exports are treated in the same way as private investment or a government deficit, i.e., they are introduced in the multiplicand, which tells us how much is to be multiplied in order to get the total increase in national income. In the second place, imports are dealt with as a "leakage" in the same way as savings. This can be done either by subtracting "induced" imports from the value of exports which enters the multiplicand, in other words taking the export surplus, or by reducing the multiplier by an amount corresponding to the propensity to import.

The essence of the approach is to consider exports as one of the determinants of the national output, and it stands in this respect in sharp contrast with the classical theory of international trade which is based on the assumption, mostly tacit, of full employment. One important aspect of the difference in assumption is seen in the treatment of the equilibrating international mechanism in case of a disturbance. The classical explanation puts the emphasis on price adjustments, while according to the multiplier theory the essential part of the mechanism is changes in total production and employment. Both explanations follow logically from their different assumptions about the utilization of the productive factors, but the multiplier theory seems to be the more realistic and straightforward under modern conditions.

The usual formulations of the foreign-trade multiplier have, however, certain shortcomings and limitations which make extensions in various directions necessary:

(1) The most important limitation is perhaps the definite static character of the theory. This either implies an instantaneous adjustment to fluctuations in exports, or that the adjustment period is of minor significance. Neither of these conditions is likely to be fulfilled. A brief analysis shows us how, with given propensities to consume and import, a change in exports will give rise to a dynamic process involving national income, savings, and imports. It also permits a more precise formulation of equilibrium conditions. One conclusion we draw from this analysis is that, contrary to common belief, there is no necessity, within the framework of the multiplier theory, for the national income to continue to rise in case of an export surplus until this has led to an elimination of the export surplus through increased imports. The explanation of this is that the export surplus in the equilibrium situation is offset by the leakage through domestic savings which is induced by a higher national income.

(2) The analysis must be carried a step further to include the case when prices and costs are rising with expanding exports and production. We can here apply various elements from the classical theory to show that under certain circumstances this facilitates an international adjustment.

(3) Furthermore, the liquidity of the banking system in our country is almost bound to change. This modifies again the multiplier theory in so far as it leads to a credit expansion or contraction.

(4) Finally, the dynamic process will probably have an impact on the domestic investment, thus further impairing the validity of the *ceteris paribus* clause on which the whole theory is based.

There is room for discussion as to how much is left of the original theory, but it is my view that it still contains a significant truth, largely because it stresses the volume of employment as an important variable in the international equilibrating mechanism.

Tuesday, July 23—The Problem of Assigning a Length to the Cycle to Be Found in a Simple Moving Average of Chance Data, EDWARD L. DODD, Professor of Actuarial Mathematics, The University of Texas.

In *The Annals of Mathematical Statistics*, Vol. 10, 1939, pp. 254-264, I described methods for assigning a length to cycles produced by most forms of linear graduation in common use when applied to independent random data. Such methods I found unsatisfactory for the cycles produced by simple averaging or summing, because of chaotic features which simple averaging does not remove.

Examination of Slutzky's figure on page 110 in his article on cycles in *Econometrica*, Vol. 5, 1937, and several similar figures based upon considerable more data has suggested to me rules for determining objectively the initial and terminal points of individual cycles or waves which appear in the moving average or moving sum of random elements.

Such averaging is often applied to monthly prices which frequently exhibit a seasonal cycle. To find the trend, we may use a twelve-month moving average to eliminate this seasonal cycle. However, if individual prices involve additively a strong chance constituent, this twelve-month averaging may *implant an unwelcome fictitious cycle*. This cycle is of length something between 30 months and 42 months—if cycles are counted by the method which I will now explain.

The method is briefly as follows: From the undulations found in a plot of a moving average, we must throw out of consideration *insignificant "ripples."* As the plot of the moving average is the same as that of the moving sum—with suitable change in the vertical scale—we use the moving sum for convenience. Now an undulation is usually lopsided—with vertical descents from the summit to the adjacent troughs unequal. As the height of the undulation, I take the least of these two vertical descents. A vertical height h is now selected as a minimum height for *waves*. If the height of an undulation falls below this h, the undulation is regarded individually as a ripple. A *wave* starts at a minimum and ends at another minimum, and between the two minima is a maximum higher than both minima by at least h. Some further conditions are needed—such as a condition to prevent two waves being counted as one.

Now if the chance elements are independent and have the same standard deviation σ, then the sum of n of such elements has the standard deviation $\sigma\sqrt{n}$. For my count of waves, I took $h = 1.5\sigma\sqrt{n}$. In Slutzky's figure on p. 110, this is just about the distance between the horizontal rulings; and for this h there are just five waves in the 126 sums plotted. This agrees with my *intuitive* count of these waves. These waves, then, have an average length of about 25, which is $2.5n$, where n is the number of items in an individual sum—here $n = 10$.

For a certain series of averages by threes, I found an average wave length of about 12—that is, $4n$. But for other series, with n taken as 9, 12, 20, 27, 40, and 80, the *average wave length lies between 2.5n and 3.5n.*

The waves are not all of equal length—that is, there is not true periodicity. But I find that usually the distribution of the dividing points of minima is decidedly *closer to that for periodicity than that for randomness.* This same feature, I found in waves produced by the Spencer 21-term formula, the waves counted from their crests or maxima—see p. 260 of my 1939 paper cited.

Moreover, I have now a formula for the variance, V, of the sum of the squares of the segments formed by the dividing random points upon the range, R. The number of segments being N,

$$V = \frac{4(N-1)R^4}{(N+1)^2(N+2)(N+3)}.$$

Tuesday, July 23—Dynamics of Labor Supply, HOWARD B. MYERS, Director of Research, Federal Works Agency, Work Projects Administration.

The question of labor supply, particularly in its dynamic aspects, has been curiously neglected by economists and by the public. It is generally assumed that, except for the operation of certain long-run factors, the supply of labor is substantially a fixed quantity.

As attempts are made to deal with the staggering unemployment of the thirties it becomes increasingly evident that the membership, characteristics, and status of the labor force are far from stable. Neither abstract economic theories nor occasional censuses provide sufficient basis for the direction of programs of public assistance, unemployment compensation, or other types of social legislation. There is need for current analyses of changes in the labor supply during periods of time, which describe and evaluate the labor force in terms of the ebb and flow of actual market conditions. Thus far few such dynamic studies have been made.

To illustrate types of information which these continuing studies may provide, some findings from recent WPA studies are presented. These indicate that the labor supply undergoes large and rapid seasonal fluctuations. The labor force increased by approximately 3 million workers between March and June, 1940, the greatest change occurring in rural areas. As a result of this influx an increase in employment of about 3¾ millions during the period resulted in a net reduction in unemployment of less than 1 million. Sample studies in urban areas suggest that considerable numbers of both workers and nonworkers change their labor-market status from one month to the next. The unemployed group is most mobile; nearly one-fourth of those unemployed and not on the Works Program in March, 1940, had secured employment or retired from the market one month later.

The size and composition of the labor force is also affected by cyclical changes in economic conditions. Little is known of these changes, but it is clear that they are more complex than is usually assumed.

Changes in the nature of unemployment and the characteristics of the unemployed are as important as changes in total volume. The experienced unemployed who are actively seeking jobs may be classified roughly into three groups, according to the duration of their unemployment: a high-turnover group, unemployed for 3 months or less, which constitutes about one-third of the total; a low-turnover group, unemployed for 12 months or more, including approximately two-fifths of the total; and a transitional group with 4 to 12 months of unemployment, forming about one-fourth of the total. The needs of these three groups require specialized treatment by agencies providing relief and rehabilitation.

Two other large groups may be identified among the unemployed. Workers who are temporarily inactive include from one-fifth to nearly one-fourth of the workers unemployed at any one time. Most of these workers are temporarily ill or disabled. Inexperienced workers, most of whom are youth, constitute another fourth of the unemployed. The youth years represent a period of very rapid shifts in labor-market status. Sample studies indicate that while 95 per cent of urban youth 18 to 25 years of age have been in the labor market at some time since leaving school, only half have remained continuously in the market since their first entry. The data further suggest that such factors as the influence of relatives and the financial status of the father are more important than innate ability in securing jobs.

The discussion has been restricted primarily to changes in the supply and composition of the labor force. Other dynamic aspects of labor supply, such as geographical and occupational mobility, are equally important. Adequate information concerning the dynamic aspects of labor problems is a prerequisite to effective action in dealing with these problems.

Wednesday, July 24—The Gold Deluge and Its Effect upon Monetary Theory, CARL SNYDER, for many years with the Federal Reserve Bank of New York.

Seven years ago, this, the richest and for long years the most prosperous country on the face of the earth, was in dire straits, despite our wealth, our industry, the extraordinary mechanical equipment of the country, our vast scheme of transportation; greater than all other countries combined. The nation was in the throes of profound depression, with unparallelled unemployment, a violent fall in prices, a tremendous decline in industrial production, apprehension and gloom, banks failing everywhere.

The doctors were called in, apparently every variety. The first product was the N.R.A.—regimentation and "share the work"; a dire failure. Then came other doctors—and economists; in foremost line what I have called the "auropaths," the "gold curists." According to this doctrine there was a great dearth of gold. Hence prices were low, industry paralyzed. This country had only four billions of gold, twice that of any other nation, one-third of the world's total; not enough, never enough!

So the idea was to turn this four billions of gold into seven, by fiat, overnight. Then there would be plenty of gold, prices would rise, food and everything would be dear, everyone happy, and the nation again prosperous; a process that once had an ugly name, "debasement of the currency." It has been a favorite trick of sovereigns and finance ministers from the beginning of coined money. What matter if plain-spoken people called it plain theft (which, of course, it was) if the treasury could be replenished and prosperity restored.

But this time (perhaps as usual) the trick did not work. Prices rose a little—not much; industry recovered a little—not much; unemployment scarcely decreased at all. Next came a colossal dole, creating a colossal debt, the greatest this country has ever known. And now, after seven years, still a blind faith that some other trick, faddery, or nostrum will turn the tide and restore prosperity; an amazing illusion, and a painful revelation of the degree of economic understanding among our rulers and guides. What actually has happened?

Raising the price of gold by seventy per cent has increased world gold production from half a billion dollars in 1933, to nearly two and a half times that figure. Most of this new gold and much from former hoards elsewhere has come to this country, until we have close to *twenty billions,* nearly three-quarters of all the monetary gold in the world. Yet—in the face of this the price level does not rise, bank credit does not greatly expand. Industry does not seek huge sums for normal expansion and improvement as in former days.

Consider the situation: Under normal conditions an increase in the money supply should mean, other things being equal, rising prices and increased business activity, wider employment, and higher money wages. What then is the trouble? Is there anything wrong with monetary theory? An endless number of answers have been given; sound, orthodox, economic and monetary theory and finance have seemed to be utterly discredited. But not for long.

These disastrous, bewildering ten years—ten years, not merely seven—have taught us a great deal. The world over, financial and monetary systems are credit systems; giving credit to purchase goods and pay debts, and having credit, faith in the soundness and honesty of our monetary systems, and those in control thereof. Destroy this confidence, this faith, this credit, and the system goes to pieces, just as it did in 1929, just as it always has. Confidence, especially, flew out of the window when the New Deal came into power. And that, as I see it, is the whole story, the entire explanation.

If, as I have predicted elsewhere, there is a change in national administration at the end of the present year, we may hope for a return to sanity, a revival of industry and business activity, a return to full employment, and therewith an end of vicious and disastrous taxation. This will mean a renewal of our normal industrial growth, our normal increase in wealth and wages and well-being.

But there is grave danger that this may come too suddenly, inaugurating a new speculative boom, and then another crash. If it should come, as a sequel to this collapse, the country will not go "New Deal" again; it will go Fascist or Communist or Totalitarian. It will be the end of democratic government, of Free America, of the splendid economic advance of these United States.

To avoid it, we must not forget the lesson that the foundation of all our prosperity, all our well-being, is not alone intelligent and sound economic procedure, but that illusive thing we call Confidence, Credit, Faith. Destroy this, and this wondrous and fertile scheme of capitalistic industry breaks down.

We cannot undo what is past. But we need have, I believe, no fear that this prolonged depression has had any lasting and profound effect upon the American character or the American system. The only real danger is from the loss of Faith, the loss of confidence, from the artless and arrogant ignorance which has ruled this country now for seven years.

I do not believe that our great gold stock is in itself a danger, nor do I believe that this flood of gold threatens our economic stability—if only it can be managed and directed with sufficient intelligence, courage, and understanding.

Wednesday, July 24—War and Commodity Prices,* HOLBROOK WORKING, Economist, Food Research Institute, Stanford University.

The historical observation that great wars have commonly been accompanied by great advances in commodity prices has perhaps fostered a mistaken impression of uniformity in the relation between wars and price advances; yet the facts admit no doubt that a significant relation exists. The irregularity of the relation between the scope, severity, and duration of war and the magnitude of the associated price advance, however, suggests the hypothesis that war has little direct effect on the general level of commodity prices, but that it creates a condition favorable to the generation of independent forces tending toward great price advances.

Many statistical and theoretical analyses of changes in the general price level have been devoted to intensive examination of the effects of particular influences. At the present time it seems especially appropriate to take a broad view of price changes in time of war and to attempt to estimate quantitatively the magnitude of the price effects of each of the major influences. To this end, I undertake first to appraise the effects of several major influences associated with price advances at the time of the last great war.

In order to measure roughly the effects of different price influences during the last great war, resort is had to a form

* The analysis of price developments of the last great war presented in this lecture followed the lines of a paper published under the same title in the *Journal of the American Statistical Association*, June, 1940, pp. 309-324.

of differential analysis. Using price index numbers computed for each country from prices of the same 40 "basic" commodities, with like weighting for all countries,[1] the *differences* between price movements are studied in relation to the differences between price influences operating in the several countries.

It appears from this analysis that strategic governmental controls of individual commodity prices, supported by some regulations on consumption, are capable of preventing large advances in the general price level so long as the political and financial position of a government remains strong. So far as the evidence on monetary influences goes, it tends to support the contention that in the absence of extreme currency inflation, the money and credit situation is a conditioning factor rather than an active causal influence on the general price level. Several specific influences appear to have had appreciable effects on the general price level in at least one of the countries studied. Those influences included changes in costs of ocean transportation; weather, in connection with its effects on crops; and special demands for war materials.

Price movements during nearly eleven months of the present European war have on the whole reflected tendencies similar to those of the last great war, but there have been two noteworthy differences in the important causal influences, both springing from experience in the last war. Governmental price controls in the belligerent countries were in this war not deferred until great price advances had occurred but were imposed promptly. And popular expectation of great price advances led to sharp upward movements of prices in most speculative markets immediately after the outbreak of the present war, although conditions seem generally not to have warranted such advances.

[1] Index numbers computed by Warren and Pearson and published in *World Prices and the Building Industry*, New York, 1937.

Thursday, July 25—Problems Confronting the Federal Reserve System, E. A. GOLDENWEISER, Director of Research and Statistics, Board of Governors of the Federal Reserve System.

It is difficult in the midst of radically changing world conditions to define the problems that confront the Federal Reserve System or any other economic, political, or social institution. It seems futile to look at the future in terms of problems of the past, and it is beyond the powers of most to visualize in realistic terms the vital problems of the future, even of the near future.

Taking for granted that this group is familiar with conventional central-bank considerations and techniques, one may ask how much of this is still relevant, particularly in the next few years. Discount rates and discount policy are almost obsolete terms in the face of immense excess reserves. Open-market operations have been demoted from an instrument of positive action on the supply of reserves to the status of a gyroscope functioning as a mild stabilizer in the market for Government securities and indirectly in the capital market as a whole.

Problems of gold and silver, of currency and foreign exchange, of Treasury finance and taxation have crowded the usual central banking functions off the field. It is only those who seek for the solution of fundamental economic problems by manipulating mechanical devices that still look for panaceas for national and international ills in the monetary or central-bank field. Some still believe that the root of all good and all evil is in gold or in silver, others that all depends on currency in circulation, still others that it is pivoted on bank deposits or their turnover. With my nose to the grindstone, I am unable to see any vital problems of today that can be solved by monetary magic.

I think that for the present perhaps the greatest service that the Federal Reserve System can render the country, aside from purely routine functions in supplying currency and check-clearing facilities, is to utilize its contacts, its authority, and its staff to analyze and diagnose the country's economic needs. The System has 37 contact points in its banks and branches; it has 260 directors drawn from different fields of activity, and 11,000 officers and employees scattered over the country, many of whom have valuable training and experience. This equipment can be used to collect and interpret currently information on our economic life—and to help formulate a rational program for the country to cure its ills and to meet its current needs. This might look like a novel central-banking function—but it is the most vital one today. Beside it the others seem much less urgent.

Nevertheless there are more direct problems that we must face. We must find new means of regulating the volume of bank reserves—when, if, and as a runaway situation should develop. Existing means are not adequate. We must also study means of more direct control of speculative expansion in various directions lest another boom and crash should once more shatter our economic life. We must find means to equalize the status of different groups of banks—national, State member, insured nonmember, and uninsured. We must also devise means for consolidating or co-ordinating the numerous agencies of bank supervision, State and Federal, and we must be informed and competent on the monetary phases of gold, silver, foreign exchange, and fiscal policies.

And that leads us back to our starting point. With little direct authority over economic forces that determine monetary developments today, the Federal Reserve System at present must seek to serve the country chiefly by analysis and diagnosis, by competent interpretation and by courageous advice. If it can do that effectively, then the time may come when more direct and conventional monetary action will once more become the central function of our central-banking organization.

Thursday, July 25—Statistics of Employment and Unemployment, ARYNESS JOY, Assistant to the Commissioner, Bureau of Labor Statistics, United States Department of Labor.

The European War and the rapidly developing American defense program have, for the time being, taken unemployment off the front pages and out of the minds of many people as the nation's leading problem. But it is still a grim reality to millions of people; and it is clear that the defense program, because of its specialized character, will by no means completely eliminate unemployment. In order to plan effectively, the need for realistic information on unemployment and employment is even more pressing, if possible, than it has been for the past decade. It is of the utmost importance that we cease to think and talk of unemployment as one great, homogeneous mass of people represented by a round figure of 9,000,000-10,000,000.

A variety of information is needed. For industrial planning, and for an analysis of the general unemployment situation, it is important to know how many people are not working who are able and willing to work, and who have certain skills, as well as *where* they are. For planning policies of relief, public assistance, and unemployment insurance, additional information on industries laying off workers and on sources of support for workers other than public aid is needed. We should know how many are sick or otherwise not equipped to enter the labor market. Separate data for urban and rural areas are a prime requisite.

For these varied purposes, the mass totals of unemployment, such as the estimates of the American Federation of Labor, the Congress of Industrial Organizations, and the National Industrial Conference Board, are not adequate, although they are very valuable in reminding the public constantly of the gravity of the unemployment problem. For May, 1940, they show a range from roughly 8,830,000 for the National Industrial Conference Board to 10,670,000 for the Congress of Industrial Organizations.

All are subject to considerable error both as regards the totals of and seasonal variations in unemployment, because of the unavoidable lack of certain basic statistics. All use essentially the same procedure and materials, with slightly varying assumptions. They start with the now long outdated Census of Population for April, 1930 as a benchmark.

First, a base employment figure is obtained by subtracting the number unemployed in 1930 from the number "gainfully occupied," and monthly changes in employment to date are then estimated, by industries, to obtain total employment for subsequent months. Since necessary Census data and current indexes of employment to bring them up to date are satisfactory for only about 40 per cent of the working population, an error of 1 to 2 million in these figures is not unlikely in the direction of understatement.

Although the Censuses of 1940 will provide up-to-date bench marks, and new reports from the Social Security Board will give comprehensive current employment totals for many industries for which data have been unavailable, so many gaps will remain that any figures of total employment can only be approximate, even in the future.

Second, changes in the total number "gainfully occupied" are estimated on the basis of the increase in population since 1930 for specific age-sex groups, assuming the same ratios of workers to total population in each group that prevailed in 1930 except for the very young and the very old, for whom reduced employment is assumed. The estimates show a growth during the ten-year period ranging from 5,294,000 as reported by the American Federation of Labor to 6,057,000 for the National Industrial Conference Board. None take full account of the women whom depression has forced into the labor market, which might give an increase of more nearly 7,000,000. They may include, however, many workers not *actively* in the labor market, and the resulting total of 53,900,000-55,900,000 in the labor supply may be high by as much as 1,000,000. These estimates may be seriously criticized because they assume seasonally unvarying labor supply. A sample survey conducted by the Work Projects Administration shows a seasonal increase of 3,000,000 in the labor supply from March to June of 1940 alone, as school children and others enter the labor market.

Finally, employment estimates are subtracted from the estimates of the number "gainfully occupied" to obtain a figure called "unemployment." The possible errors of 1 or 2 million in each of the two principal components represent 2-3 per cent, and are permissible in such large totals. Combined, however, they make too wide a possible error—(2 to 4 million) in unemployment, amounting to 20-40 per cent on a total of about 10 million. This disregards the large error in seasonal fluctuation.

Partly for these technical reasons, but chiefly because any total, however accurate, would not give an indication of the *nature* of the problem, we need not *one* total, but a comprehensive balanced current analysis of the employment and unemployment situation, providing statistics descriptive of several segments of the total. An enlarged statistical program along the following lines is suggested:

1. A direct sample survey of unemployment, employment, and the labor supply for urban and rural areas separately, giving certain characteristics of the unemployed such as the proportion which is ill, not active in the market, etc. The Work Projects Administration has recently been conducting valuable experiments in this field.

2. More adequate use of reports from state Unemployment Compensation agencies on the number unemployed who are either waiting for or receiving unemployment insurance payments, in the several states and the United States as a whole. These reports will never measure all unemployment, because the insurance systems do not include workers in agriculture, domestic service, and casual occupations, or proprietors, etc., nor do the reports on unemployment cover workers who have exhausted their benefit rights. These statistics are administratively significant, however, and they are similar to many of the measures of unemployment used abroad.

3. More detailed statistics of *employment*, by industries and industry subdivisions, stated in terms of number of employees rather than index numbers, and by states and large industrial areas wherever possible, supplemented by indications of the intensity of employment as measured by manhours. This involves the not very difficult extension and improvement of monthly reports to the Bureau of Labor Statistics, used in conjunction with employers' reports to the

Social Security agencies on the number of employees on their pay rolls.

With these new tools, it should be possible to make a much more realistic analysis of the unemployment problem which recognizes its complexity, an analysis directed toward effective programs for many groups which make up the so-called "unemployment."

Friday, July 26—The Problem of Stabilization of the Residential Construction Industry, ERNEST M. FISHER, Director of Research in Mortgage and Real Estate Finance, American Bankers Association.

1. By "stabilization of the residential construction industry" is meant a reduction in the amplitude of cyclical fluctuations in the annual volume of residential construction, measured by number of dwelling units constructed or by amount of expenditure for labor and materials.

2. The construction industry as a whole has been subject to wide fluctuations; no segment of the industry probably more than the residential. Measured by number of nonfarm dwelling units, the amplitude of the fluctuation in the last two decades is represented by a decline from an estimated 900,000 dwelling units in 1925 to some 55,000 in 1933.

3. These fluctuations influence the condition of the whole national economy, the rise in volume contributing to prosperity and wide employment; the decline, conversely, reducing employment and exaggerating the depths and prolonging the period of depression.

4. Reduction in the amplitude of the fluctuations in residential construction would probably tend to reduce both the amplitude and duration of general business cycles.

5. But residential building proceeds as a means of meeting demand for (a) increments to the supply of dwelling facilities or (b) the replacement of portions of the supply with units of different quality.

6. The demand for increments to the supply may arise from (a) changes in the number of households or (b) migration of households.

7. Demand for replacement arises from (a) changes in the composition of households or (b) changes in the standard of living.

8. Both migration and rise in the standard of living are in large measure a function of changes in income or prospects for earning income. Demand is, then, in some measure dependent upon general prosperity.

9. Supply, on the other hand, is relatively fixed and fluctuates, generally speaking, only in one direction, that of increase.

10. The largest annual increment to the supply was in 1925 when between 800,000 and 900,000 units or something less than 4 per cent was added to the "stock."

11. A reduction in incomes may easily cause a shrinkage in demand by as much as 4 per cent through "doubling up," migration from urban to rural areas, or postponement of long-term expenditure on durable goods. Thus the whole of the demand for increment dries up.

12. But with shrinkage in volume of building, costs usually decline, and the opportunity for investment in building for a long-term return is enhanced; the prospect for immediate return vanishes.

13. The more important the expectation of immediate (speculative) return, therefore, as the objective in building, the more likely is volume to decline. Substitution of long-term investment as the objective will tend to encourage building during periods of low costs and to discourage it in periods of high costs. It would tend, therefore, toward stabilization.

14. To the extent that public funds go into construction of housing facilities, like all other public-works programs, housing-construction programs should be timed so as to be at a peak in depression and at a low point during periods of prosperity.

Friday, July 26—The Economic Interpretation of History, HAROLD T. DAVIS, Professor of Mathematics, Northwestern University, and Research Associate, Cowles Commission.

The first suggestion that the principal variable in the interpretation of historical movements was economic, rather than one depending upon some current ideology, is found in H. T. Buckle's *History of Civilization* (1857) and in the *Misère de la philosophie* (1847) of Karl Marx, later repeated and given currency in the third volume of *Das Kapital* (1894). Unfortunately neither of these writers, nor their interpreters, have been able to establish a statistical basis for this thesis. But the recent accumulation of reliable data, particularly as they relate to price movements over long periods of time, affords some possibility of making a test of certain aspects of the economic interpretation of history.

The data published in 1934 by E. J. Hamilton on Spanish prices over the period from 1500 to 1650, combined with reliable data on the importation of treasure into Europe from A. Soetbeer's estimates (1891) and those of Hamilton, make it possible to explain and interpret the historical events of that century and a half. Moreover, the comparative sterility of Europe from the time of the fall of Rome to the discovery of America is explained by the low and almost unchanging level of prices as revealed by J. E. T. Roger's classical study of agricultural prices in England.

The sudden expansion of literature and science in Europe at the end of the sixteenth century is explained by the great increase in economic well-being which took place during that period. This well-being is not to be attributed to the rise in prices, but rather to the increase in trade which is revealed by an application of the equation of exchange, namely, $MV = PT$. In this equation M is money, defined statistically by the data on treasure, V is the velocity of money, assumed to be nearly constant since the lack of a modern banking system restrained circulation to a hand-to-hand exchange, P is price defined by Hamilton's data, and T is trade, computed in the form of a relative from the equation of exchange itself. From this statistical study it was found that the effect of the defeat of the

Spanish Armada upon Spanish trade was relatively unimportant, but that the ultimate decline of the Spanish empire began with an economic collapse around 1624 of a magnitude comparable with that of the American depression which followed the stock-market break in 1929. There is also evidence from historical sources that political experiments similar to the AAA, the NRA, and the sales tax were attempted during this period of Spanish history.

One method suggested for the statistical investigation of the economic factor in history is a study of the equation obtained by the differentiation with respect to time of the equation of exchange, namely,

$$V \frac{dM}{dt} + M \frac{dV}{dt} = T \frac{dP}{dt} + P \frac{dT}{dt} .$$

Major wars are always attended by dP/dt , and an index of the intensity with which wars affect individual economies was suggested by this consideration. Also great inflationary movements not related to wars, such, for example, as the tulip mania in Holland around 1637, the South Sea Bubble in England in 1720, the Mississippi Bubble in France in 1720, and the American speculation in 1929, are accompanied by a positive value for dM/dt , or by a positive value in dV/dt, without an increase in dT/dt . The first is illustrated by the experiment of John Law, father of the Mississippi Bubble, and the second by the stock-market inflation of 1929.

Golden ages in history are not caused by increasing prices, but by increasing trade. These increases appear to be logistic in character. In Spain during the sixteenth century and in Europe and the United States in the nineteenth and twentieth centuries, these trade logistics appear to have been about one hundred years in length. The Spanish logistic was occasioned by expanding commerce with the new world; the recent American and European logistics were caused by the industrial revolution brought about through the development and application of scientific discoveries.

American political history was interpreted in terms of the price index from 1797 to the present time. The thesis was advanced that violent reactions to the political party in power occur when dP/dt is strongly negative. No real exceptions are to be found to the proposition that the party in power is re-elected when the derivative of price is strongly positive. The American presidents, Van Buren, Buchanan, Johnson, and Hoover, suffered severe repudiation in periods of violent down-swings of the price index.

ABSTRACTS OF PUBLIC LECTURES

Tuesday, July 16—The Cost of the New Deal and Its Approaching End, CARL SNYDER, for many years with the Federal Reserve Bank of New York.

We live in a strange era, unique in the history of this country. Following a thirty-year period of prosperity and advancement never surpassed and perhaps rarely equalled by this nation or any other nation in the history of the world, we have had, since 1930, the longest and undoubtedly the most severe depression which this country, at least, has known, characterized by the largest number of unemployed and people on relief yet recorded, even percentagewise. This is in contrast to the steady growth during the 140 years previous, broken, it is true, by recurrent depressions sometimes severe but always followed by fairly rapid and sustained recovery.

In the midst of this severe depression the American people seemed deliberately to vote to continue the economic and political policies which were responsible. The "New Deal," of course, was not accountable for the collapse following 1929, but rather the "New Era," and the unrestrained stock-market speculation of that period. The "New Deal" policies were not in the platform of 1932 but were originated after the party came into power. The cost of these policies, in unrealized industrial gain, in the last ten years, has been prodigious, and almost unbelievable. The writer's estimate is of the order of one hundred and fifty billion dollars. Mr. Leon Henderson, of the Securities and Exchange Commission, has similarly estimated the cost of the depression as one hundred thirty-five billion dollars. Although the "New Deal" did not originate the depression, the fabulous cost seems largely due thereto, for if there had been a strong recovery in '33-'34 and a resumption of normal industrial growth, as was the usual course of events after past depressions, this would soon have made up the deficit of the preceding four, five, or six years. In other words, incredible as it may seem, we should now have a national income or, if you please, an income per capita, twice that of the present year and one-half greater than that of 1928 or '29.

What was the underlying cause for this prolonged and baleful blight? Practically all the wealth, the comforts, the conveniences, and, mainly, even the enjoyments of this modern world are due to machinery. These machines and contrivances and means of transport and all the rest are the product of a rare race of inventors, discoverers, contrivers, enterprisers, and organizers, who constitute perhaps less than one per cent of the population. In the last six or eight years we have not exterminated this race, but we have so utterly discouraged it that we have had no net gain in total product, no increase in goods and enjoyment and wealth. This has been due chiefly to cutting off of the lifeblood of the nation, industrially and economically, that is, the capital supply. There has been little opportunity for unfettered, free private enterprise. —How long is this to last?

On the basis of statistical studies of political contests and of national elections, through the last 60 or 80 years, the end may, with high probability, be predicted at the next election. This method is based on the observation of recurrent political cycles of twenty years each, and on a study of the rate of change of votes in the Congressional elections intermediate between the Presidential contests. By this method the Republican success in 1896 could have been predicted, as could even the curious result in 1912. By it the following predictions were made privately to friends, but not published, in November, 1934:

1. That in 1936 Mr. Roosevelt would be re-elected President by a plurality as large as or larger than in 1932. It was actually slightly larger.

2. That in 1938 there would be one of the heaviest swings in the opposite direction since 1894. This was statistically a fact.

3. That if this swing took place it would undoubtedly mean the return of the Republican party to power in 1940. This last prediction I still hold to.

Supposing these prophecies fulfilled, what may we reasonably expect within, say, the next ten years in an economic way? Probably, a period of great prosperity, tempered by endless strikes and labor controversies, in a last tyrannical struggle of the labor magnates to continue their deadly and throttling hold on American industry and therefore upon the whole life of the nation. If the political domination of this element is reasonably curtailed, we may have years of industrial peace. Labor, it seems, has yet to learn that the only possible way to increase wages is through an increase in the product per worker.

There is a yet more ominous sign, the danger of wild and unrestrained speculation, such as characterized the 1920's and led directly to the disastrous collapse of 1929, and therefore to the "New Deal" and its great cost. We know now that speculation can be controlled easily and simply, without injury to trade and enterprise, by sole control of credit. Unfortunately, the political heads of our financial administrations have not in the past ever evidenced the slightest understanding of this simple but all-important problem. Let us hope that we may now have a more intelligent financial and credit policy, that will not be under the malign notion that successful treasury policy is to keep down the Government interest rate.

Granting then some small hope that labor domination of industry may be duly curbed and that crazy speculation will likewise be reasonably restrained, we may, I think, look for such an economic expansion as we have not had, for more than very brief periods, in a hundred years or more. Immediately the forces of industrial progress and well-being are released from the fetters of the last eight years, all the splendid technical advance of the period—much of this resulting from corporation expenditures on scientific research, made because such expenditures offered some relief from income tax, will bear magnificent fruit.

Monday, July 22—Which Way to Peace? IRVING FISHER, Professor Emeritus of Economics, Yale University.

Today it is more true than ever before that the world is full of wars and rumors of war. Yet few want war. Why, then, these wars in East and West? Why any war? How can we rid ourselves of the terrible recurrent curse of war?

Like every other event, a war is the result of many causes. Some are very superficial. Some are very fundamental. Some are intermediate in importance.

Among the superficial causes is "Hitlerism." But this word does not explain how Hitler came to power, nor how, after this Hitler is stopped, we are to prevent some new Hitler from coming to power.

Among the intermediate causes of this war are the world-wide depression and the monetary deflation which largely caused the depression. The most neglected and hidden influence operating in human history is undoubtedly fluctuating money, i.e., inflation and deflation. Inflation raises prices so that people complain of a "high cost of living" and blame it on the "profiteer." Deflation lowers prices so that employers lose money, stop producing, and discharge their workmen, who then complain of unemployment and blame it on the employer. Just as deflation put Roosevelt into office in America in 1932, it put Hitler into office in Germany in 1930.

Another intermediate cause of war is restriction of international trade. So far as there was any substantial justification for Germany's wars, it was to cut herself loose from interferences with her international trade which, she rightly felt, bound her hand and foot.

We should at least try to understand Hitler and also understand why the German people applaud him. We must do both if we are ever to act intelligently about their misbehavior. Had we understood them better ten and twenty years ago, there might have been no such misbehavior today nor any Hitler.

In 1918 Germany laid down her arms in accordance with an agreement for which President Wilson's fourteen points were the essential bases. She trusted America to carry out this pre-Armistice agreement. But it was a promise which

America did not and could not fulfill. Instead came the Treaty of Versailles which Germany, after being rendered helpless, was forced to accept as a dictated peace instead of the negotiated peace she had been led to expect. An impossible reparation bill was drawn up and handed to Germany. Her colonies were taken away. Nevertheless, at first and for some fifteen years, Germany, even in her resentment at what she felt to be injustice and perfidy, tried to carry out policies of "fulfillment," appeasement, conciliation. She even joined the League of Nations. But Germany's neighbors cut off her trade. So did the United States.

Germany's desperation culminated in 1930 when the acute world-wide depression of that year reached Germany. This blow was largely due to our American "deflation," following the stock-market crash of 1929. In a like position, we should probably have been as resentful as Germany was.

To destroy Hitlerism is a military problem. To destroy unstable money and trade restrictions are economic problems. But to destroy war as an institution is a political problem. It consists in substituting law for war. Law and war are the two great alternative methods of settling disputes.

Disputes are inevitable. But war is not, as long as there is another way that men like better to settle disputes. The history of civilization is the history of the development of law as the preferred method of settling disputes and so obtaining peace.

The earliest peace group was the family, where the patriarch laid down the law and enforced it, thus keeping peace within the family group. Then came the village community, a cluster of families with its government and justices of the peace. Then came the state, a cluster of communities. Then the nation, a cluster of states.

The process of the enlargment of the peace group from one stage to another has always consisted of four steps. First, isolation gave way to more frequent contacts. Next, contacts led to disputes, especially over trade. Disputes led to fighting as the only way available for settling them. Finally laws were set up and observed, after the futility of war was recognized.

This substitution of law for war was either because one party defeated the other and thereupon set up its own laws, or because both parties agreed voluntarily to observe certain conditions to keep the peace and to provide machinery for deciding disputes by submitting them to a disinterested third party as a tribunal.

After the World War of 1914-1918, we made a noble attempt to substitute law for war through the League of Nations and the World Court. But Wilson, just because he was the greatest statesman of his time, was ahead of his times, and his efforts were defeated by Clemenceau and Lloyd George and by our own Senate. The United States never joined the League, and the dominant members of the League refused to give effect to Article 19 and make needed revisions in the Treaty. Oppressed Germany turned to Hitler.

It was a ghastly tragedy that the world was not ready in 1918 for a world organization. Had it been, had Wilson's great idea had the enthusiastic support it deserved, there would be no peace problem today. Perhaps the world is not ready even yet. But we are making progress. The isolationists today even in the United States are few and hesitant.

But again we ask: After this war, what? I have no doubt that in some form the fourth step or some attempt at it will be taken. The present wars in Europe and Asia are, like all wars, temporary expedients. They can't go on forever.

Now the fourth step, the substitution of law for war, may be taken, as it has been taken in the past, in either one of two ways.

Either the victor will annex the vanquished and impose his laws on the vanquished, or there will be voluntary agreement, through some sort of a League of Nations, as after the last war.

It is time for us to see clearly that isolation is no longer possible, and that we merely have the choice between being a member of a voluntary world organization, on an equal footing with other members, or an involuntary member of a world empire as a conquered nation.

I cannot see any third permanent solution of our present problem of peace and war. No "balance of power" is likely to stay in balance long. All nations must eventually unite. We may unite with other nations voluntarily or involuntarily; that is all. Unite we must in any case. And if we are to avoid the wrong way of uniting, we must not delay too long finding the right way.

Herbert Spencer was right in saying that the economic organism bears some analogy to the biological organism. The whole world is now well developed economically, i.e., in what corresponds to the body's circulatory system. But its regulatory system has not caught up. War represents the growing pains we suffer until it can catch up.

As long as the growth of the regulatory system lags behind that of the circulatory system, one of the symptoms of this maladjustment is the race in armaments. This race cannot be avoided as long as we have no international organization to prevent it. The pacifist who advises us to "set a good example" is not facing facts. His advice amounts to asking us to bare our breast to the foreigner's dagger.

It would be far better, of course, for all concerned, if all would disarm. But as long as others will not, we cannot. The only practical disarmament is by agreement—we will if you will.

We may put this whole problem of disarmament and its relation to an international organization to keep the peace into an aphorism: Either we must compete with the rest of the world in armament or combine with the rest of the world in disarmament.

At present we must accept the first alternative. Later, when the economic burden of keeping up this insane competition sufficiently gets through the heads of the public, including the followers of Hitler and the other dictators, that terrible burden with its taxes and debts will act as a powerful incentive toward a world organization as an escape, providing as it will a cheaper, surer, and better way to keep the peace.

Our own immediate concern is the military problem of preventing Hitler from possibly including America in his scheme. Next will come the great economic problems of world money and trade. But we shall not finish until we have solved the great political problem of world peace through world organization. A world governmental organization is the most fundamental principle for the establishment of permanent peace. Tennyson foresaw this a hundred years ago in "Locksley Hall":

Till the war-drum throbbed no longer, and the battle-flags were furled
In the Parliament of man, the Federation of the world.

There the common sense of most shall hold a fretful realm in awe,
And the kindly earth shall slumber, lapped in universal law.