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A STATISTICAL STUDY OF  
LIVESTOCK PRODUCTION  
AND MARKETING

*By*

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In co-operation with the Bureau of Agricultural Economics,  
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## PREFACE

The study reported here was undertaken at the Cowles Commission for Research in Economics in the fall of 1950 with the cooperation of the former Bureau of Agricultural Economics, United States Department of Agriculture, and the agricultural economics research group of the University of Chicago. Work on the project was substantially completed in 1952, but the pressure of other activities and the change in locations of the authors have somewhat delayed the preparation of a manuscript for publication.

The reason for the study was, in the first instance, methodological. It was desired to try various methods, recent and traditional, of problem formulation and statistical analysis in an important and promising practical setting. Useful empirical results were sought, but the main emphasis was placed on the development, application, and testing of methods that might prove effective in analyzing interrelated segments of economic activity. The livestock complex seemed an appropriate field for study because of its importance, the availability of data, and the possibility of obtaining guidance from previous studies and from experienced research workers familiar with one or more aspects of the livestock industry.

As the work progressed, the authors sought advice from various sources, and many helpful suggestions were received. However, divergent views were not infrequent, and so it cannot be assumed that any particular procedure followed would be endorsed by all those whose assistance was obtained. At the stage of building a theoretical model, a number of discussions were held with Professors T. W. Schultz and D. Gale Johnson of the University of Chicago, with Roland Welborn of the commercial research department of Swift and Company, and with Karl A. Fox, Richard J. Foote, and Harold Breimyer, who were at that time attached to the division of statistical and historical research of the Bureau of Agricultural Economics. The Agricultural Marketing Service has subsequently taken over this part of the work formerly done by the Bureau of Agricultural Economics.

In tabulating and interpreting data from published sources, substantial help was obtained from R. D. Jennings and Malcolm Clough, formerly of the Bureau of Agricultural Economics. Through the Bureau, access to some unpublished materials was also provided. An earlier draft of the manuscript was read by Professor George Kuznets of the Giannini Foundation, University of California, and substantial portions were read by Professors T. C. Koopmans and Roy Radner of the Cowles Com-

mission and by Dr. Fox. The present version benefits from several of their suggestions, and, in addition, they have provided some criticisms that are likely to be useful in future studies.

The authors are also indebted to the former Bureau of Agricultural Economics for the main financial support for the project, which was conducted with contract funds under the Agricultural Marketing Act of 1946, and to O. V. Wells, then chief of the Bureau of Agricultural Economics and now administrator of the Agricultural Marketing Service, for his continuing interest in the work. Computations were performed at the Cowles Commission under the supervision, at various times, of Dan Waterman, Jagna Zahl, Totaro Miyashita, and Francis Bobkoski.

*March 1955*

CLIFFORD HILDRETH  
F. G. JARRETT

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## CHAPTER I

### INTRODUCTION

#### ULTIMATE PURPOSES OF STUDY

The ultimate purposes of studies like the present one are probably sufficiently apparent that little discussion of them is required here. The investigators have tried to obtain quantitative approximations to some of the underlying relations determining quantity and price of livestock products produced and sold in the United States each year. If accurate and reliable approximations to such relations could be obtained, useful applications to both private and public policy would readily suggest themselves. If farmers could more accurately forecast price and cost conditions, they could more efficiently adjust their production.<sup>1</sup> If processors of livestock products had better notions of the amounts of various animals and products that would be forthcoming, savings in processing costs would undoubtedly be possible.<sup>2</sup> If the government were considering sending large amounts of grain or other feedstuffs abroad, a knowledge of the relations considered here would enable one to forecast the effects of such a diversion on domestic prices of feed and livestock and on production of livestock products. Similarly, responses to price supports, taxes, subsidies, or other measures depend on the nature of the underlying economic and technical relations.

As for the application of the results of this study to the kinds of

<sup>1</sup> For the forecast to be useful in this respect it would have to be widely known and seriously considered. This would mean that, to make a good forecast, the forecaster would have to allow for the influence of his forecast on the behavior of various participants in the market. Although learning to take account of the effects of a forecast presents problems that have not been very thoroughly considered, the recognition that a forecast may influence action does not make good forecasting a logical impossibility.

<sup>2</sup> For many purposes processors of livestock would be interested in forecasting amounts of particular types of livestock to be marketed in a given period rather than the aggregates considered here. They would also be interested in forecasts for periods shorter than a year. The present study is limited to annual variations in aggregate variables, partly because of the complexity of relations among disaggregated variables over shorter time periods and partly because of the nature of the available data.

problems indicated above, the authors must suggest caution. As will be seen in later chapters, sampling variations alone are sufficient to give considerable dispersion of observed variables around the values that can be forecast from our fitted relations. Still more troublesome are the possibilities that some of the assumptions on which statistical analysis is based are unrealistic and that these have led to various kinds of specification error<sup>3</sup> in the results. In the present state of econometrics such difficulties are typical, and economists have generally regarded the results of empirical studies of economic relations with proper reservations.

#### INTERMEDIATE PURPOSES

Although one ordinarily cannot feel very hopeful that a particular study will yield empirical relations of the accuracy and reliability necessary to answer many of the practical questions that might be posed, there is still, we believe, much to be gained from efforts to obtain the best empirical relations possible. If we are ever to obtain good approximations a certain amount of trial and error may be unavoidable. Frequently something can be conjectured by noting the circumstances under which a particular fitted relation fails to hold. Preliminary examination of data incidental to such a study is often suggestive, and even implausible results may lead to new and useful hypotheses. Part of the difficulty in a typical study lies in inadequacies of the available data. Attempts to use existing data should provide useful suggestions concerning the kinds of additional data that are needed. Similarly, attempts to apply existing methods should provide better judgments of the kinds of improvements in methodology that are most needed.

<sup>3</sup> For discussions of some types of specification error see the following:

D. Cochrane and G. H. Orcutt, Application of Least Squares Regression to Relationships Containing Autocorrelated Error Terms, *Journal American Statistical Association*, Vol. 44, pp. 32-61, 1949.

G. H. Orcutt and D. Cochrane, A Sampling Study of the Merits of Autoregressive and Reduced Form Transformations in Regression Analysis, *Journal American Statistical Association*, Vol. 44, pp. 356-372, 1949.

S. G. Allen Jr., An Example of Loss of Efficiency in Structural Estimation, and Jean Bronfenbrenner, Sources and Size of Least Squares Bias in a Two-Equation Model, both in *Studies in Econometric Method*, Cowles Commission Monograph 14, William C. Hood and T. C. Koopmans, editors, John Wiley & Sons, New York, 1953.

Leonid Hurwicz, Some Specification Problems and Applications to Econometric Models, *Econometrica* (abstract), Vol. 19, pp. 343-344, 1951.

Egon S. Pearson, The Analysis of Variance in Cases of Non-Normal Variation, *Biometrika*, Vol. 23, pp. 114-133.

M. S. Bartlett, The Effect of Non-normality on the *t* Distribution, *Proceedings Cambridge Philosophical Society*, Vol. 31, pp. 223-231.



In many studies perhaps the most important gains accrue because the investigator is forced to formulate his assumptions about the part of the economy being studied more completely and more precisely than on other occasions. These then become available for general criticism and discussion. Out of such discussion, new hypotheses may arise. Though discussion can and does often proceed without reference to data, we believe the intention to incorporate data into the analysis imposes a discipline that should sharpen theoretical discussion and make it an increasingly useful aid in forming practical judgments.<sup>4</sup>

#### EMPHASIS IN PRESENT STUDY

In the course of the study the authors have tried to keep in mind both the ultimate objectives mentioned at the outset (i.e., predicting future values of economic variables and the consequences of economic policies) and the intermediate objectives of formulating ideas about market behavior, testing them as far as possible against empirical data, and making reformulations when these seem necessary. We are interested in the intermediate objectives because we hope that success in pursuing them will eventually make a significant contribution to our ability to predict. However, one can only guess how much work may need to be done before the significant contribution may be realized. Fortunately the ultimate and intermediate objectives cited above are to some extent complementary. A process of trial and error is not likely to be fruitful unless one makes good tries. On the other hand, some choices have to be made. If an investigator wanted to make the best possible forecasts for next month or next year, he would use his resources rather differently than if his main objective were to improve the groundwork on which forecasts might later be based. In particular, if immediate forecasts are the main objective, one is likely to rely more on expert opinion and informal methods of interpreting past experience. If an eventual contribution to forecasting is sought, one may be interested in experimenting with more elaborate and formal methods and one is likely

<sup>4</sup> Of course, theoretical analysis can be compared with data without estimation of parameters in assumed relations or without applying formal statistical inference at all. Examples of such procedure may be found in:

J. S. Duesenberry, *Income, Saving and the Theory of Consumer Behavior*, Harvard University Press, Cambridge, 1949.

A. G. Hart, *Money, Debt and Economic Activity*, Part III, pp. 257-300, Prentice-Hall, New York, 1948.

J. Viner, *Canada's Balance of International Indebtedness 1900-1913*, Harvard University Press, Cambridge, 1924.

Colin Clark, *The Conditions of Economic Progress*, Macmillan & Co., London, 1951.

to spend more time theorizing and trying to understand the fundamentals of the activities one considers. The reader will have no difficulty recognizing that the emphasis here is on intermediate objectives and that much work remains to be done if the results are to be very useful in practical forecasting.

Our decision to investigate livestock products as an aggregate rather than as individual products (such as milk, eggs, beef) is an example of the emphasis on intermediate objectives. For some purposes (e.g., see footnote 2), even good forecasts of aggregate livestock production and an index of livestock prices would have limited usefulness. The greater heterogeneity of aggregates also presents an additional barrier to the attainment of good forecasts.<sup>5</sup> However, to have considered the important food livestock products individually and with any thoroughness would have been beyond the resources of the present study. To have centered attention on a single product would not have eliminated the difficulties. Livestock products are so interrelated both in supply and in demand that any reasonable analysis of one product must take into account the conditions of supply and demand for the others. It is hoped that the results of the present study will be helpful in later treatments of individual commodities or types of livestock.

#### PRODUCTS STUDIED

The individual products included in the livestock products aggregate are cattle, calves, hogs, chickens, turkeys, sheep, milk, and eggs. The basic price and quantity data used to construct aggregate price and quantity indices (construction of the indices and sources of data are discussed in some detail in Chapter III) relate to prices received by farmers and quantities sold by farmers to dealers or processors or consumed on farms. Thus demand in the present study refers to the sum of demand for home consumption plus demand of the commercial sector of the economy for these products. Behavior of final consumers and behavior of dealers and processors are combined in the demand relation studied.<sup>6</sup> Since the individual commodities listed tend to be substitutes in both production and consumption, there should be a tendency for their prices to change proportionally over time. To the extent that this has in fact been the case, some of the disadvantages of aggregating are mitigated. Some notion of the strength of this tendency can be obtained by examining the plotted price series in Figure 1.

<sup>5</sup> This may to some extent be counterbalanced by the greater stability of aggregates. They may be disturbed relatively less by accidental factors on which the investigator has no data.

<sup>6</sup> For further explanation of this point see pp. 107 and 108.

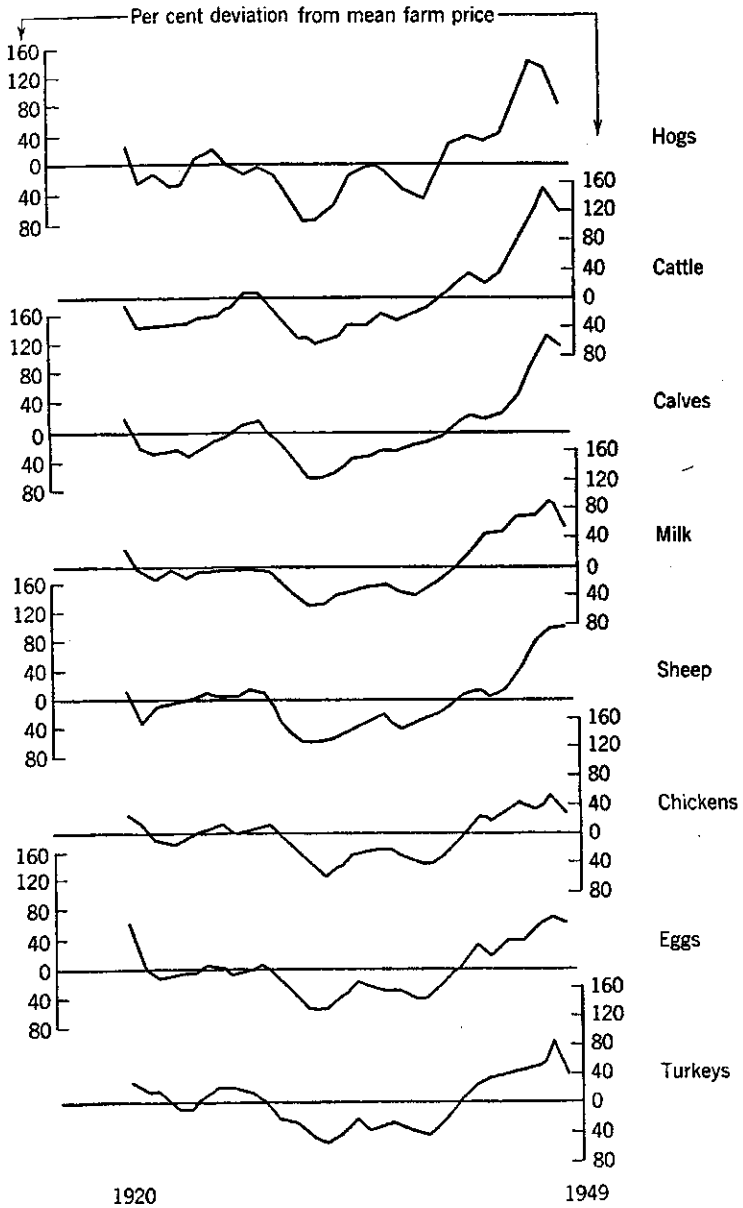


FIG. 1. Price movements for individual products. Based on price data from *Agricultural Statistics*, 1942-50.

The principal livestock omitted from the study are horses and mules. They have been excluded because of their declining importance<sup>7</sup> and because their inclusion in an aggregate would have greatly increased its heterogeneity. Similarly, sheared wool has been excluded since it is a minor product, contributing in recent years about 1% to the value of livestock products sold, and since its price is strongly influenced by a number of factors that are not present for the other commodities. The other livestock and livestock products omitted—poultry other than chickens and turkeys, goats, mink, foxes, dogs, etc.—are of negligible importance. Chapter III includes some discussion of problems arising from the exclusion of horses and mules. The data used in the study cover operations during the calendar years 1920–49, inclusive.

#### STATISTICAL SPECIFICATION, STRUCTURE, MODEL

In any application of statistical analysis, the inferences that can be drawn depend both on the data and on the a priori assumptions the investigator makes about the statistical universe from which the data are drawn. These a priori assumptions are called the statistical specification. Choosing an appropriate statistical specification is a particularly critical and difficult task in most applications of statistics to economics. Most applications to economics involve nonexperimental data so that the investigator cannot make his data conform to a previously selected specification. He cannot design an experiment but must endeavor to choose a specification that will be reasonably consistent with the process by which his data were generated in the real world.

#### ECONOMIC AND STATISTICAL MODELS

In the language that has been developed to consider statistical analysis of economic relations, the process by which a set of economic variables is generated is called a structure. The variables whose values are explained by the structure are called endogenous variables whereas those whose values are determined outside the structure are called exogenous. The set of structures compatible with the investigator's statistical specification is called a model.<sup>8</sup> In the present study we find it

<sup>7</sup> Horses and mules on farms declined from 26 million in 1920 to 8 million in 1949. Data from *Agricultural Statistics, 1942-51*, U. S. Department of Agriculture, Washington, D. C.

<sup>8</sup> For a fuller discussion of the role of structures and model see T. C. Koopmans and William C. Hood, *The Estimation of Simultaneous Linear Economic Relations, Studies in Econometric Method*, Cowles Commission Monograph 14, William C. Hood and T. C. Koopmans, editors, John Wiley & Sons, New York, 1953.

J. Marschak, *Statistical Inference in Economics: an Introduction, Statistical*

useful to distinguish between what we shall call the economic model and the statistical model.

From economic theory and from a priori knowledge of the workings of the sector of the economy being considered, the investigator can usually form some idea of the kinds of relations that are included in the structure under consideration and the variables that are important in each relation. He thus has some guide as to the appropriateness of assumptions he might make about these aspects of his model. If these assumptions are questioned, economic theory or knowledge of institutional arrangements can usually be brought to bear. Economic theory and knowledge of institutional arrangements may also make it possible to specify certain qualitative restrictions on the nature of the relations. We apply the term economic model to the set of structures consistent with the assumptions that the investigator develops from considerations of economic theory and knowledge of existing institutions.

In the present state of economic and statistical theory, the investigator typically finds it necessary to make additional assumptions for which economic and institutional considerations offer little if any guide. He must typically specify the algebraic form of his relations and the way in which the relations are affected by unobserved influences. Although economic considerations may sometimes exclude certain possibilities, they do not usually provide very strong grounds for preferring a particular set of assumptions. The specifications made about these aspects of the structure are often chosen partly to simplify the statistical analysis and are to some extent arbitrary. In our study we refer to the set of structures consistent with all the assumptions of the investigator as the statistical model.

Although the above distinction is not always a sharp one, we believe it has usefulness. If the investigator always makes explicit the basis for the assumptions underlying his economic model, these can be more readily evaluated by economists and by persons well informed about the relevant institutions. There is perhaps more incentive to experiment with alternative sets of assumptions that apply only to the statistical model, and it may be hoped that experience will help in forming preferences among them.

#### CONTENT OF LATER CHAPTERS

The development of the economic model used in the present study is described in Chapter II. Results using a particular statistical model and the observations described in Chapter III are given in Chapter IV. In

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*Inference in Dynamic Economic Models*, Cowles Commission Monograph 10, T. C. Koopmans, editor, John Wiley & Sons, New York, 1950.

succeeding chapters, particular relations are discussed in more detail and results obtained from different statistical models (and in a few instances from different economic models) are presented. Chapter VIII contains some comparisons of the estimated relations with data for 1950, designed to give a preliminary notion of the possible predictive value of the relations. A sample computation is given in the appendix.

## CHAPTER II

### THE ECONOMIC MODEL

#### AN INITIAL MODEL

As an expository device, the economic model underlying the estimates that follow in Chapter IV will be developed by first considering a crude but simple model and then considering various difficulties in the model and possibilities for improving it. Though admittedly crude, the model does seem to represent the operation of the livestock economy to a first approximation and closely resembles descriptions of the market that have been taken as first approximations in other studies.<sup>1</sup>

In this simple model we regard production of livestock products in a given year as primarily determined by the amount of feed fed to livestock in that year and the number of animals on farms at the beginning of the year. This implies a relation among three variables—production of livestock products, quantity of feed fed, and number of animals in the beginning inventory.<sup>2</sup> This will be called the production relation. Each of the three variables is an aggregate of various products, feeds, or kinds of animals and can be regarded as observable only by the construction of an index number. It has also been recognized that there have been improvements in breeds of animals and in feeding practices that have enabled producers gradually to increase their output from given herds and feed supplies. One might expect that, to a first approxi-

<sup>1</sup> James H. Lorie, Causes of Annual Fluctuations in the Production of Livestock and Livestock Products, *Journal of Business*, University of Chicago, Vol. 20, no. 2, Part 2, 1947.

Mordecai Ezekiel, The Cobweb Theorem, *Quarterly Journal of Economics*, pp. 255-279, Feb. 1938.

<sup>2</sup> The inventory variable is relevant because increasing quantities of fodder fed to a given herd are subject to diminishing returns. For evidence of this see the following:

Einar Jensen et al. Input-Output Relationships in Milk Production, *Technical Bulletin 816*, U. S. Department of Agriculture, Washington, D. C., 1942.

L. Jay Atkinson, Feed Consumption and Marketing Weight of Hogs, *Technical Bulletin 894*, U. S. Department of Agriculture, Washington, D. C., 1945.

Aaron G. Nelson, Relation of Feed Consumed to Food Products Produced by Fattening Cattle, *Technical Bulletin 900*, U. S. Department of Agriculture, Washington, D.C., 1945.

mation, the net effect of these improvements might be regarded as a smooth function of time and could be allowed for in an empirical study by introducing time into the production relation.

The quantity of feed fed to livestock in a given year is approximately equal to the quantity produced, and this is primarily influenced by weather. Hence we do not try to explain feed consumption within the model but regard it as exogenous, i.e., determined outside the set of relations being considered. The number of animals on hand at the beginning of a year will depend on past prices of livestock, past prices of feeds, and other factors affecting producers' expectations concerning the profitability of current livestock production. Animal numbers also depend to some extent on weather, but this influence is not significant in most years.

The price of livestock products depends on quantity produced and on such demand factors as population, consumer incomes, and prices of related commodities. The price of feed is undoubtedly strongly influenced by the price of livestock products. Since we have indicated in the production relation that the efficiency with which various quantities of feed can be converted into livestock products depends on the number of animals fed, it would also seem reasonable that the price of feed should also be influenced by the quantity of feed fed and the number of animals on hand at the beginning of the period.

To consider the relations described above more fully and to consider possible modifications, it will be convenient to represent them symbolically. The following notation will be used:

$h_t$  = number of animals on hand at the beginning of the  $t$ th time period

$p_t$  = price of livestock products (the  $t$  subscript will be used to denote the  $t$ th time period throughout)

$q_t$  = price of feed

$l_t$  = quantity of livestock products produced during period (at this stage assumed equal to quantity sold)

$f_t$  = quantity of feed fed to livestock (at this stage assumed exogenous and equal to quantity produced)

$y_t$  = consumer income

$n_t$  = population



$r_t$  = index of other prices

$s_t$  = unspecified factors affecting expectations of producers about conditions at  $t + 1$

$t$  = time

The discussion so far can now be summarized in the following four relations:

$$(2.1) \quad h_t : p_{t-1}, q_{t-1}, s_{t-1} \quad (\text{inventory relation})$$

$$(2.2) \quad l_t : h_t, f_t, t \quad (\text{production relation})$$

$$(2.3) \quad p_t : l_t, y_t, n_t, r \quad (\text{demand for livestock products})$$

$$(2.4) \quad q_t : p_t, f_t, h_t \quad (\text{demand for feed})$$

A colon may be read "depends on"; a comma may be read "and." We would thus read (2.1) as "The number of animals held at the beginning of the  $t$ th period depends on the price of livestock products in the previous period, the price of feed in the previous period, and unspecified factors of the previous period which influence producers expectations for the present period."

#### HISTORICAL CHANGES IN EFFICIENCY

The use of time as a variable in the production relation may properly cause some uneasiness. At best, time is a vague representation of various influences whose net effect during the period of observation has been a fairly steady increase in the efficiency with which feed is converted into livestock products. Unless these influences are understood rather well, there is little basis for conjectures concerning their persistence in the future. There is also the question of how well the variable time represents these influences during the sample period. Some evidence on the latter point may be obtained by examining data for the sample period.<sup>3</sup> A superficial inspection of the data shows them not to be inconsistent with the assumption that increases in efficiency developed fairly smoothly over time when some allowance is made for random disturbances. Relevant observations are summarized in Table I, where the data are ag-

<sup>3</sup> It is, of course, recognized that to use the sample observations as an aid in making statistical specifications modifies the interpretation that may later be made of confidence regions or statistical tests based on these specifications. We believe, however, that in this case, as in many economic investigations, uncertainty about the underlying specification is a sufficiently prominent feature of the analysis that a fairly high price in terms of qualifications on subsequent analysis can be paid for anything that might aid in achieving a reasonable specification.

TABLE I  
GENERAL MOVEMENT OF PRODUCTION VARIABLES

Time Period	Variable				
	$l_t^*$	$f_t^\dagger$	$h_t^\ddagger$	$\frac{l_t}{f_t}$	$\frac{l_t}{h_t}$
(1) 1920-22	17.69	1.104	17.85	16.02	0.99
(2) 1923-25	19.24	1.101	17.83	17.48	1.08
(3) 1926-28	20.03	1.109	16.76	18.07	1.20
(4) 1929-31	20.81	1.066	17.21	19.52	1.21
(5) 1932-34	21.01	1.070	18.76	19.64	1.12
(6) 1935-37	19.77	1.002	16.81	19.73	1.18
(7) 1938-40	22.22	1.148	17.70	19.36	1.26
(8) 1941-43	26.73	1.392	20.18	19.20	1.32
(9) 1944-46	27.67	1.420	20.75	19.48	1.33
(10) 1947-49	27.15	1.336	18.97	20.33	1.43

\* Figures are in billions of dollars' worth of product at average prices.

† Figures are in trillions of pounds of total digestible nutrients.

‡ Figures are in billions of dollars' worth of estimated potential production.

Data are taken from Tables III and IV, pp. 60, 61. Original sources and methods of aggregation are explained in Chapter III.

gregated into three-year totals to reduce the number of figures that the reader needs to compare.

An alternative explanation of the over-all increase in ratios of output to input in the last two columns of Table I might be increasing returns to scale in aggregate livestock production. If one were to use this explanation, the three points (periods 1-2, 5-6, 9-10) at which both inputs decrease and the ratio of output to input continues to increase would be difficult to explain. If other influences on production could be identified and observed, there is no question but that substituting them for time in the relation would make statistical analysis more promising. An attempt in this direction is reported in Chapter V (pp. 85 to 87). Meanwhile, the model is developed with the provisional acceptance of time in the production relation.

#### QUALITATIVE RESTRICTIONS ON FORM OF PRODUCTION RELATION

The algebraic forms of the relations are as yet unspecified. In this study as in most studies of economic relations, economic considerations do not give the investigator strong grounds for choosing a particular form. Within fairly wide limits, the choice is made on grounds of simplicity or convention, and must be regarded as to some extent arbitrary.

In these circumstances, the investigators believed that it might be useful to experiment to some extent with alternative forms. Although such experimenting may not give conclusive indications of the relative merits of the alternatives tried, it would seem reasonable to prefer one algebraic form to another if it consistently yielded more plausible estimates and more accurate predictions. Though economic considerations do not typically specify algebraic forms of relations very narrowly, they do often give certain presumptions about directions of variations and other qualitative restrictions. For example, in relation (2.1) we would expect

$$(2.5) \quad \frac{\Delta h_t}{\Delta p_{t-1}} > 0, \quad \frac{\Delta h_t}{\Delta q_{t-1}} < 0$$

and in (2.2) we would expect

$$(2.6) \quad \begin{array}{lll} \frac{\Delta l_t}{\Delta h_t} > 0, & \frac{\Delta l_t}{\Delta f_t} > 0, & \frac{\Delta l_t}{\Delta t} > 0 \\ \frac{\Delta^2 l_t}{\Delta h_t^2} < 0, & \frac{\Delta^2 l_t}{\Delta f_t^2} < 0, & \frac{\Delta^2 l_t}{\Delta h_t \Delta f_t} > 0 \end{array}$$

Analogous statements that might be made about (2.3) and (2.4) are fairly obvious. Although these do not tell the investigator which form to use, he will ordinarily wish to avoid a form that is inconsistent with these restrictions. The equations actually used in this study will be specified in subsequent chapters.

#### POSSIBLE APPLICATION OF LEAST SQUARES PROCEDURES

For the remainder of this chapter relations will be specified to the extent of indicating the observable variables believed to enter them, which variables are regarded as endogenous and which as exogenous, and the kinds of technical restraints or behavior patterns that the relations are intended to represent. In (2.1)–(2.4) the variables  $f_t$ ,  $y_t$ ,  $n_t$ ,  $r_t$ ,  $t$  are regarded as exogenous. If one assumes that unobserved random disturbances that enter the relations are serially independent, then lagged values of the variables may be regarded as predetermined.<sup>4</sup> If (2.1) were then written as an expression linear in known functions<sup>5</sup>

<sup>4</sup> A variable is predetermined at time  $t$  if the random disturbances that enter the relations at time  $t$  are distributed independently of the variable. In models with serially independent disturbances, both exogenous and lagged endogenous variables are predetermined. See T. C. Koopmans, *When Is an Equation System Complete for Statistical Purposes?*, *Statistical Inference in Dynamic Economic Models*, Cowles Commission Monograph 10, p. 406, and T. C. Koopmans and William C. Hood, *op. cit.*, pp. 120–125.

<sup>5</sup> Logs of observed variables or squares and cross products of observed variables are the most common examples.

of the observed variables and an additive random disturbance, unknown parameters in the expression could be estimated by least squares. If a similar expression is written for (2.2) and the disturbance in (2.2) is assumed to be serially independent and independent of the disturbance in (2.1), then  $h_t$  may be regarded as predetermined in (2.2), and least squares methods will give unbiased estimates of parameters in (2.2). Likewise, if the disturbances in linear expressions for (2.3) and (2.4) are independent of each other and of disturbances in the preceding equations, then  $l_t$  may be regarded as predetermined in (2.3),  $p_t$  and  $f_t$  may be regarded as predetermined in (2.4), and least squares methods are appropriate for these equations also.<sup>6</sup>

Reasoning like the above could be applied to a large number of economic relations pertaining to agricultural commodities.<sup>7</sup> Although counterarguments could typically be produced to the effect that all of the assumptions needed to justify least squares procedures are not strictly true, it is quite possible that they sometimes characterize the underlying relations sufficiently well that there is not much to be gained by applying methods based on more refined statistical specifications. Questions on the adequacy of various possible specifications and the statistical procedures associated with them are difficult to evaluate a priori, so that considerable interest attaches to comparisons of results and particularly of predictions. In this study we have applied both least-squares and alternative procedures to the various relations studied and have made such comparisons<sup>8</sup> as seemed practical at the time. Further checks of the predictive value of the relations should be made as new observations become available. Though such comparisons from a single study or even from several studies cannot be treated as conclusive, they are relevant as an aid to judgment in choosing the most hopeful approach for a similar problem.

#### CRITICISMS OF INITIAL MODEL

Potentially useful extensions and refinements of the model given by (2.1)-(2.4) are easy to suggest. Making use of them in an empirical

<sup>6</sup> The estimation problems that would be encountered if the investigator did not assume independence of disturbances have been discussed by Lawrence R. Klein in *A Textbook of Econometrics*, Row Peterson & Co., 1953. See pp. 112-117.

<sup>7</sup> Karl A. Fox, *Relations Between Prices, Consumption and Production*, *Journal American Statistical Association*, Vol. 46, 1951.

James Tobin, *A Statistical Demand Function for Food in the U. S. A.*, *Journal Royal Statistical Society, Series A*, Vol. 113, Part II, 1950.

Henry Schultz, *The Theory and Measurement of Demand*; University of Chicago Press, Chicago, pp. 133-136, 1938.

<sup>8</sup> See pp. 74, 75.

study is rather harder. We have, in fact, only attempted a few modifications of the model. It may be useful to discuss briefly both the criticisms of the model that were taken into account in our reformulation and some that were not. The latter should be kept in mind as possible sources of specification error in the present study and as indicators of directions for future research.

#### TREATMENT OF LIVESTOCK FEED

One respect in which the model given by (2.1)–(2.4) is a crude approximation is in the treatment of livestock feed. Feed fed to livestock in a given year is treated as exogenous. Though it is not hard to think of grounds for regarding feed production of a given year as exogenous,<sup>9</sup> there are important types of feeds for which current domestic consumption need not equal current domestic production.

Feed grains can be economically stored or shipped abroad. Much of the domestic production of linseed cake and meal is exported in a typical year. Wheat is not ordinarily regarded as animal feed; yet the amount of wheat fed has constituted between 0.3 and 4.4 % of total livestock feed during our period of observation.<sup>10</sup> A rough comparison of total feed produced with feed consumed by livestock during the period 1926 to 1948 shows an average difference of about 6 %. For comparison, the average annual change in feed consumption is about 4 %.<sup>11</sup>

It thus seems useful to distinguish between feed consumed and feed produced in our model. This requires a revised explanation of the determination of feed fed. Consider first a fixed physical supply with alternative uses, say storage, export, and current feeding. The amount that

<sup>9</sup> See James H. Lorie *op. cit.*, pp. 17–40.

<sup>10</sup> The total quantity of feed fed was expressed in number of pounds of total digestible nutrients (TDN). Total digestible nutrients for any feedstuff represents the nutrients available for maintaining farm animals. The nonutilized portion of a feed will be mainly fiber which is incompletely broken down in digestion. For each year the quantity of wheat fed in TDN was obtained as a percentage of total feed fed in TDN.

<sup>11</sup> The total production of feed was computed in TDN for each year from 1926 to 1948. For the roughages, production and consumption were treated as identical. However, since the consumption of hay, pasture, corn, and oats by horses and mules was excluded from  $f_i$ , a similar adjustment was made to total production. In addition, the quantity of nonfeed uses of corn, oats, and barley was also deducted from total production, to arrive at a production figure that would be comparable with the consumption variable  $f_i$ . All data are from Feed Statistics, *Statistical Bulletins 85, 95*, U. S. Department of Agriculture, Washington, D. C. 1949, 1950. The years 1926–48 were used since data on non-feed uses of corn, oats, and barley are available for this time period. The absolute difference between the adjusted production figure and  $f_i$  was expressed as a percentage of  $f_i$ .

will be taken for each of these uses will depend on price, and the equilibrium price is the one at which the sum of the amounts going into the three uses equals the physical supply. We can think of a total demand relation obtained by adding the amounts for the three uses at each price. We then think of price as determined by the intersection of total demand and the inelastic physical supply. Alternatively, we may think of supply and demand for feed for current feeding. In this case the supply relation is obtained by subtracting from the physical supply the amounts going into storage and export for each possible price.

Since we are primarily interested in the behavior of persons closely connected with the livestock market, we choose the latter point of view and consider demand and supply of feed for current feeding. Both the demand and supply relations include, of course, variables other than quantity and price of feed. As indicated in (2.4), demand depends also on the price of livestock products and on the inventory or "herd" of livestock available at the beginning of the period. Supply depends on the factors that influence demand for feed for storage and demand for feed for export. Unfortunately these factors are difficult to specify and observe. We would expect demand for export to depend on level of income, price of livestock products, price of alternative feeds, and import restrictions in importing countries. In general data for other countries are not so complete as data for the United States, and one would have to know the structure of the market in these countries pretty well to make best use of such data as are available. Demand for feed for storage depends very much on anticipations of dealers and producers, and these are difficult to relate to observable variables. Government policy has also been important in determining demand for feed for storage and export during most of our period of observation. Because of these and other complications, we decided not to try to specify completely the supply of feed for current feeding in the present study. Our model is incomplete in this respect. It is to be hoped that future research will reveal more about the nature of this relation.

#### FIRST REVISION OF MODEL

The modifications indicated above are incorporated in the model represented by (2.7)-(2.11) below. Necessary changes in definitions of symbols are indicated.

$f_t$  = quantity of feed fed to livestock (now regarded as endogenous  
and generally not equal to quantity produced)

$f_t^*$  = physical supply of feed (regarded as exogenous)

$s_t'$  = unspecified or unobserved factors affecting supply of feeds for

current feeding

$$(2.7) \quad h_t : p_{t-1}, q_{t-1}, s_{t-1} \quad (\text{inventory relation})$$

$$(2.8) \quad l_t, f_t ; h_t, t \quad (\text{production relation})$$

$$(2.9) \quad f_t, q_t, p_t ; h_t \quad (\text{demand for feed})$$

$$(2.10) \quad l_t, p_t ; y_t, n_t, r_t \quad (\text{demand for livestock products})$$

$$(2.11) \quad J_t, q_t ; f_t^*, s_t' \quad (\text{supply of feed})$$

A semicolon may be read "appear in a relation with." The variables to the left of the semicolon are current endogenous variables, those to the right are regarded as predetermined in the particular relation being viewed. (2.8) could be read "Current endogenous variables  $l_t$  and  $f_t$  appear in a relation with predetermined variables  $h_t$  and  $t$ ." Regarding  $h_t$  as predetermined in (2.8) and (2.9) is proper if random disturbances appearing in (2.7) are statistically independent of disturbances in the other relations.<sup>12</sup> In this case, the model can be separated into two statistically independent parts, (2.7) being thought of as determining the probability distribution of  $h_t$ , given  $p_{t-1}$ ,  $q_{t-1}$ , and  $s_{t-1}$ , and (2.8)–(2.11) as determining a multivariate distribution of  $l_t$ ,  $f_t$ ,  $q_t$ ,  $p_t$  for given values of  $h_t$ ,  $t$ ,  $y_t$ ,  $n_t$ ,  $r_t$ ,  $f_t^*$ ,  $s_t'$ . This may be contrasted with the model given by (2.1)–(2.4) in which each relation determines a univariate distribution of a single endogenous variable for given values of certain predetermined variables.

#### SIMULTANEOUS DETERMINATION OF ENDOGENOUS VARIABLES

This aspect of the transition from (2.1)–(2.4) to (2.7)–(2.11) can perhaps be stated more simply by temporarily assuming that no random disturbances enter the relations. In a certain sense we may then say that each of the relations (2.1)–(2.4) determines the value of one endogenous variable. If the relations determining the exogenous variables and the lagged variables remain fixed, then we may consider the effects of changing the relations in the model one at a time. If (2.1) is changed,  $h_t$  is the only variable in (2.1) that will be altered. Similarly, if (2.2) is changed, only  $l_t$  (of the variables in 2.2) will change.  $p_t$  and  $q_t$  would respond to changes in (2.3) and (2.4), respectively.

In this sense each relation may be said to have one dependent and several independent variables. This does not hold for the system (2.7)–(2.11). (2.7) is the same as (2.1) and has one dependent variable  $h_t$ . However, such an association of a particular variable with a particular relation cannot be made for the other relations. If any one of the rela-

<sup>12</sup> See T. C. Koopmans, *op. cit.*, pp. 402–406.

tions (2.8)–(2.11) is altered, the variables  $l_t$ ,  $f_t$ ,  $q_t$ ,  $p_t$  all change. We say that they are simultaneously determined by the relations 2.8–2.11. Under suitable assumptions about the algebraic form of the relations and the distribution of random disturbances, least-squares theory would apply to any of the relations 2.1–2.4 but not to 2.8–2.11. The kind of mutual interdependence illustrated by the latter set is common among economic relations and gives rise to the economist's interest in simultaneous equations methods of analysis.<sup>13</sup>

#### HETEROGENEITY OF LIVESTOCK FEEDS

The original model has thus far been modified to allow for the fact that the quantity of feed fed to livestock depends on a market adjustment and is not determined by feed production alone. When we look at the nature of the markets for individual types of feed, we find rather wide differences. Pasture, for example, can be marketed or stored in only a very restricted fashion. Corn on the other hand can be sold in an almost worldwide market and can be economically stored for several years if conditions warrant. These differences seemed to offer some incentive for disaggregating the feed-consumption variable. To a considerable extent, differences in marketability, conditions of production, and chemical composition of feeds tend to coincide, leading to a rather natural classification of livestock feeds into roughage, feed grains, and protein feeds. The latter two classes are sometimes treated together under the heading of concentrates.

Roughage includes pasture, hay, straw, corn fodder and corn stover. Although hay can be stored and traded to a larger extent than pasture, these operations are costly for hay compared with concentrates. Roughage consumed does not necessarily equal roughage produced in a given year, but there is no well-organized market on which roughage consumption is brought into equilibrium with supply. Roughage may be wasted in one part of the country while it is very scarce and being economized in another. Hogs and poultry can make only limited use of roughage, whereas cattle can be fed roughage exclusively and in certain regions it is economical to follow this procedure. To a fairly close approximation, it seemed to us that roughage consumption could be regarded as deter-

<sup>13</sup> For more comprehensive discussions see the following:

Trygve Haavelmo, *The Probability Approach in Econometrics*, *Econometrica*, Vol. 17, Supplement, 1944.

———, *The Statistical Implications of a System of Simultaneous Equations*, *Econometrica*, Vol. 11, 1943.

Jacob Marschak, *op. cit.*

Herbert A. Simon, *Causal Ordering and Identifiability*, *Studies in Econometric Method*, Cowles Commission Monograph 14, *op. cit.*



mined by the aggregate and geographic distribution of roughage production and the numbers and geographic distribution of various types of animals. These factors seemed sufficiently independent of the relations in our model to justify us in regarding roughage consumption as exogenous.

The most important feed grain is corn. During our period of observation corn fed to livestock averaged 23.2% of the total livestock feed and 67.4% of feed grains fed. Other important feed grains are oats, barley, wheat, and grain sorghums. Feed grains average by weight about 7.9% total digestible protein and 67.9% of other digestible nutrients. This compares with 3.1% protein and 16.2% other digestible nutrients for roughage and 30.8% protein, 42.4% other nutrients for protein feeds.<sup>14</sup> For the most part, the items we have classed as protein feeds result as by-products in the processing of agricultural products. These include such items as tankage, various oil cakes and meals, fish meal, and dried-milk products. There are, however, a number of feed crops that resemble protein feeds in chemical composition and a number of by-product feeds whose composition is like feed grains. We elected to classify these according to composition rather than origin. As will be seen in Chapter III, this resulted in our putting wheat and rice mill feeds in with feed grains and calling soybeans a protein feed. However, with minor qualifications, we can say that feed grains have a fairly homogeneous chemical composition, are farm-produced, and are commonly fed to livestock on the farms where the feed is produced. They can be readily stored and transported but have rarely been imported or exported in large quantities by the United States. Protein feeds have a different chemical composition, are produced as by-products in processing operations, can be stored somewhat less easily than feed grains, but have sometimes entered relatively more into international trade of the United States.

#### SECOND REVISION OF MODEL

These differences would seem to make it worth while to distinguish supply-and-demand relations for feed grains and for protein feeds. A model reflecting these decisions with respect to the disaggregation of the feed variables is given below.

$a_i$  = amount of feed grains fed to livestock

$b_i$  = amount of protein feed fed to livestock

<sup>14</sup> Total digestible nutrients (TDN) are equal to total digestible protein (TDP) plus other digestible nutrients (ODN). The data and sources used in calculating these percentages are more fully discussed in Chapter III.

$c_t$  = quantity of roughage fed to livestock

$d_t$  = price of feed grains

$g_t$  = price of protein feed

$a_t^*$  = physical supply of feed grains

$b_t^*$  = physical supply of protein feeds

$s_t'$  = unspecified or unobserved factors affecting supply of feed grains  
for current feeding

$s_t''$  = unspecified or unobserved factors affecting supply of protein  
feeds for current feeding

$$(2.12) \quad h_t : p_{t-1}, q_{t-1}, s_{t-1} \quad (\text{inventory relation})$$

$$(2.13) \quad l_t, g_t, b_t ; h_t, c_t, t \quad (\text{production relation})$$

$$(2.14) \quad a_t, d_t, g_t, p_t ; h_t, c_t \quad (\text{demand for feed grain})$$

$$(2.15) \quad b_t, d_t, g_t, p_t ; h_t, c_t \quad (\text{demand for protein feed})$$

$$(2.16) \quad l_t, p_t ; y_t, n_t, r_t \quad (\text{demand for livestock products})$$

$$(2.17) \quad a_t, d_t ; a_t^*, s_t' \quad (\text{supply of feed grains})$$

$$(2.18) \quad b_t, g_t ; b_t^*, s_t'' \quad (\text{supply of protein feeds})$$

Perhaps some further explanation of the use of the variables  $s$ ,  $s'$ ,  $s''$  in the model is in order. As these are being used, an  $s$  (with or without primes) in a relation indicates that important factors that influence the behavior represented by the relation are omitted. The omissions arise because the investigators do not know what the other influences are or because data on the other factors have not been assembled. Of course, there are unspecified factors influencing all the relations. If we are willing to assume that the most important factors have been specified and that the influence of the others can be approximated by drawings from a probability distribution with certain properties,<sup>15</sup> then the relation may be analyzed statistically. This is how the statistical treatment of the relations that do not contain an  $s$  is rationalized. In effect, then, the inclusion of an  $s$  in a relation is an indication that, in the investigators' judgment, the neglected factors are so important that it would be useless

<sup>15</sup> These properties, of course, help to determine what statistical procedures are appropriate. In general, in this study we have assumed additive random disturbances with stable distributions over time and with finite first and second moments.

to attempt statistical analysis of the relation without further specification of variables.

#### MULTIPLE ROLES OF ANIMALS IN HERD

One other important aspect of livestock production is not shown in the system (2.12)–(2.18). A given animal at a given time may be viewed as (a) a finished good, (b) a good in process or (c) a piece of fixed capital. This is perhaps most dramatically apparent for a young heifer, say 16 to 20 months old, of a beef or dual-purpose breed. If the animal has been well fed, she may be immediately marketable as medium or possibly good beef. Alternatively she might profitably be fed intensively for a short period with a consequent increase in weight and possibly in grade. A third alternative would be to retain her in the breeding herd to produce calves (or calves and milk if she were a dual-purpose heifer). Though a narrower range of alternatives exists for most other animals, it is typically true that selling livestock for slaughter reduces the productive capacity of the farm herd. Thus an individual producer or all producers as a group can increase current marketings either by increasing feeding and production or by decreasing the productive potential of their herds. For a given level of production, an increase in marketings means a decrease in production capacity retained.

If there existed only one type of animal and if we measured inventory, production, and sales in a common unit, say pounds, the connection indicated above would appear in the form of an exact accounting identity. Ending inventory for any period would be equal to beginning inventory plus production minus sales. If we have various animals and products and do not employ common weights in forming indices of inventories, production, and sales, the accounting relation will not be exact. The discrepancy will depend on the size of various components in each of the indices. In our present model we have fairly strong reasons for not using common weights in forming our various aggregates. In (2.16), the demand for livestock products relation, we wish to represent the reactions of consumers and processors to various levels of marketings of livestock products for given levels of income, population, other prices. To a processor or consumer an 800-lb heifer and an 800-lb steer are almost perfect substitutes. To a beef producer they are quite different.

It is clear that, for the production relation to be accurate,  $h_i$  should reflect the productive potentials of the various types of animals in the herd. For the demand relation to be accurate,  $l_i$  should combine animals and products sold as nearly as possible according to their typical marginal rates of substitution in consumption. Thus one of the difficulties of an aggregate study is that (1) an inappropriate index must be used

for  $h_t$  in the production relation (and the demand for feed relations), (2) an inappropriate index must be used for quantity sold in the livestock demand relation, or (3) our natural accounting identity must be made inexact. The last alternative seems the least of the three evils; hence (2.26) below contains a random component, reflecting changes in composition of inventories, production, and sales.

On the average, during our period of observation, the difference between livestock products produced and livestock products sold was 2.5% of production. This may seem too small to bother with, unless we recognize that we are seeking an explanation for annual variations in our variables. The average annual variation in livestock produced was 3.7%.<sup>16</sup>

### THIRD REVISION OF MODEL

The livestock demand relation (2.16) makes no allowance for the influence of past incomes and prices on present consumption. Most arguments for recognizing such influences<sup>17</sup> are based on the belief that consumers do not immediately adjust consumption to changed incomes and prices. Although a variety of influences are undoubtedly present, it has seemed to us that consumption of the immediately preceding period was perhaps as good a simple indicator of the influence of past circumstances on present consumption as we could find.<sup>18</sup> Lagged consumption is therefore included in (2.23), the livestock demand relation in the revised model presented below.

$l_t$  = quantity of livestock and livestock products produced

$l_t^*$  = quantity of livestock and livestock products sold

$k_t$  = price of farm labor in period  $t$

$\alpha$  = a constant reflecting average adjustment for differences in weighting in measuring  $h_t$  and  $l_t, l_t^*$

<sup>16</sup> These percentages are computed from data in Table III on p. 60.

<sup>17</sup> Some discussion of the effects of past experience on current behavior are contained in the following:

J. S. Duesenberry, *Income, Saving and the Theory of Consumer Behavior*, Ch. 5, Harvard University Press, Cambridge, 1949.

F. Modigliani, Fluctuations in the Saving-Income Ratio: a Problem in Economic Forecasting, *Studies in Income and Wealth*, Vol. XI, National Bureau of Economic Research, New York 1949.

M. J. Farrell, Irreversible Demand Functions, *Econometrica*, Vol. 20, 1952.

James Tobin, *op. cit.*

<sup>18</sup> See also Chapter VII, pp. 112, 113.

$v_t$  = a disturbance representing the effects of variations in composition of  $h_t, l_t^*, l_t$

- (2.19)  $l_t, a_t, b_t; h_t, c_t, t$  (production relation)  
 (2.20)  $a_t, d_t, g_t, p_t; h_t, c_t$  (demand for feed grain)  
 (2.21)  $b_t, d_t, g_t, p_t; h_t, c_t$  (demand for protein feed)  
 (2.22)  $l_t, d_t, g_t, p_t, l_t^*; h_t, k_t$  (supply of livestock products)  
 (2.23)  $l_t^*, p_t; y_t, n_t, r_t, l_{t-1}^*$  (demand for livestock products)  
 (2.24)  $a_t, d_t; a_t^*, s_t'$  (supply of feed grains)  
 (2.25)  $b_t, g_t; b_t^*, s_t''$  (supply of protein feeds)  
 (2.26)  $h_{t+1} = h_t + \alpha(l_t - l_t^*) + v_t$  (inventory relation)

(2.26) and (2.22) replace (2.12) of the previous model.  $h_{t+1}$  is the inventory of livestock on farms at the end of the  $t$ th period. An additional relation has been added along with the new endogenous variable,  $l_t^*$ . It seems convenient at this stage to state the inventory relation in terms of beginning and ending inventories of the  $t$ th period since the other relations all refer to activities of the  $t$ th period.

Relations 2.20-2.22 represent the behavior of producers of livestock. These will sometimes be called the farm decision relations and will be discussed more fully in Chapter VI. Briefly, the view taken is that the producers start a period with given herds and decide during the period how to feed and dispose of them. The amount of roughage to be fed to the given herds is regarded as a function of the composition of the herds and the roughage produced (primarily determined by weather) and not a matter for current choice on the part of the producers. The producers do make current decisions on amounts of grain and protein feeds to be consumed, quantity of livestock products to be produced, quantity to be marketed, and numbers of livestock of various types to be retained for future production. Producers are free to decide these five quantities, subject to the restrictions given by the production relation 2.19 and the inventory relation 2.26. There are then, in terms of the variables used here, three free decisions which together with the two restrictions determine the five variables indicated. (2.20)-(2.22) show the factors on which these three decisions depend. Logically, of course, the decision relations could have been written in various equivalent ways. For example, the meaning of the model would be unchanged if  $h_{t+1}$  were substituted for  $l_t^*$  in (2.22). As (2.22) stands, we are likely to think of producers deciding on sales and the resulting ending inventory being determined by (2.26).

It would be no different to think of producers deciding on their ending inventory and sales being determined by (2.26). Given (2.26) and  $l_t$ , one decision is sufficient to determine  $l_t^*$  and  $h_{t+1}$ .

In the previous model,  $s_{t-1}$  represented unobserved factors affecting the desires of producers to hold animals at the beginning of period  $t$ . In the new model the subscripts have been advanced one period (this is a trivial change since the inventory relation is assumed to hold for all values of  $t$ ) and the price of labor  $k_t$  has been substituted for  $s_t$ . Although there is little doubt that the price of labor does affect willingness to retain animals for future production, there may still be important factors that have been neglected. The authors decided, in view of the difficulty in specifying additional observable variables<sup>19</sup> to experiment with (2.22) in the form given above. However, it is quite possible that an  $s_t$  should have been retained in this relation. It should also be noted that there are inputs other than animals and feed which should in principle enter our production relation. Unfortunately these are not observable. We have elected to analyze the production relation as given, but should keep the missing inputs in mind as possibly qualifying the results. Analyses of alternative forms of the production relation are given in Chapter V.

#### TOPICS FOR FURTHER STUDY

In Chapter IV the relations are assumed to have a particular parametric form, and estimates of parameters in (2.19)–(2.23) are obtained. Before these are presented, it may be well to review briefly a number of possible criticisms of the model given by (2.19)–(2.26) that the authors have not found it possible to take into account in the present study. Difficulties in treating aggregates and the gaps in the model indicated by  $s'_t$ ,  $s''_t$  have already been mentioned. Difficulties that arise in connection with the use of a time variable in the production relation have also been noted. In addition, our model does not explain variations in feeds produced. Though there are some grounds for regarding year-to-year fluctuations as predetermined, one should recognize that, for many uses to which the relations may be put, longer-run responses of feed production to economic conditions may have to be considered.

Regarding consumer income and other prices as exogenous is probably not strictly justified. Since income generated by livestock production and processing is a component of total income, it is clearly not entirely correct to regard total income as independent of the relations of the livestock model. The assumption of independence was made in this study to

<sup>19</sup> For further discussion see pp. 104, 105.

limit the scope of the study, in the hope that biases resulting from such dependence as does exist would be small.<sup>20</sup>

The demand relation that has been specified relates farm price for livestock products (including live animals) to quantity of livestock products sold from farms. It thus combines the behavior of consumers and processors of livestock products. To have specified separate relations representing the behavior of consumers and the behavior of processors would have been preferable in some respects but would have required rather more study of the behavior of processors than was possible.<sup>21</sup> Certainly this aspect of the market should also receive more detailed attention in future studies.

<sup>20</sup> A discussion of the possible bias in treating general variables such as income and the general price level as exogenous in models directed towards specific markets has been given by T. C. Koopmans in *The Bias in Single Equation Methods of Estimating Behavior Equations Relating to a Small Sector of the Economy, Cowles Commission Discussion Paper, Statistics 336*, Cowles Commission for Research in Economics, University of Chicago, Chicago, 1949 (unpublished). Work on the interconnection of particular economic sectors with the general economy is being undertaken at the Survey Research Center, University of Michigan. See *Research Seminar in Quantitative Economics, Annual Report, 1951-52*, Survey Research Center, University of Michigan, Ann Arbor, 1952 (mimeographed).

<sup>21</sup> We may think of (2.23) as being derived from a consumer demand relation and a relation representing behavior of processors, say

$$(2.23') \quad l_i^*, p_i^* ; y_i, n_i, r_i, l_{i-1}^* \quad (\text{consumer demand})$$

$$(2.23'') \quad p_i^*, p_i, l_i^* \quad (\text{behavior of processors})$$

where  $p_i^*$  represents prices paid by consumers for livestock products and  $p_i^* - p_i$  is the processors' margin. If (2.23'') were solved for  $p_i^*$  and substituted into (2.23'), our relation 2.23 would result. Of course, if other variables enter (2.23''), they will be introduced into (2.23) in this process.

In a recent study by Been,  $p_i, p_i^*, l_i$  appeared to be the more important variables determining total margins of processors. See Richard O. Been, *Price Spreads between Farmers and Consumers, Agricultural Information Bulletin 4*, U. S. Department of Agriculture, Washington, D. C., p. 7, 1949.

For the relation between the farm and the commercial sector for all food, Girshick and Haavelmo also suggest the use of quantity of sales, and farm and retail prices. See M. A. Girshick and Trygve Haavelmo, *Statistical Analysis of the Demand for Food, Examples of Simultaneous Estimation of Structural Equations, Econometrica*, Vol. 15, pp. 91-93, 1947.

## CHAPTER III

### THE OBSERVATIONS

#### CHOICE OF SAMPLE PERIOD

In the various statistical analyses to be presented, our sample will consist of observations over time of all of the variables in the economic model given on p. 23, except  $s'_i$ ,  $s''_i$ , and  $v_i$ . These variables have been fairly well defined by their verbal descriptions and by the roles that they play in the model. Some of the variables do not correspond very closely to any regularly compiled data. It is necessary to construct from the available data measurements that will correspond as closely as possible to the concepts employed in the model.<sup>1</sup> The purposes of this chapter are to describe the measurements that were used, to indicate the sources of data, and to present some rationalization of the choices of measurements. The observations used in the study are tabulated at the end of the chapter.

The sample period in the study extended from 1920 through 1949. The choice of 1920 as the initial observation was mainly conditioned by the availability of data. Though production and marketings data were available before 1920 for cattle and calves, sheep and lambs, hogs, chickens and eggs, a breakdown of January 1 numbers by sex, age, and purpose for cattle and calves, sheep and lambs, and hogs was only available from 1920 on. As for the feeds consumed by livestock, separate observations were available for the important feed grains (corn, oats), and for the other feed grains combined, before 1920. For the by-product feeds, quantity data were less complete, series on most of the by-product feeds beginning about 1926. The investigators believed that estimates could be made for missing observations from 1920 on with some degree of confidence, but that attempting to extend some series back beyond 1920 would involve considerably more error in the observations.

Another early choice that had to be made was the time interval to which the observations should refer. Many of the data on livestock, particularly of livestock inventories on farms, are available only for

<sup>1</sup> For a discussion of the general problem of the relations between theoretical models and measurements see T. Haavelmo, *The Probability Approach in Econometrics*, *Econometrica*, Supplement, 1944.



calendar years, whereas many of the data on feed consumption are available only for crop years defined to take account of the normal harvest period for various crops. After some examination of the data, it was decided to use the calendar year as the basic time interval for the study and to adjust the observations accordingly. Whatever basic time interval is used, there will always be developments during the interval not fully reflected in data for the full interval. These contribute to the disturbances in the relations. Thus, a year in which there was a shortage of feed in the first or last half is really dissimilar to a year in which feed was more uniformly available; yet our data do not reflect such differences. In principle, it should be possible to construct a model with different time periods for different variables. Complications in the construction and interpretation of such a model made it seem advisable to forego such an attempt in the present study and to adjust measurements to relate to a common time period.

#### WEIGHTS FOR QUANTITY AGGREGATES

Observations on the production, marketings, births and deaths of cattle and calves, sheep and lambs, and hogs are available in *Meat Animals—Farm Production and Income, 1924-44*, and annual releases for 1945-50 published by the U. S. Department of Agriculture. Production of eggs, milk, turkeys, broilers, and chickens is from *Agricultural Statistics* published annually by the U. S. Department of Agriculture. Slaughter of cattle and calves and farm prices for the components of  $l_i$  and  $l_i^*$  are from this latter source. The data on January 1 numbers are from *Livestock on Farms and Ranches, January 1, Statistical Bulletin 88* of the U. S. Department of Agriculture and from *Agricultural Statistics, 1942-51*. The most detailed data on feed consumption by livestock are contained in two bulletins by R. D. Jennings. The first, *Feed Consumption by Livestock, 1910-41, Circular 670, U. S. Department of Agriculture*, contains data on a calendar-year basis. The second, *Consumption of Feed by Livestock 1909-47, Circular 836, U. S. Department of Agriculture*, presents data on an October-year basis, that is a year beginning and ending on October 1.

The problem of constructing aggregates from heterogeneous components is always difficult and involves subjective judgments of the researcher. Some guides for judgment can usually be found by examining the components of the aggregate and the relations of the aggregate to other variables in the model. Consider the quantity aggregates  $l_i$ ,  $l_i^*$  in the model on p. 23.  $l_i^*$  represents the quantity of livestock and livestock products sold from farms during the  $t$ th year (home-consumed products were treated as sold).  $l_i$  represents the quantity of livestock

and livestock products produced. Each of these is an aggregate of quantities of cattle, calves, hogs, sheep and lambs, chickens, broilers, turkeys, milk, and eggs. As was noted in Chapter I, these products can, generally speaking, be expected to be substitutes both in production and in consumption. If they were perfect substitutes in production, the equilibrium levels of their relative prices would equal their constant rates of substitution in production, and these rates of substitution would furnish natural weights for aggregating production of the various commodities.

Similarly, if the commodities to be aggregated were perfect substitutes in consumption, their relative prices would, in equilibrium, be equal to their constant rates of substitution in consumption, and these rates of substitution would furnish natural weights for aggregating quantities sold. Although the commodities entering  $l_i^*$  and  $l_i$  are not perfect substitutes, either in production or consumption, there would seem to be a high degree of substitutability on the consumption side, and, at least at the margin, considerable substitutability on the production side. It thus seems that observed prices over a fairly long period should roughly indicate marginal rates of substitution over the same period. In view of these considerations, we have elected to use average farm prices over the sample period 1920-49 as weights in forming  $l_i$  and  $l_i^*$ .

The remainder of the present chapter is devoted to the details of the construction of indices of these and other variables. The observations used in computations are given in Tables III, IV, and V at the end of the chapter. Some readers may wish to omit the details of adjustment and aggregation and refer immediately to the tabulated data.

#### DATA ON PRODUCTION AND SLAUGHTER

Published data on production were available for all the commodities that entered  $l_i$ . For hogs, cattle and calves, sheep and lambs, the published series on liveweight production of these animals in the United States were constructed as follows. Liveweight production for each state was obtained by deducting the weight of livestock shipped into the state from the total pounds of marketings (including both slaughtered animals and animals shipped in for feeding and breeding) plus farm slaughter, and adding or subtracting the difference in inventory poundage between the beginning and end of the year. State figures were summed to give total production for the United States. For chickens and turkeys, the published series on liveweight production were somewhat similarly defined. For chickens, the number produced consisted of the total number raised (equals hatchings minus death loss of chicks and young chickens during the year), less the death loss during the year of mature birds

on hand January 1. The number of chickens produced was then converted to a liveweight basis.

For turkeys, production consisted of the number raised (equals poults hatched minus death loss of poults and young turkeys during the year), less the death loss of breeder hens on hand January 1. Numbers produced were then converted to a liveweight basis. However, for turkeys, no observations on liveweight production were available for the period 1920-28. Production was estimated for these years. In the period 1929-38, turkey production increased about 60%. We have assumed that turkey production showed a similar increase in the period 1920-28 and that this increase occurred at the constant rate of  $5\frac{1}{2}\%$  per year. Commercial broiler production in pounds liveweight was assumed zero before 1934 when broiler production first began to make an important contribution to total poultry production. Farm production of eggs is reported in million dozen. Farm production of milk in million pounds, as reported, excludes milk produced by cows not on farms.<sup>2</sup> It may be doubted whether milk produced by cows not on farms enters commercial channels, and so, where possible, we have excluded from the feed variables those quantities of feed going to livestock not on farms. Furthermore, since our livestock inventory variable refers to livestock on farms, the exclusion of milk production by cows not on farms seemed appropriate. For all the above commodities, it would appear that the published data on production correspond reasonably well with the theoretical concept of production for the commodities entering  $l_t$ .

As for cattle and calves and sheep and lambs, separate farm prices were reported for each of these four categories. However, liveweight production data were reported for cattle and calves combined, and for sheep and lambs combined. The farm price of calves per pound was appreciably higher in most years than the corresponding price for cattle. A similar situation was found in comparing the farm price of lambs per pound with the farm price of sheep per pound. It appeared to the investigators that these differentials in price should be taken into account in the weighting of the combined production data for cattle and calves and the combined production data for sheep and lambs. The production of cattle and calves in pounds liveweight was weighted by the average price of cattle over the sample period. To the resulting dollar value was added a calf "bonus" in dollars, which allowed for the heavier contribution of the calf price to the production in dollar terms. The calf

<sup>2</sup> Farm production of milk for the years 1920-23 was taken from R. D. Jennings, *Consumption of Feed by Livestock 1909-47, Circular 886, U. S. Department of Agriculture, Washington, D. C., p. 57, 1949.* Subsequent years were from *Agricultural Statistics, 1942-51, U. S. Department of Agriculture, Washington, D. C.*

"bonus" was calculated in the following manner. Net births of calves (births minus deaths) were computed for each year. Each calf was assumed to weigh 200 lb at slaughter, so that the calf "bonus" is 200 times (average farm price of calves per pound, minus the average farm price of cattle per pound) times (net births).<sup>3</sup>

For sheep and lambs, a farm price per pound for each year for the combined commodities was obtained by dividing cash receipts in dollars from the sale of sheep and lambs by marketings in pounds.<sup>4</sup> The average farm price for sheep and lambs per pound was the mean of the series so resulting, and this was the weight used in forming  $l_i$ .

For  $l_i^*$ , the contribution that turkeys, broilers, eggs, and milk make to sales was taken to be the same as the contribution of these four groups to  $l_i$ . For broilers, eggs, and milk, the influence of inventory changes is negligible, whereas, for turkeys, relatively few data exist for earlier years which would enable the value of inventory changes to be considered in this class.  $l_i^*$  represents a sales variable, where sales are to the commercial sector for consumption purposes. Not all published data gave a series of livestock and livestock products that entered into the commercial sector, and it was necessary to isolate from the observations available a sales component which corresponded with the theoretical component of  $l_i^*$ .

#### ADJUSTMENTS IN INDIVIDUAL SERIES

1. *Hogs.* Marketings in thousand pounds liveweight included inshipments for feeding and breeding purposes. Inshipments were excluded from the marketings data for two reasons. First, animals that are shipped in, fed, and sold during the calendar year would appear to be counted both as inshipments and also as subsequent slaughtering. Second,  $l_i^*$  represents a consumption variable, since we are primarily interested in the demand for livestock products for use by consumers and not in the demand for feeding and breeding purposes. Inshipments in thousand pounds liveweight were computed by dividing the cost of inshipment in a given year by the farm price per thousand pounds for that year. The live weight of inshipments was then deducted from liveweight market-

<sup>3</sup> The slaughter weights which are used throughout this chapter were suggested by the weights of animals slaughtered under Federal inspection from *Livestock Meats and Wool. Market Statistics and Related Data*, 1942, and *Livestock Market News. Statistics and Related Data*, 1949, both published by U. S. Department of Agriculture, Washington, D. C.

<sup>4</sup> Data on cash receipts for sheep and lambs together with the gross income and cost of inshipments figures used for  $l_i$  are from *Meat Animals—Farm Production and Income 1924-44 and 1945-50* from U. S. Department of Agriculture, Washington, D. C. The inshipments data for hogs under  $l_i^*$  were from this same source.

ings. To the figure so resulting was added farm slaughter in thousand pounds liveweight. Average weights for farm slaughter in each year were obtained by dividing marketings in pounds by marketings in numbers and using these average weights to convert farm slaughter in numbers to thousand pounds liveweight. The final "sales" figures were then weighted by average farm price per pound for hogs over the sample period to yield a dollar value of "sales" at an average price.

2. *Cattle and calves.* Total slaughter of cattle in thousands including farm slaughter was converted to thousand pounds liveweight, using weights of Federally inspected slaughter. Total slaughter of calves in numbers was similarly converted to pounds liveweight.

As a check on the accuracy of the data we employed the following identity.

$$(3.1) \quad x_t^* - x_t = X_{t+1} - X_t$$

where  $x_t^*$  = production in the  $t$ th period of a given commodity entering  $l_t$

$x_t$  = sales of the same commodity in the  $t$ th period

$X_t$  = inventory at the beginning of the  $t$ th period of this commodity

$X_{t+1}$  = inventory at the end of the  $t$ th period of this commodity

These checks were performed for cattle and calves and hogs. For cattle and calves we employed pounds liveweight as the unit of measure for (3.1). The production and slaughter data were already in pounds liveweight, and to convert the numbers of the various classes of cattle and calves on hand January 1 to pounds liveweight we assumed various weights. These weights were suggested by those published in Morrison.<sup>5</sup> The check for cattle and calves revealed that total liveweight slaughter for cattle and calves combined tended to be greater than production plus or minus the change in inventory. It was decided to adjust the series on both cattle slaughter and calf slaughter downward. The decision to adjust the slaughter data rather than the production or inventory data was made after discussion with various advisers had made it appear that the production and inventory data were somewhat more reliable than the reported figures on total slaughter of cattle and calves. Numbers of Federally inspected slaughter seemed to be more reliably reported than numbers of other slaughter. To adjust the liveweight slaughter of both cattle and calves, we used the ratio of total 1920-49

<sup>5</sup> F. B. Morrison, *Feeds and Feeding*, 2d ed., Morrison Publishing Co., New York, p. 615, 1943.

production of cattle and calves combined to the total 1920-49 slaughter of cattle and calves combined. For hogs, equation 3.1 was again used with pounds liveweight as the unit of measure. Using the adjusted marketings data employed for hogs in  $l_i^*$ , we found a close agreement between production minus sales and the inventory change. Ideally, the data check indicated by (3.1) should also be performed for sheep and lambs and chickens. However, the resources available precluded such a check, and, particularly for chickens, the nature of the enterprise permits marked fluctuations in inventory throughout the year, so that the check is somewhat less meaningful than for the longer-lived animals. In each case the adjusted pounds liveweight of slaughter were weighted by the appropriate average farm price, to obtain an index in dollars of cattle "sales" and calf "sales," and these latter two combined to form an index of cattle and calf "sales."

3. *Sheep and lambs.* For sheep and lambs, the marketings data included inshipments for feeding and breeding purposes as they did for hogs. In view of the differential between the farm price of sheep and the farm price of lambs, the following procedure was adopted to obtain a "sales" figure in pounds liveweight. Slaughter income (gross income from all sales of sheep and lambs minus the cost of inshipments) was divided by the farm price per pound of the combined sheep and lambs category (described under  $l_i$  above). Pounds liveweight of sheep and lambs were then weighted by the average farm price of sheep and lambs over the sample period to yield a "sales" figure in dollars.

4. *Chickens.* An average weight for chickens was computed for each year from data on farm price per head and farm price per pound. Numbers sold plus numbers consumed in farm households were weighted by the calculated average weights to obtain sales in pounds liveweight. Pounds liveweight were then weighted by the average farm price per pound over the sample period to lead to a sales figure in dollars.

#### CONSTRUCTION OF PRODUCTION, SALES, AND PRICE INDICES

$p_t$  is an index of the price of livestock products sold during the year. To obtain this index, sales data in physical measure (pounds liveweight, dozen, etc.) for each year were multiplied by the appropriate farm price for that year, and summation yielded the value of sales at current prices for that year. Division of the total value of sales at current prices by  $l_i^*$  (sales at average prices) resulted in the price index for that year. This procedure insured that the relation price times quantity equals value would hold for our aggregates as it does for an individual commodity.

We may summarize the procedures for  $l_t$ ,  $l_t^*$ , and  $p_t$  as follows:

$$(3.2) \quad l_t^* = \sum_i \bar{w}_i x_{it}$$

$$(3.3) \quad l_t = \sum_i \bar{w}_i x_{it}^*$$

$$(3.4) \quad V_t = \sum_i w_{it} x_{it}$$

$$(3.5) \quad p_t = \frac{V_t}{l_t^*}$$

where  $l_t^*$  = sales in dollars of livestock and livestock products in the  $t$ th period

$\bar{w}_i$  = average farm price over the observation period of the  $i$ th component of  $l_t^*$

$x_{it}$  = sales in physical measure (liveweight pounds, dozen, etc.) of the  $i$ th component of  $l_t^*$  in the  $t$ th period

$l_t$  = production in dollars of livestock and livestock products in the  $t$ th period

$x_{it}^*$  = production in physical measure of the  $i$ th component of  $l_t$  in the  $t$ th period

$V_t$  = total value of livestock and livestock products sold in the  $t$ th period

$w_{it}$  = price of the  $i$ th component of  $l_t^*$  in the  $t$ th period

$p_t$  = ratio of value of sales at current prices to value of sales at average prices

#### DATA ON LIVESTOCK INVENTORIES

$h_t$  is an index of the quantity of livestock held on farms at the beginning of the  $t$ th period. The inclusion of  $h_t$  in the production relation recognizes the fact that the efficiency of the feeding operation depends on the beginning inventory. If increasing quantities of feed are fed to a given herd, one would expect diminishing returns to operate. An increase in the numbers fed so that the feed inputs are now spread over a larger number of animals would be expected to increase the efficiency of the feeding operation and increase production. In the decision relations 2.20 and 2.21 the role of  $h_t$  is again a question of the efficiency of the feeding operation. One would expect that, for an efficient feeding program, the quantity of feed demanded would be larger, the greater the beginning inventory. In both cases an increase in the number of animals on hand January 1 will increase the marginal productivity of feed inputs, so that production will be extended, and increased quantities of feed will be fed. To be consistent with the construction of  $l_t$ , we will require that prices also be used as weights in combining the components that enter  $h_t$ . However, one must realize that, whereas, in consumption, the sex of an animal does not affect the substitutability of a commodity, this

is not true so far as the role of the initial inventory in future production is concerned. Similarly, although age of an animal affects its usefulness both in production and in consumption, the two effects need not coincide. The components of  $h_i$  should be combined in such a way as to recognize the possible contribution of an animal to production. To this end, an average potential production for an individual animal in each category was estimated, and these estimates were used as weights in forming  $h_i$ . Data were obtained on the number of livestock on farms, January 1, by classes. The types of livestock that entered this aggregate were cattle and calves, hogs, and sheep and lambs.

With chickens and turkeys the investigators encountered some difficulty in weighting the numbers on hand January 1 by an average potential production. For chickens, data on hens and pullets and other chickens on hand January 1 were available over the sample period. However, the time required to enter and leave poultry production is short; production can be expanded within a few months and laying flocks reduced rapidly by the sale of laying hens for meat. The problem that arises is to estimate the possible contribution that chickens on hand January 1 make to total calendar-year production of meat and eggs. Chickens are predominantly hatched in the months of March, April, and May.<sup>6</sup> The seasonality of marketings has not changed much in the last twenty years, according to Christensen and Mighell.<sup>7</sup> These authors state that, for the four months of heaviest marketings, July through October, sales made up 51 % of the total in 1945-49, compared with 17 % during the four months of lightest marketings, January through April. Chickens are sold at approximately 3 months old to older ages, but it would appear, from the seasonality of hatchings and marketings, that hatchings after January 1 contribute most to annual marketings. Since the proportion of total chicken production that goes into storage is approximately 3.8 to 6.5 %, the bulk of production in a calendar year would appear to come from chickens hatched after January 1. The situation in egg production was less clear than that for meat production. In the early years of the sample, egg production displayed a marked seasonality, with peak production in the months of March through June. For the years 1945-49, production in the same months was approximately 43 % of total production.

<sup>6</sup> Monthly data on hatchings and production are from *The Poultry and Egg Situation*, February 1949 and August-September 1950, published monthly by the U. S. Department of Agriculture, Washington, D. C.

<sup>7</sup> R. P. Christensen and R. L. Mighell, *Competitive Position of Chicken and Egg Production in the United States*, *Technical Bulletin*, 1018, U. S. Department of Agriculture, 1950.



The decline in seasonality has been attributed mainly to the increased rate of lay in fall and winter, rather than to increased numbers of layers on hand January 1.<sup>8</sup> This means that layers on hand January 1 could be sold off in the early months of the year, without impairing total production of eggs, to a greater extent than was previously possible. In view of the difficulty of obtaining data on the contribution that the January 1 inventory made to total production of meat and eggs, and the evidence that the January 1 inventory was of little importance in total annual meat production, but of somewhat greater importance in total annual egg production, the investigators decided to exclude January 1 chickens from  $h_t$ . Though a case can be made for attributing some weight to the January 1 inventory, in our opinion the omission of chickens from  $h_t$  seemed to involve as little error as the use of small but inaccurate weights. For turkeys, the total number on farms January 1 was available from the year 1929 on. A breakdown of January 1 numbers by sex was available from 1937 on. In view of the incomplete data and the difficulty of assigning weights of the beginning inventory, turkeys were also excluded from  $h_t$ . For categories of animals that were included in  $h_t$  the following procedures were used.

#### CONSTRUCTION OF INDEX OF LIVESTOCK ON HAND

1. *Cattle and calves* on farms January 1 are divided into animals for milk production and those not for milk production. In the for-milk-production category, the classification is cows and heifers 2 years old and over, heifers 1 to 2 years old, and heifer calves.

(a) Cows and heifers 2 years and over for milk. Animals in this category are possible milk producers and also may produce calves during the year. Total milk production over the sample period was divided by the total numbers of animals in this classification to obtain an average milk-production figure. This latter figure was weighted by the average farm price of milk over the sample period, to give an average potential milk-production estimate of \$81.27.

The weight for potential calf production was calculated in the following manner. Total calves saved (births minus deaths) was divided by the total cows and heifers 1 to 2 years old and over in the for-milk category, plus total cows in the not-for-milk category, plus 0.6 times heifers 1 to 2 years in the not-for-milk category, to yield a net birth rate  $X_1$ . The figure 0.6 was used, on the assumption that approximately 60% of the heifers in the category concerned are bred, the remaining 40% being fattened. The net birth rate was 0.62. The net birth rate

<sup>8</sup> *Ibid.*

times the total number of cows and heifers 1 to 2 years old and over for milk leads to the total number of calves saved in dairy herds. The total number of calves slaughtered, divided by the total number of calves saved in dairy herds, gives the average percentage (54%,  $X_2$ ) of calves sold for slaughter, if it is assumed that all slaughtered calves are from dairy herds. The total value of calves slaughtered divided by the total number of calves slaughtered results in an average value of a slaughtered calf of \$21.58 ( $X_3$ ). The average weight for a calf that is saved and slaughtered during the year is then the product of  $X_1$ ,  $X_2$ ,  $X_3$ : namely, \$7.23. Some calves will not be slaughtered in a given period but will be carried over into the next period. This potential contribution is computed as  $(1 - X_2)$  times the liveweight of a heifer calf January 1 (295 lb from unpublished Bureau of Agricultural Economics data) times the average farm price per pound for calves over the sample period. This yielded an estimate of \$14.40. The sum  $\$81.27 + \$7.23 + \$14.40 = \$102.90$  is the average potential contribution of an animal in the class 1a.

(b) Heifers 1 to 2 years for milk potentially possess all the producing capacities of animals in 1a, but also may gain weight in moving from 1b to 1a. The average gain in weight in so moving is 265 lb (unpublished BAE data), and so the value component due to gain in weight is 265 times the average farm price for cattle per pound over the sample period. The estimate yielded was \$23.45. The total average potential production estimate was then  $\$102.90 + \$23.45 = \$126.35$  for animals in 1b.

(c) Heifer calves potentially can gain weight in moving from 1c to 1b. The gain in weight in so moving is 280 lb (unpublished BAE data), and 280 times the average farm price of cattle per pound over the sample period was the appropriate weight (\$24.78) for animals in class 1c.

(d) Cows and heifers 2 years old and over not for milk. Animals in this category may produce calves, and, in line with the assumption that all slaughtered calves are from dairy herds, calves from the class 1d will be carried over into the next period. The average weight of a calf in this class on January 1 is 305 lb (unpublished BAE data), so that the estimated value component was given by  $X_1 \times 305 \times$  average farm price of calves per pound = \$20.06.

(e) Heifers 1 to 2 years not for milk. In 1a, it was assumed that only 0.6 of this class were bred, so that the potential production of a calf carried over was  $(0.6)(20.06) = \$12.04$ . A contribution to production may also be made by the gain in weight in moving from 1e to 1d. This gain in weight was estimated as 265 lb (unpublished BAE data), so that the average potential production in this class was  $\$12.04 + \$23.45 = \$35.49$ .

(f) Calves not for milk. Potential production in this class consists of

a gain in weight. Animals in this category include heifer and bull calves, with the latter tending to show larger weight gains than the former. It was estimated that an average gain in weight would be 335 lb, and the potential production estimate was then  $335 \times$  average farm price of cattle = \$29.65.

(g) Steers. Allowance was made for a gain in weight only of 225 lb. The average weight of a steer January 1 was taken as 795 lb (unpublished data from BAE), and the average slaughter weight was estimated as 1020 lb. The estimate of potential production was  $225 \times$  average farm price of cattle = \$19.91.

(h) Bulls. The average potential production of a bull is a possible weight gain, and the estimate was arbitrarily taken as 10% that of cows 2 years and over not for milk; namely, \$2.01.

2. Hogs on farms January 1 are classified as under 6 months old; sows and gilts 6 months and over and others 6 months and over. Following Jennings, pigs under 6 months were treated as fall pigs, pigs over 6 months as spring pigs.<sup>9</sup> Potential production of pigs under 6 months consists in the value of farrowing plus any gain in weight, since gilts that are capable of farrowing during the coming year are included in the under-6-months category. Sows and gilts 6 months and over may farrow and also gain weight. Pigs over 6 months are potential weight gainers only.

(a) Fall pigs. The average gain in weight during the year of a fall pig was given as 140 lb by Jennings.<sup>10</sup>  $140 \times$  average farm price of hogs per pound during the sample period = \$14.53 was the component of the potential production estimate due to the gain in weight. It was assumed that one half of the total fall pigs saved were farrowed by gilts under 6 months on January 1. To obtain a fall birth rate from new sows, 0.5 times fall pigs saved was divided by fall pigs on hand January 1 ( $X_3$ ). It was estimated that deaths subsequent to the pigs-saved enumeration were approximately 10% of all pigs saved. The average weight of a fall pig on January 1 was given by Jennings as 90 lb. The component due to gilts under 6 months on hand January 1 which subsequently farrow was  $0.9 \times X_3 \times 90 \times$  average farm price of hogs per pound = \$4.71. The total average potential production of fall pigs was then  $\$4.71 + \$14.53 = \$19.24$ .

(b) Spring pigs. The average gain in weight of a spring pig from the

<sup>9</sup> R. D. Jennings, Units of Livestock Production. A Measure of Grain Consumption at the National Level, *F. M. 63*, Bureau of Agricultural Economics, U. S. Department of Agriculture, Washington, D. C., 1948.

<sup>10</sup> The weights used throughout this section on hogs were suggested by the weights in Table 4 of R. D. Jennings, *ibid.*

previous crop was assumed to be 50 lb during a given calendar year, as indicated by Jennings. The estimated average production potential was then  $50 \times$  average farm price of hogs per pound = \$5.19.

(c) Sows and gilts. Animals in this class may produce a spring litter and a fall litter, both of which will show a subsequent weight gain, as well as a gain in weight during the year by animals in this class on hand January 1. This latter weight gain was assumed to be 60 lb, so that this component of the estimated potential production was  $60 \times$  average farm price of hogs per pound = \$6.23. A spring birth rate was calculated by dividing the total spring pigs saved by the total number of sows and gilts on hand January 1. This was 5.3 ( $X_1$ ). Deaths subsequent to the pigs-saved enumeration were again assumed at the rate of 10% of pigs saved. The average weight of a spring pig on hand January 1 was given as 190 lb by Jennings, so that the component due to a spring litter was  $0.9 \times X_1 \times 190 \times$  average farm price of hogs per pound = \$94.07.

A fall birth rate for animals in 2c was computed by assuming that 0.5 times total fall pigs saved were farrowed from sows and gilts in this category and by dividing 0.5 times the total fall pigs saved by the total number of sows and gilts on hand January 1. This birth rate was 1.54 ( $X_2$ ). Since the average weight of a fall pig on hand January 1 was given by Jennings as 90 lb, the component due to a fall litter was  $0.9 \times X_2 \times 90$  times average farm price of hogs per pound = \$12.95. The total average production potential estimate for animals in this category was  $\$6.23 + \$94.07 + \$12.95 = \$113.25$ .

3. *Sheep and lambs* on farms January 1 are given by the following classes: sheep on feed and stock sheep, the latter category including ewe and ram lambs, ewes 1 year and over, rams and wethers. The latter are predominantly wool producers, and, since wool did not enter  $l_i$ , wethers were excluded from  $h_i$ . Similarly, no weight was given for sheared wool production by other sheep and lambs.

(a) Sheep and lambs on feed are potential weight gainers. The average weight gain as indicated from unpublished data from the BAE was 12 lb, so that the average potential production estimate in this class was  $12 \times$  average farm price of lambs over the sample period = \$1.31.

(b) Ewe and ram lambs. Ewes in the class were taken to be too young for breeding (Morrison<sup>11</sup>), so that this class exhibits a gain in weight only. The gain in weight in moving from 3b to 3c was given as 40 lb (unpublished BAE data) so that the average potential production estimate was  $40 \times$  average farm price of sheep per pound over the sample period = \$2.30.

(c) Ewes one year and over may produce a lamb but probably show

<sup>11</sup> F. B. Morrison, *op. cit.*, pp. 778-779.

little gain in weight. Total lambs saved divided by the total ewes 1 year and over gave a lambing rate of 0.85 ( $X_1$ ). Deaths subsequent to weaning were approximately 10% of lambs saved. The average weight of a lamb on January 1 was assumed to be 75 lb, so that the average potential estimate was  $0.9 \times X_1 \times 75$  times average farm price of lamb per pound = \$6.25.

(d) Rams. The estimate used for rams was arbitrarily assumed to be 10% that for ewes 1 year and over. The estimate used was \$0.63.

Animals in the various classes were weighted by the relevant computed weights, and summation over the various elements in 1, 2, 3 yielded  $h_i$  in dollars.

#### MEASUREMENT OF FEED FED TO LIVESTOCK

$f_i$  represents the pounds of total digestible nutrients contained in feeds fed to livestock. No single measure of the feeding value of different feeds is acceptable under all circumstances. Total digestible nutrients is the common denominator most often used. The use of total digestible nutrients in forming an aggregate of feed fed may be most useful in combining feed grains or the various types of hay. Jennings considers that the use of total digestible nutrients as a basis for combining all feeds may tend to undervalue the protein content of some of the high-protein feeds, since protein exercises a strategic role in the feed ration, being essential for reproduction and growth.<sup>12</sup> In Chapter V an experiment was attempted in which the amount of total digestible nutrients was broken down into total digestible protein and other digestible nutrients. In such an experiment, explicit recognition was given to the protein in the diet. The results reported in Chapters IV and V, together with a comparison of the sources of nutrients in Table XII, p. 87, lead the investigators to settle on total digestible nutrients as the measure of relative feeding values in most of the relations. One further consequence of using total digestible nutrients is that, from the production relation, the productivity of a pound of total digestible nutrients could be computed, and, from this, the productivity of individual components of  $f_i$ .

Sources of data and adjustments applied to obtain quantities consumed of the 47 different feeds that entered our aggregate are presented below. The feeds are listed in Table II, p. 51.<sup>13</sup>

<sup>12</sup> R. D. Jennings, *Feed Consumption by Livestock, 1910-41. Relations between Feed, Livestock, and Food at the National Level, Circular 670, U. S. Department of Agriculture, 1943.*

<sup>13</sup> Unless a different source is given all calendar-year data on feed consumption are from R. D. Jennings, *Feed Consumption by Livestock 1910-41, op. cit.* All October year data on feed consumption by livestock are taken from R. D. Jen-

## FEED GRAINS

Corn and oats consumption were available on a calendar-year basis for all livestock, excluding horses and mules, for the years 1920-41. For the years 1942-47, figures were available on an October-year basis. Calendar-year figures for the latter years were obtained from a moving average of the October-year data. Consumption of corn for calendar year 1944, for example, was taken to be 0.75 times published consumption for the year beginning October 1, 1943, plus 0.25 times consumption for the year beginning October 1, 1944. The ratio 0.75:0.25 was indicated by the pattern of quarterly consumption figures, where it appeared that approximately 25% of total consumption of corn on an October-year basis occurred in the October-December quarter.<sup>14</sup> Observations for 1948 and 1949 for corn were obtained as follows. The ratio of calendar-year consumption of corn by livestock, excluding horses and mules, for 1926-41 to consumption by all livestock for the same years was applied to adjust downward the October-year observations for consumption by all livestock for 1948 and 1949, and the moving average was then applied. The corn consumption figures refer to consumption as grain and include the grain in corn silage and corn hogged off. The approximate grain content of corn silage is given by Jennings as 12%.<sup>15</sup> For oats, exactly the same procedure was followed to obtain calendar-year observations for 1942-49 as was used for corn. Corn and oats seemed to be the only feeds, apart from pasture and hay, for which an adjustment due to the exclusion of horses and mules seemed practicable. This may result in overestimation of other feed inputs in the earlier years of the sample period.

Data on barley and sorghum grains separately and on wheat and rye combined for the calendar years 1927-49 are from unpublished BAE data. For the years 1920-26, observations on each of these three categories were secured as follows. Jennings' calendar-year data included a category "other grains," which consisted of barley, wheat, and additional grains fed. These calendar-year observations on "other grains" for 1920-26 were partitioned between barley and sorghum grain separately and wheat and rye combined. The proportionate weights given to each of these three classes were taken from their ratios in consumption on an

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nings, Consumption of feed by Livestock, 1909-47, *op. cit.* The series on by-product feeds from this latter source were extended to 1949 from data published in *The Feed Situation*, July 1951, U. S. Department of Agriculture, Washington, D. C. monthly.

<sup>14</sup> Feed Statistics, *Statistical Bulletin 96*, U. S. Department of Agriculture, Washington, D. C., 1950.

<sup>15</sup> R. D. Jennings, Feed Consumption by Livestock, 1910-41, *op. cit.*, p. 3.

October-year basis for each of the years 1920-26. We extended the process to obtain observations on wheat and rye combined for the additional years 1927-34. Each of the observations on wheat and rye combined for 1920-34 was now taken, and, for each of these years, the ratio of wheat fed on farms where produced to rye fed on farms where produced<sup>16</sup> for that year was employed to break the combined wheat and rye category into separate observations for wheat and rye. After 1934, wheat fed to livestock is reported separately.<sup>17</sup> To obtain rye separately, these observations were subtracted from the unpublished BAE data on wheat and rye combined.

Wheat mill-feeds consumption on an October-year basis was given for the years 1926-47.<sup>18</sup> The series was completed to 1949. To place these data on a calendar-year basis, a moving average using the weights 0.75:0.25 was employed. For wheat mill feeds, one would expect production and consumption to move together to a great extent, and the quarterly production figures indicated that approximately 25 % of production on an October-year basis took place in the October-December quarter. Consumption in the years 1926-30 was approximately 104 % of production, both on an October-year basis. From unpublished BAE data on calendar-year production, the calendar-year consumption for 1920-26 was estimated by taking 104 % of production. For rice mill feeds the same pattern of consumption was assumed as for wheat mill feeds. The same moving average was used on October-year data to obtain calendar-year observations from 1927-49. From 1920-26, unpublished BAE data on calendar-year production were assumed equal to calendar-year consumption; stocks of rice mill feeds are undoubtedly small. The other by-product feeds, which are listed as other grains in Table II, p. 51, include tentative estimates of hominy feed, oat mill feed, molasses, and screenings, and were placed on a calendar-year basis by using a moving average with weights of 0.75:0.25 on October-year data for the years 1927-49, the pattern for these feeds being assumed the same as for wheat mill feeds. Before 1927 consumption was assumed constant at 2000 tons per year.

Data for dried and molasses beet pulp for 1927-49 were treated with a moving average with weights of 0.75:0.25 to convert an October-year

<sup>16</sup> The data on wheat and rye fed on farms where produced are from *Agricultural Statistics, 1942-51, op. cit.*

<sup>17</sup> Feed Statistics, *Statistical Bulletin, 85, U. S. Department of Agriculture, Washington, D. C., 1949.*

<sup>18</sup> For most of the by-product feeds, quarterly or monthly production data are given in Feed Statistics, *Statistical Bulletin, 85, 95, op. cit.* Where quarterly production figures are referred to, these two bulletins are the source, unless otherwise specified.

basis to a calendar-year basis. For the years 1928-32, October-year consumption of pulp was approximately 0.027 times total production of sugar beets.<sup>19</sup> October-year observations for 1920-26 were estimated, using this conversion factor and data on production for these years. The consumption of brewers' dried grains was assumed zero before 1936. For 1936-49, October-year data were placed on a calendar-year basis, using a moving average with weights of 0.75:0.25, these weights again being indicated by quarterly production figures. A moving average with weights of 0.75:0.25 was used to convert October-year data on distillers' dried grains to a calendar-year basis for the years 1936-49. Before 1936 unpublished BAE data on calendar-year production were taken equal to consumption.

Most data<sup>20</sup> on miscellaneous feeds fed on farms where produced were on a crop-year basis and had to be converted to a calendar-year basis. As was done previously, a system of moving averages was employed. For rice the weights that were used were 0.70:0.30, which gives a heavier weight to the period subsequent to harvest, when feeding is probably somewhat heavier than in later periods. For soybeans and cowpeas, feeding was assumed heaviest in the fall months, since the beans and peas are probably of poorer quality than those marketed and do not store easily, and the moving average weights were 0.40:0.60. For the years 1920-24, when no observations were available for soybeans and cowpeas, estimates were obtained as follows. The ratio of calendar-year consumption to acreage harvested for 1925-29 was used on acreage harvested for 1920-24 to obtain estimated consumption for 1920-24.<sup>21</sup> Peanuts fed on a crop-year basis were taken to be the same as on a calendar-year basis on the grounds that the bulk of the peanuts fed are fed in the period between harvest and the end of the year. Buckwheat data on a crop-year basis were treated with a moving average with weights of 0.75:0.25 to convert them to a calendar-year basis. Such weights assume that consumption is approximately equal in each quarter. For cottonseed, the data were lagged one year to place crop-year data on a calendar-year basis.<sup>22</sup> Such a procedure was based on the assumption that little feeding takes place between the harvesting of the crop and the beginning of the following calendar year. For 1924-49, calendar-year data on liquid whole milk fed to calves were available.<sup>23</sup> Consumption

<sup>19</sup> *Agricultural Statistics, 1942-51, op. cit.*

<sup>20</sup> *Feed Statistics, Statistical Bulletins, 85, 95, op. cit.*

<sup>21</sup> Data on acreage harvested for soybeans and cowpeas were from *Agricultural Statistics, 1942-51, op. cit.*

<sup>22</sup> The October-year data were completed with crop-year data from *Agricultural Statistics, 1942-51, op. cit.*

<sup>23</sup> *Feed Statistics, Statistical Bulletins, 85, 95, op. cit.*



in 1920-23 was assumed the same as in 1924. October-year data for 1927-49 for velvet beans were converted to a calendar year by a moving average with weights of 0.40:0.60, on the grounds that feeding is heaviest in the fall quarter as with soybeans and cowpeas. For 1920-26, the ratio of production (equals consumption) to acreage harvested for 1927-31 was used as for soybeans and cowpeas.

#### PROTEIN FEEDS

For the by-product feeds most of the data were on an October-year basis. A moving average was used to convert an October year to a calendar year. For soybean cake and meal, linseed cake and meal, and copra cake and meal from 1927 to 1949 the weights used were 0.70:0.30, on the basis that protein feeds are fed heaviest in the fall and winter quarters, and from unpublished BAE data on quarterly consumption of cottonseed cake and meal, which was assumed indicative of the pattern of feeding for most of the oil cakes and meals. Unpublished BAE data on the quarterly consumption of cottonseed cake and meal indicated that, for the years 1926-42 the percentage distribution of total consumption for these years was October-December 41 %, January-March 28 %, April-June 14 %, July-September 17 %. For cottonseed cake and meal, data indicated that the October-December quarter was the quarter of heaviest production. For soybean cake and meal, linseed cake and meal, and copra cake and meal, quarterly production data indicated a more even distribution of production throughout the year. In view of this more even distribution, consumption in the October-December quarter was not weighted so heavily as the quarterly consumption data on cotton seed cake and meal would indicate. For years prior to 1927, in which no observations were available, estimates were made as follows.

Considerable import and export takes place in the oil cake and meals.<sup>24</sup> For four years of the sample period, a comparison was made between net exports and quantity fed for some of the by-products feeds for which data were available. Net exports of dried grains, cottonseed cake and meal, soybean cake and meal, linseed cake and meal, peanut cake and meal, tankage, fish meal and soybeans were compared with total consumption of the by-product feeds for the years 1922, 1927, 1936, and 1944. The comparison was made in terms of TDN for net exports and consumption. For 1922, net exports were approximately 9 % of consumption; for 1927, 11 %; for 1936, 1 %; and for 1944, 1 %. For soybean cake and meal, in the early years of the sample period imports predominated over exports. For 1923-26, consumption was assumed equal

<sup>24</sup> Data on calendar-year imports and exports of by-product feeds are available in *The Feed Situation*, June 1945, U. S. Department of Agriculture, Washington, D. C.

to calendar-year production plus 0.9 times imports. For 1922, consumption in the calendar year 1922 was assumed equal to production (year beginning October 1, 1922) plus 0.9 times imports. Observations for 1920 and 1921 were assumed equal to that of 1922. For linseed cake and meal, exports tended to outweigh imports, but data were available on both from 1922, and on exports from 1920. Consumption for the years 1922-26 was assumed equal to production less net exports, the production data being completed with unpublished BAE data.<sup>25</sup> For 1920 and 1921, consumption was assumed equal to production minus 0.9 times exports. For copra cake and meal, for the years 1920-26 consumption was estimated as production plus net exports. For peanut cake and meal, the moving average employed in October-year consumption data had weights of 0.60:0.40 for the years 1926-49. These weights differ from those used on the oilseed cakes and meals considered up to this point. Peanut cake and meal are fed predominantly in the southern parts of the United States, and the weights used assume that the quarterly feeding pattern was the same as that exhibited by cottonseed cake and meal. Before 1927 calendar-year consumption of peanut cake and meal was assumed equal to calendar-year production.

The principal animal protein feeds are tankage and meat scraps, fish meal, and dried-milk products. For 1936-49, October-year data were available. The weights in the moving average to convert data for tankage and meat scraps and fish meal to a calendar-year basis were 0.70:0.30. For tankage and meat scraps for 1921-30, calendar-year consumption was assumed equal to calendar-year production plus average imports of 22,500 tons for each year. The average import figure was for the years 1931-35, when data on imports were first reported. For 1931, consumption was estimated as equal to production plus imports, January-September 1931, plus 0.25 times imports for the year beginning October 1931. For 1932-35, consumption was assumed equal to production plus imports. The latter were for an October year and were placed on a calendar-year basis by employing a moving average with weights of 0.75:0.25.<sup>26</sup> An estimate for 1920 was obtained by computing the fraction that consumption of tankage and meat scraps for 1921 was of total animal slaughter for 1921, and using this same fraction of total animal slaughter for 1920. For fish meal, consumption in the years 1931-35 was assumed equal to calendar-year production plus net cal-

<sup>25</sup> For linseed cake and meal, copra cake and meal, peanut cake and meal, tankage and meat scraps, the production data in Feed Statistics, *Statistical Bulletin*, 85, 95, *op. cit.*, were supplemented by unpublished BAE data.

<sup>26</sup> October-year data on import and export of by-product feeds are available in Feed Statistics, *Statistical Bulletins*, 85, 95, *op. cit.*

endar-year imports. In the years 1921-30, consumption was taken as equal to production plus average net imports 1931-35. The observation for 1920 was assumed equal to that for 1921.

For the years 1936-49, October-year data on dried-milk products were converted to a calendar-year basis by employing a moving average with weights of 0.80:0.20. Heavier weights were given to the preceding October year for dried-milk products than for the other animal-protein feeds, on the grounds that dried-milk products are, to some extent, fed to young animals, and quarterly production data indicated that the production of dried-milk products was heaviest in the months of April, May, and June. This would seem to imply a somewhat heavier weight to spring feeding. For 1920-35, average consumption for the years 1935-39 was utilized for each year. October-year data on skim-milk consumption were for the years 1927-49. The moving average employed had weights of 0.60:0.40 to convert these data to a calendar-year basis. Before 1927, consumption was estimated by using the ratio of skim milk fed to livestock to milk production for the years 1927-31 on production data for the earlier years.

For cottonseed cake and meal, amounts fed to livestock on a calendar-year basis for the years 1927-49 were from unpublished BAE data. Before 1927, data were available on an August 1 year, and, for these data, consumption was taken as supply for domestic consumption, minus ending inventory, minus amounts used for fertilizer.<sup>27</sup> The August 1 year was converted to a calendar year, using unpublished estimates from the BAE on quarterly consumption as follows.  $C_1$  equals consumption for January-March plus April-June plus 0.3 times July-September for the years 1927-31.  $C_2$  equals 0.7 July-September consumption plus October-December consumption.  $r_1$  equals  $C_1/C_1 + C_2$ ,  $r_2$  equals  $C_2/C_1 + C_2$ . The August 1 year was converted to a calendar year by use of a moving average with weights  $r_1:r_2$  for each year for 1920-26. For gluten feed and meal, October-year data on production were available for the entire sample period. Production was assumed equal to consumption, and the October-year data were converted to a calendar-year basis by employing a moving average of 0.75:0.25, the weights being indicated by quarterly production figures. Alfalfa meal is mainly fed in the late fall and winter months as a substitute for pasture and range, and the weights employed in the moving average to convert October-year data for the years 1928-49 were 0.60:0.40. Before 1928, consumption for each year was assumed as the average consumption for 1928-32.

<sup>27</sup> Quantities of cottonseed cake and meal used for fertilizer are from Feed Statistics, Supplement to the February 1940 issue of *The Feed Situation*, U. S. Department of Agriculture, Washington, D. C.

## ROUGHAGE

The total quantity of hay fed to livestock, excluding horses and mules and livestock not on farms, for the years 1920-44 was on a calendar-year basis. For the years 1945-47, consumption was on a May 1 year.<sup>28</sup> Observations for 1948-49 were obtained by deducting an assumed 18,000 tons for horses and mules and 1750 tons for livestock not on farms from consumption by all livestock. To convert a May 1 year to a calendar year, a moving average with weights of 0.65:0.35 was employed for the years 1945-49. The weights used were to some extent arbitrary, since no information was available on quarterly or monthly feeding rates of hay. However, it was assumed that little hay would be fed during the grazing period and that hay would be heavily fed during the winter months. An assumed constant monthly feeding rate would imply weights of 0.33 and 0.67; so the weights actually used gave considerable emphasis to the quantities of hay fed in the early part of the calendar year. To obtain consumption of individual types of hay, the total consumption of hay was divided amongst individual hays, using the percentage contribution that each hay makes to total hay production.<sup>29</sup> This procedure applies to the individual hays, beginning with clover and timothy hay and ending with cowpea hay in the list given in Table II on p. 52.

For sorghum, forage data were available for the years 1929-49, the production series being used as consumption.<sup>30</sup> Although the data given were on an October-year basis, it was assumed that most of the sorghum forage is fed off before the advent of winter with little feeding after December. On these grounds, October-year data were used as calendar-year data. For the years 1920-28, consumption was estimated by employing the ratio of sorghum-forage production to the production of all sorghums for grain, for the years 1929-33, on data on sorghum-grain production for these earlier years.<sup>31</sup> For sorghum silage, consumption was again taken as equal to production. The series were for 1929-49 on an October-year basis. This series was first completed back to 1919, by employing the ratio of sorghum-silage production to the production of all sorghums for grain, and proceeding as for sorghum forage. To place the October-year series on a calendar-year basis, a moving average was

<sup>28</sup> The observations for 1945-47 were from R. D. Jennings, *Consumption of Feed by Livestock, 1909-47*, *op. cit.* This series is the same as the disappearance data on a May 1 year reported in *Feed Statistics, Statistical Bulletins, 85, 95, op. cit.*

<sup>29</sup> The production of types of hay is reported in *Feed Statistics, Statistical Bulletins, 85, 95, op. cit.*

<sup>30</sup> The October-year data for sorghum forage, sorghum silage, and wet beet pulp were completed with data from *Agricultural Statistics, 1942-1951, op. cit.*

<sup>31</sup> *Feed Statistics, Statistical Bulletin, 85, op. cit.*

used with weights of 0.70:0.30, which imply a heavier rate of feeding in the October–December quarter. For wet beet pulp (production equals consumption), October-year data were available for 1930–49 and were weighted by a moving average with weights of 0.50:0.50. The October-year series was first completed back to 1919 by using the ratio of wet beet-pulp production to total sugar-beet production for the years 1929–34. To obtain corn-silage consumption on a calendar-year basis, the procedure was that used by Jennings in obtaining calendar-year data for 1910–41. Namely, 40 % of production was fed in the year produced, and 60 % in the following year. This series included the grain in silage, and, to obtain the consumption of silage as roughage, an average grain content of 12 % was deducted. Corn hogged off as roughage was obtained by multiplying acreage hogged off by the yield of corn silage per acre by 0.88.<sup>32</sup> It was assumed that most of the corn is hogged off before the end of December. Data on peanuts hogged off were available for the years 1937–49.<sup>33</sup> The ratio of total acreage hogged off in the southeast area to total consumption in the United States for 1937–49 was applied to data on acreage hogged off in the southeast area, to yield estimates for 1920–36.<sup>34</sup>

For pasture two pieces of information were used. October-year data on pasture consumption in feed units<sup>35</sup> by all livestock, excluding horses and mules, were available up to 1949. These data were based on average consumption by the various types of livestock and were adjusted by the index of pasture condition.<sup>36</sup> Clough<sup>37</sup> had constructed an index of pasture and range consumption based on acreage and condition, but not taking into account animal numbers. Jennings' data on pasture consumption allowed for animal numbers and availability, the latter being indicated by the condition index. Clough's data took into account the acreage of pasture and the condition index. We were inclined to combine both these sets of data, on the grounds that pasture consump-

<sup>32</sup> The acreage and yield figures are from *Agricultural Statistics, 1942–51, op. cit.*

<sup>33</sup> *Farm Production, Farm Disposition and Value of Principal Crops, 1937–49*, U. S. Department of Agriculture, Washington, D. C.

<sup>34</sup> Peanuts in Southern Agriculture, *F. M. 65*, U. S. Department of Agriculture, Washington, D. C., 1947.

<sup>35</sup> The feed unit employed by Jennings takes corn as the base: One pound of corn is one feed unit, and the nutritive value of other feeds is then expressed as equivalent to so many feed units. For a more detailed discussion, see R. D. Jennings, *Consumption of Feed by Livestock 1909–47, op. cit.*, p. 54, footnote 1.

<sup>36</sup> The index of pasture and range condition is from Feed Statistics, *Statistical Bulletins, 85, 95, op. cit.*

<sup>37</sup> Malcolm Clough, *Feed Production in the United States: Index Numbers, 1909–43*, unpublished manuscript, U. S. Department of Agriculture, Washington, D. C.

tion by livestock is affected by animal numbers, total acreage of pasture, and condition. The first problem that arose was to convert feed units into physical measure. The net energy<sup>38</sup> of pasture grasses and clovers in comparison with corn (the base of the feed unit standard) indicated a conversion factor of 0.165; that is, if a pound of corn is one feed unit, then one pound of pasture is 0.165 feed unit. Each observation was divided by 0.165 to convert feed units to physical measure, that is, tons of pasture. On the assumption that little pasture and range are fed after October 1, the observation for a year beginning October was taken as an observation for the subsequent calendar year.

The data from Clough were recalculated, using additional data.<sup>39</sup> The difference between our procedure and that of Clough was that the latter computed an index of plowable pasture equivalent in acres, using conversion factors to change woodland pasture and pasture other than plowable (excluding pasture not in farms) to plowable pasture equivalent. We also computed plowable pasture equivalent and used the same conversion factor on woodland pasture, but we included pasture not in farms and changed the conversion factor from 0.25 to 0.1, the smaller conversion factor allowing for the poorer-quality pasture in the category pasture not in farms. A further difference was that Clough used an average value in acres for woodland pasture and pasture other than plowable, whereas we obtained observations on each of the three categories of pasture, using a linear interpolation for the intercensus years. The interpolation was performed in the following manner. For two successive census years, total pasture acreage plus cropland harvested was obtained.<sup>40</sup> A linear interpolation yielded observations on this combined acreage for intercensus years. Similar interpolations yielded acreages of woodland pasture and other than plowable pasture. Subtraction of these latter two acreages plus cropland harvested for that year resulted in an estimate of plowable pasture for the given year. To allow for the pasture-range condition on consumption, average pasture condition was given a weight of 0.66, and average range condition a weight of 0.34.

<sup>38</sup> The net energy values for corn and pasture are taken from R. D. Jennings, *Consumption of Feed by Livestock 1909-47*, *op. cit.*, p. 52. Net energy refers to the worth of a feed as a source of energy for productive purposes when the feed is used properly in a well-balanced ration. Since no feed unit value was available for pasture, the procedure given above was adopted.

<sup>39</sup> Total pasture in the United States consists of plowable pasture, woodland pasture, pasture other than plowable, and pasture not in farms, the first three categories being pasture in farms, and the last category being idle grassland and woodland and brushland. See *Land Utilization in the United States*, U. S. Department of Agriculture, Washington, D. C., 1947.

<sup>40</sup> These acreage data are from *Agricultural Statistics, 1942-51*, *op. cit.* and *Land Utilization in the United States*, *op. cit.*

The weights are based on estimated amounts of feeding coming from pasture and range, respectively.<sup>41</sup> The condition figure arrived at times plowable pasture equivalent resulted in a production-condition figure in acres.

To convert acreage to pasture in tons, an average "yield" was computed from total (1920-49) pasture consumption in tons from Jennings and total (1920-49) production-condition in acres from Clough. For each year, the production-condition figure in acres was multiplied by the average "yield," to form production-condition estimates in tons. The production-condition estimates in tons will tend to overestimate pasture consumption in the early years and underestimate consumption in later years because of a component due to horses and mules having to be excluded. October-year data on pasture in physical tons fed to horses and mules were obtained for 1920-49.<sup>42</sup> For each year the deviation of pasture fed to horses and mules in physical tons from the mean of the series was obtained, and this deviation was added to the production-condition estimates in tons. As would be expected, these deviations are negative in early years and positive in later years of the sample. Finally, our estimate of pasture consumption in tons was obtained by giving a weight of 0.7 to the estimates based on Jennings and a weight of 0.3 to the estimates based on Clough. The weights used were somewhat arbitrary, but it was believed by the investigators that animal numbers were more important in determining pasture consumption, so that a heavier weight was given to data based on animal numbers.

#### CONSTRUCTION OF FEED AGGREGATES

To actually obtain  $f_i$ , the average total digestible nutrients (TDN) content of the feed considered was necessary. Average composition figures for the feeds considered were obtained.<sup>43</sup> To allow for the differ-

<sup>41</sup> Malcolm Clough, *op. cit.*

<sup>42</sup> The available October-year data for horses and mules were completed by a personal communication from R. D. Jennings to the authors.

<sup>43</sup> Average composition figures of most of the feeds listed in Table II for Poultry and livestock other than poultry are available in R. D. Jennings, Consumption of Feed by Livestock, 1909-47, *op. cit.* p. 52. However, for the wheat millfeeds, we have used Jennings figures on wheat bran and middlings and have weighted these composition figures by the percentages of bran and middlings in by-products from the milling operation. These percentages are 41% bran and 59% middlings. See F. B. Morrison, *op. cit.*, p. 342. For the rice mill feeds, those by-products from the milling operation which are available as feed consist of 62% bran, 23% polish, and 15% brewers' rice. See F. B. Morrison, *op. cit.*, p. 354-355. An average composition figure for rice mill feeds was obtained, using these weights and the composition figures for the three components in F. B. Morrison, *op. cit.*, p. 988. For rice, buckwheat, peanuts, and peanuts hogged off, the composition figures are from F. B.

ence in availability between poultry and livestock other than poultry for some feeds the published values were weighted. For corn, oats, wheat and rye, soybean cake and meal, peanut cake and meal, copra cake and meal, tankage and meat scraps, fish meal, dried-milk products, gluten feed and meal, brewers' dried grains, and distillers' dried grains, the percentages of these feeds going to poultry and other livestock were used as weights.<sup>44</sup> Where less than 10% of a feed went to poultry, the composition figures used were those for livestock other than poultry. The average composition figures actually employed in this study are shown in Table II.

In Chapter V, one version of the production relation that was fitted involved a disaggregation of  $f_i$ . One of the important changes that has occurred in livestock feeding practices since the early years of the sample in this study is the increased feeding of protein concentrates. An attempt was made to estimate the separate effects of protein and other nutrients in the ration. For this purpose, the total digestible protein content (TDP) of the feeds was obtained, using the composition figures in Table II. The total pounds of other digestible nutrients (ODN) in feeds fed to livestock was obtained by subtracting the total TDP figure from  $f_i$ , which was measured in pounds TDN. The total pounds of digestible protein and the total pounds of other digestible nutrients contained in feeds fed to livestock are reported as  $y_9$  and  $y_{10}$ , respectively, in Table IV, p. 62.

As was mentioned in Chapter II,  $f_i$  was also disaggregated into three components, feed grains, protein concentrates, and roughage. Table II contains a list of the feeds included in  $f_i$  and their classification into the three types of feed. Roughages were classified mainly on physical properties and origin. In the other two classes, the investigators could have differentiated feed grains and protein feeds on the basis of origin, that is, farm-produced or produced as the result of some processing activity.

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Morrison, *op. cit.*, pp. 976-986. The reader will note that TDN figures for peanuts exceed 100. This is possible for peanuts since TDN include fat, which is multiplied by 2.25, since the energy value for animals is approximately 2.25 times that of protein or carbohydrate. See F. B. Morrison, *op. cit.*, p. 42.

For the other grains category in Table II, since these contained oat mill feed and hominy, principally the latter, Jennings' figure on hominy feed was adjusted downward to allow for the poorer quality feedstuffs. The wild hay composition figure is from R. D. Jennings, *Feed Consumption by Livestock, 1910-41, op. cit.*, p. 12. For the category other hay, since this contains wild hay, grains, cut green, and sweetclover hay, the composition figures in Table II were somewhat arbitrarily chosen, using the composition figures for these three categories as guides.

<sup>44</sup> These percentages were taken from R. D. Jennings, *Consumption of Feed by Livestock, 1909-47, op. cit.*, pp. 20-23.



TABLE II  
COMPOSITION PER 100 LB OF COMMON FEEDS FOR ALL  
LIVESTOCK, INCLUDING POULTRY

Feed	ODN	TDP	TDN	$\frac{\text{TDP}}{\text{ODN}}$
<i>Feed Grains</i>				
Corn.....	72.1	6.8	78.9	0.094
Wheat.....	63.8	12.0	75.8	0.188
Rye.....	56.9	9.5	66.4	0.167
Barley.....	68.3	9.9	78.2	0.145
Oats.....	59.6	8.9	68.5	0.149
Sorghum grains.....	72.3	8.4	80.7	0.116
Other grains.....	55.0	5.0	60.0	0.091
Rice.....	62.9	6.2	69.1	0.099
Buckwheat.....	68.0	9.7	77.7	0.143
Wheat mill feeds.....	62.6	14.0	76.6	0.224
Rice mill feeds.....	65.2	8.4	73.6	0.129
Dried and molasses beet pulp.....	63.5	4.3	67.8	0.068
Cowpeas.....	56.8	19.2	76.0	0.337
Peanuts.....	112.8	27.1	139.9	0.239
Cottonseed.....	73.7	17.1	90.8	0.232
Velvet beans.....	62.7	19.0	81.7	0.303
Copra cake and meal.....	57.0	16.1	73.1	0.282
Liquid whole milk.....	12.9	3.3	16.2	0.256
<i>Protein Feeds</i>				
Soybeans.....	53.9	33.7	87.6	0.625
Skim milk.....	49.5	31.2	80.7	0.630
Cottonseed cake and meal.....	37.2	32.6	69.8	0.876
Soybean cake and meal.....	40.6	35.7	76.3	0.879
Linseed cake and meal.....	47.7	29.3	77.0	0.614
Peanut cake and meal.....	47.6	30.6	78.2	0.748
Tankage and meat scraps.....	32.8	30.6	63.4	0.933
Fish meal.....	20.1	51.9	72.0	2.582
Dried-milk products (as for skim milk).....	49.6	27.9	77.5	0.563
Gluten feed and meal.....	53.2	24.0	77.2	0.451
Brewers' dried grains.....	44.8	19.6	64.4	0.438
Distillers' dried grains.....	56.2	22.0	78.2	0.391
Alfalfa meal.....	32.1	12.2	44.3	0.380
<i>Roughages</i>				
Pasture.....	12.6	2.9	15.5	0.230
Wild hay.....	35.0	1.6	36.6	0.046
Corn silage.....	14.9	1.1	16.0	0.074
Corn hogged off (as for corn silage).....	14.9	1.1	16.0	0.074
Sorghum forage (as for kafir fodder).....	45.6	4.0	49.6	0.088
Sorghum silage.....	15.9	1.1	17.0	0.069
Wet beet pulp.....	8.0	.8	8.8	0.100

TABLE II—Continued

Feed	ODN	TDP	TDN	$\frac{TDP}{ODN}$
Clover and timothy hay . . . . .	46.4	4.8	51.2	0.103
Lespedeza hay . . . . .	41.1	6.4	47.5	0.156
Peanut hay . . . . .	45.3	6.6	51.9	0.146
Grains cut green (as for corn silage with ears) . . . . .	18.7	1.3	20.0	0.070
Other hay . . . . .	42.0	8.0	50.0	0.190
Peanuts hogged off . . . . .	83.3	20.2	103.5	0.242
Alfalfa hay . . . . .	39.8	10.5	50.3	0.264
Soybean hay . . . . .	39.4	9.6	49.0	0.244
Cowpea hay . . . . .	39.1	12.3	51.4	0.315

Alternatively, feed grains and protein concentrates could have been classified on the basis of chemical composition. On the demand side, chemical composition would probably have resulted in an aggregate that was more homogenous from the viewpoint of rations fed to livestock. On the supply side, origin would probably have resulted in an aggregate that was more meaningful. Since in the model 2.19–2.26 the demand relations for feeds have been more completely specified than the corresponding supply relations, it was decided to differentiate the feed grains and the protein feeds on the basis of chemical composition. For this purpose the ratio of total digestible protein to other digestible nutrients was obtained for each feed, not classified previously as a roughage. For feeds classified as protein concentrates, the ratio of TDP to ODN was 0.380 for alfalfa meal and 2.582 for fish meal. These figures were the minimum and maximum, respectively, of the ratios computed for protein feeds. In the group of feeds classified as “feed grains,” the minimum value was 0.068 for dried and molasses beet pulp, and the maximum value was 0.337 for cowpeas.

$a_t$  is the number of pounds of TDN in feed grains fed to livestock. These feeds classified as feed grains were weighted by the average TDN content given in Table II.

$b_t$  is the number of pounds of TDN in protein concentrates fed to livestock during the year. These feeds classified as protein concentrates were weighted by the average TDN content given in Table II.

$c_t$  is the number of pounds of TDN in roughages fed to livestock. For pasture, since the total quantity of pasture fed was based on plowable pasture equivalent, the composition figures used were those given for pasture grasses and clovers in fertile pastures.

## PRICE OF FEEDS

$d_t$  is an index of the price of feed grains expressed as dollars per thousand pounds of TDN. Since the unit of measure employed for the feeds is total digestible nutrients, it is appropriate to express the price variables in the model 2.19–2.26 in terms of this unit of measure. An estimated value for each year of all feeds in this category was divided by  $a_t$  to yield  $d_t$ . The estimated value was obtained from the number of physical tons fed and average prices received by farmers, or average wholesale prices for obvious by-product feeds. Since the calendar year was the basic time unit employed, calendar-year prices rather than season average prices were used in forming estimated value of feeds fed, except where only a season average price was available. For corn, wheat, rye, barley, oats, buckwheat, rice, and cottonseed, the calendar-year average price was obtained for the years 1920–37, except for rice when an observation was available for 1938 also.<sup>45</sup> For later years a calendar-year average price was obtained by weighting monthly prices by the percentages of each crop sold in the various months.<sup>46</sup> The exceptions were cottonseed, rice, and buckwheat, where a simple average of monthly prices was used, since the percentage distribution of sales by months was not available.<sup>47</sup> For sorghum grains, the prices used for 1920–32 were for December 1, whereas for 1933–49 weighted monthly prices were utilized.<sup>48</sup> The other grains category included hominy feed, and the price used for other grains was the average wholesale price per ton at Chicago for hominy feed. The wheat mill feeds included bran and middlings, and their relative percentages from the milling operation are given as 41 and 59% of the by-products.<sup>49</sup> These were the weights used to obtain a weighted average price of wheat mill feeds from the average wholesale prices per ton, Chicago, of bran and middlings. For the rice mill feeds, it was assumed that the relevant price was 0.96 times the wheat mill feeds price; the estimate being based on the relative TDN figures for the two mill feeds.

For dried and molasses beet pulp for 1930–49, the price used was a

<sup>45</sup> *Income Parity for Agriculture*, Part I—Farm Income, Sec. 1, 5, 7, 9, 10, 15. U. S. Department of Agriculture, Washington, D. C., 1938–41.

<sup>46</sup> Data on monthly prices and percentages of each crop sold in the various months are from Feed Statistics, *Statistical Bulletins*, 85, 95, *op. cit.* Unless otherwise stated, all monthly price data are from these two sources.

<sup>47</sup> For early years in the sample, the source was *Yearbook of Agriculture*, 1930–1932, U. S. Department of Agriculture, Washington, D. C. For the subsequent five years the source was *Crops and Markets*, Vol. 28, U. S. Department of Agriculture, Washington, D. C., 1951.

<sup>48</sup> The December 1 prices are from *Agricultural Statistics*, 1942–51, *op. cit.*

<sup>49</sup> F. B. Morrison, *op. cit.*, p. 342.

simple average of monthly prices for molasses beet pulp, San Francisco. For earlier years, the price was estimated as 0.9 times the corn price, the conversion factor being suggested by the relative TDN figures for pulp and corn. For liquid whole milk, the price per hundred weight received by farmers for milk was used. To allow this full price for milk fed would seem to have overestimated a price for milk fed, so that an arbitrary factor of 0.8 was used on the price data. For velvet beans, the December 1 price was used for the years 1924-49.<sup>50</sup> For earlier years 0.6 times the price of cottonseed was the estimate. A comparison of the relative TDN contents of velvet beans and cowpeas had suggested a conversion factor which appeared to overestimate the price of velvet beans. This overestimation was suggested by a comparison of the calculated price, using the conversion factor and published data on prices of both cowpeas and velvet beans. The factor of 0.6 was suggested by a comparison of the prices of cottonseed and velvet beans for years where both these series were available. For copra cake and meal for 1933-49, a simple average of monthly prices, Los Angeles, was used. For earlier years, 0.95 times the price of gluten feed was the estimate employed. For cowpeas and peanuts, a simple average of monthly prices was used.<sup>51</sup>

$g_t$  is an index of the price of protein feeds in dollars per thousand pounds of TDN. An estimated value for each year of all feeds in this class was divided by  $b_t$  to lead to  $g_t$ . The prices used were predominantly average wholesale prices for the by-product feeds. For soybeans, the prices received by farmers were obtained, a simple average of monthly prices being used.<sup>52</sup> For cottonseed cake and meal, the price utilized was a simple average of monthly prices paid by farmers for cottonseed meal for the years 1920-38.<sup>53</sup> Annual average prices for years subsequent to 1938 were used. For soybean cake and meal, the annual average wholesale price for soybean meal, Chicago, was used for the years 1930-49. For years prior to 1930, price was estimated as 1.2 times the price for cottonseed meal. In general, for soybean cake and meal, peanut cake and meal, fish meal, distillers' dried grains, and alfalfa meal, where

<sup>50</sup> The December 1 price is from *Agricultural Statistics, 1942-51, op. cit.*

<sup>51</sup> For both cowpeas and peanuts, the earlier price data were from the *Yearbook of Agriculture, 1920-32*, and for later years from *Agricultural Statistics, 1942-51, op. cit.*

<sup>52</sup> For soybeans, the monthly price data were from the *Yearbook of Agriculture, op. cit.*, for the earlier years, and from *Agricultural Statistics, op. cit.*, for later years.

<sup>53</sup> For the years 1920-38, monthly price data were from the March 1944 issue of *Agricultural Prices*. For 1939, the annual average price was from the December 1945 issue, and for 1940-49, the annual average price was from the March 1950 issue of this source, published monthly by the U. S. Department of Agriculture, Washington, D. C.

price observations were missing for some years, the conversion factors used on available data were suggested by comparison with composition figures for a feedstuff for which a complete series was available and which was somewhat similar in classification to the feedstuff for which estimated prices were required. The average wholesale price for linseed meal, Minneapolis, was used as the relevant price for linseed cake and meal.<sup>54</sup> An annual average wholesale price for peanut meal was used as the price for peanut cake and meal for the years 1924-49. Observations for 1920-23 were obtained by using 1.3 times the price of cottonseed meal for those years. Price data for tankage, Chicago, were given. For fish meal for 1935-49, annual average prices and unpublished BAE estimates were used. Before 1935, an estimated price of 1.1 times that of tankage was employed. For gluten feed and meal, annual average prices were used, and also for brewers' dried grains. For distillers' dried grains, annual average prices for the years 1933-49 were employed. Before 1933 the estimated price was 1.2 times that used for gluten meal. Alfalfa-meal annual average prices for 1935-49 were available. For 1920-34, the estimated price was 0.6 times that of gluten meal. For skim milk for 1939-49, annual average prices were used. For earlier years, the ratio of the manufacturers' wholesale selling price of skim milk for animal feed to prices received by farmers for milk (1939-48) was used on data on prices received by farmers for milk. For the dried-milk products, an estimated price of 1.12 times skim-milk price was utilized. The dried-milk products are principally skim milk and buttermilk. A comparison of TDN figures for skim milk fed to all livestock including poultry and for buttermilk which is fed to livestock other than poultry suggested a conversion factor of 1.12.

We may summarize the operations on the feed variables as follows:

$$(3.6) \quad f_t = \sum_i \bar{i}_i s_{it}$$

$$(3.7) \quad a_t = \sum_i \bar{i}_i^a e_{it}$$

$$(3.8) \quad b_t = \sum_i \bar{i}_i^b q_{it}$$

$$(3.9) \quad c_t = \sum_i \bar{i}_i^c m_{it}$$

$$(3.10) \quad V_t^a = \sum_i e_{it} p_{it}^a$$

$$(3.11) \quad V_t^b = \sum_i q_{it} p_{it}^b$$

$$(3.12) \quad d_t = \frac{V_t^a}{a_t}$$

$$(3.13) \quad g_t = \frac{V_t^b}{b_t}$$

<sup>54</sup> In addition to published data for prices of linseed meal, fish meal, and distillers' dried grains, unpublished BAE estimates were used to extend some series.

where  $\bar{t}_i$  = average TDN content of the  $i$ th feed included in  $f_i$

$s_{it}$  = amount of the  $i$ th feed, in physical measure, fed to livestock

$\bar{t}_i^a$  = average TDN content of the  $i$ th feed classified as a feed grain in Table II

$e_{it}$  = quantity of the  $i$ th feed grain fed to livestock

$\bar{t}_i^b$  = average TDN content of the  $i$ th feed classified as a protein feed in Table II

$q_{it}$  = quantity of the  $i$ th protein feed fed to livestock

$\bar{t}_i^c$  = average TDN content of the  $i$ th feed classified as a roughage in Table II

$m_{it}$  = quantity of the  $i$ th roughage fed to livestock

$V_t^a$  = value of feed grains fed to livestock

$p_{it}^a$  = price in physical measure of the  $i$ th feed grain

$V_t^b$  = value of protein feeds fed to livestock

$p_{it}^b$  = price of the  $i$ th protein feed fed to livestock

$y_t$  = consumer disposable income in the United States<sup>55</sup>

$d_t$  = price of feed grains per 1000 lb TDN

$g_t$  = price of protein feeds per 1000 lb TDN

$n_t$  = U.S. population<sup>56</sup>

#### OTHER PRICES

$r_t$  represents the influence of "other prices" in the commercial demand relation. Ideally,  $r_t$  should not contain the components contained in  $p_t$ . The index used for  $r_t$  was the Bureau of Labor Statistics' index of wholesale prices excluding farm products.<sup>57</sup> This index appeared to give the widest coverage of both commodities and areas. However, excluding all farm products meant that some of the "other prices," primarily the fibers, have been omitted, and, without considerable disaggregation and re-

<sup>55</sup> Consumption of Food in the United States 1909-48, *Misc. Pub. 691*, U. S. Department of Agriculture, Washington, D. C., 1949. *Survey of Current Business*, U. S. Department of Commerce, Washington, D. C., July 1952.

<sup>56</sup> *Outlook Charts, 1951*. U. S. Department of Agriculture, Bureau of Agricultural Economics, 1950.

<sup>57</sup> Handbook of Labor Statistics, 1947 ed., *Bulletin 916*, U. S. Department of Labor, Bureau of Labor Statistics, 1948.

construction of the published index on all wholesale prices, the index selected seemed to give as wide a coverage as possible and to appear most easily used.

#### QUANTITY OF FEEDS PRODUCED

$a_i^*$  in Chapter II represented the quantity of feed grains produced. As a result of the decision to disaggregate  $f_i$  and not to treat quantity fed as equal to quantity produced for the feeds, decision relations involving the demand for feed grains and the demand for protein feeds were constructed, and corresponding supply relations were incompletely specified. The availability of feed grains for feeding in the current period is affected not only by production in the current period but also by the carryover from previous periods. For most of the feed grains, considerable farm stocks exist at the beginning of the calendar year, and the size of these stocks influence the rate and quantity of feeding during the period. During the  $t$ th period, production from the  $t$ th period will be incompletely known until later on in the year when crop prospects may be somewhat clearer and influence the current feeding program. Since the feed grains are typically farm-produced and farm-fed, it was assumed that the supply of feed grains for feeding was influenced by January 1 stocks, and to some extent by current production. An index of the physical supply of feed grains for feeding was influenced by carryover, and to some extent by current production. An index of the physical supply of feed grains in TDN was therefore constructed. Supply was taken to be carryover, January 1, plus a fraction of the current crop. The feed grains for which significant stocks existed were corn, oats, and barley. Stocks of wheat also existed on farms January 1, but these were excluded from  $a_i^*$ . Although quantities of wheat have been fed regularly to livestock, the most notable period being the years 1943-46, the investigators have considered that factors other than the physical supply were relevant in determining the supply of wheat for feeding to livestock. The factors that determine whether wheat may be economically fed to livestock are those unobserved factors given under  $s_i$  in equation 2.24.

For corn, farm stocks, January 1, for the years 1927-49 were available.<sup>58</sup> For the years 1920-26, January 1 stocks were estimated as March 1 stocks plus two thirds of corn consumption in the first quarter.<sup>59</sup> Data

<sup>58</sup> Data on January 1 farm stocks of corn, oats, and barley are from Feed Statistics, *Statistical Bulletin 85, op. cit.*

<sup>59</sup> March 1 stocks of corn are from Corn Statistics, *Statistical Bulletin 28, U. S. Department of Agriculture, Washington, D. C., 1930.* March 1 stocks of oats and barley and August 1 stocks of the latter are from Statistics of Oats, Barley and Grain Sorghums, *Statistical Bulletin 29, U. S. Department of Agriculture, Washington, D. C., 1930.*

on quarterly consumption of corn indicated that approximately 29 % of total corn consumed in a calendar year was consumed in the January-March quarter.<sup>60</sup> January 1 stocks were then estimated as March 1 stocks plus 0.19 times calendar-year consumption of corn obtained in operations under  $f_i$ . A similar procedure was adopted for oats for 1920-27 when January 1 stocks were not available. The quarterly pattern of oats consumption was assumed to be that given for "other grains" where approximately 27 % of total "other grains" consumed in a calendar year was in the first quarter. January 1 stocks for 1920-27 were then estimated as March 1 stocks plus 0.18 times calendar-year consumption of oats from  $f_i$ . Farm stocks of barley on January 1 were available for the years 1934-49. The quarterly pattern of barley consumption was assumed to be the same as that for the "other-grains" category, namely, 27 % of total calendar-year barley consumption was in the first quarter. For 1920-30, March 1 stocks plus 0.18 times annual consumption of barley from  $f_i$  gave the estimated stock on January 1. For 1931-33, August 1 stocks plus seven-twelfths calendar-year consumption of barley from  $f_i$  resulted in January 1 stocks. The weight of seven twelfths assumed a constant monthly feeding rate. To the farm stocks, January 1, was added a fraction of production for that calendar year. The data indicated that approximately 0.4 of current calendar-year crop production is consumed in the final quarter of that year for corn, and 0.5 for barley and oats, and so these were the fractions of production used. The totals of farm stocks plus the fraction of production were converted to TDN, using average composition figures given in Table II.

$b_i^*$  is the production of protein concentrates in TDN on a calendar-year basis. For the protein concentrates which are principally by-product feeds, the distribution of production does not exhibit the marked seasonality of the feed grains, production being more evenly distributed throughout the calendar year. Data on stocks of protein concentrates were of a limited nature; in many cases the by-product feeds are not easily or economically stored, and the investigators decided to use full calendar-year production as the relevant magnitude influencing the availability of protein concentrates for livestock feeding.

For gluten feed and meal annual production, data were available for the sample period.<sup>61</sup> For alfalfa meal for the years 1928-49, data were available. For before 1928, the average consumption for the years

<sup>60</sup> Quarterly consumption of corn and "other grains" which includes oats, barley, grain sorghums, wheat, and rye, is given in Feed Statistics, *Statistical Bulletin 95, op. cit.*

<sup>61</sup> Production data on the by-product feeds are from Feed Statistics, *Statistical Bulletins 85, 95, op. cit.* These production data were supplemented in some instances by unpublished BAE estimates.



1928-32 obtained under  $f_t$  was taken as production for each year where an observation was missing. For dried-milk products, production was taken as equal to consumption, the latter being constructed for  $f_t$ . Peanut cake and meal production figures were available, as were production figures for brewers' dried grains, distillers' dried grains, and fish meal. The production of brewers' dried grains was assumed zero before 1936. For fish meal, 1920 production was assumed equal to 1921 production, as no observation was available for 1920. For skim milk, production was assumed equal to the consumption series constructed under  $f_t$ . For 1921-49, production figures on tankage and meat scraps were available. An observation was obtained for 1920 by multiplying 1921 production by the ratio of total animal slaughter for 1920 to total animal slaughter for 1921. Calendar-year production of cottonseed cake and meal was given for 1927-49. For years prior to 1927, October-year production data were available and were put on a calendar-year basis by employing a moving average with weights of 0.56:0.44. Quarterly production data indicated that approximately 44 % of total annual production takes place in the October-December quarter. For soybean cake and meal, calendar-year production was available for 1923-49. For 1920-22, October-year production was available and was taken to be equal to calendar-year production. To avoid counting soybeans both as farm produced and as soybean cake and meal, it seemed preferable to use cake and meal production rather than soybeans production, since there are uses other than livestock feeding for soybeans. The protein feeds were converted to TDN, using average composition figures from Table II.

#### PRICE OF FARM LABOR

$k_t$  is the price of farm labor in period  $t$ . Some evidence exists, for example Hoselitz et al., that, at least for some livestock, the price of farm labor may influence the current supply of livestock for slaughter.<sup>62</sup> The liquidation of sheep herds that took place between 1940 and 1945 and in later years has been attributed by some observers to the high price of farm labor. As a measure of the price of farm labor we used the cash farm wage rate per hour.<sup>63</sup>

The observations that were obtained by the processes described in the foregoing are tabulated in Tables III, IV, and V. The first symbol at the head of each column identifies the variable in the notation of Chapter II. The second symbol refers to a revised notation introduced in Chapter IV.

<sup>62</sup> Bert F. Hoselitz, J. M. Letiche and T. W. Schultz, *Lessons to be Drawn from War-Time Experiences with Regard to Foreign Trade*, unpublished manuscript, University of Chicago, May 1945.

<sup>63</sup> *Outlook Charts*, 1951, *op. cit.*

TABLE III  
LIVESTOCK AGGREGATES CONSTRUCTED FOR THE  
COMPUTATIONS

Year	$l_t, y_{1t}$	$l_t^*, y_{4t}$	$\frac{l_t^*}{n_t}, \frac{y_{4t}}{z_{6t}}$	$p_t, y_{6t}$	$h_t, z_{1t}$
1919	....	5880	55.95	....	....
1920	5999	5655	53.10	1.218	5957
1921	5831	5699	52.52	0.821	5881
1922	6257	6037	54.83	0.798	6008
1923	6458	6459	57.72	0.811	6252
1924	6445	6606	57.90	0.800	5977
1925	6342	6417	55.41	0.945	5605
1926	6543	6560	55.88	0.973	5525
1927	6747	6615	55.59	0.935	5588
1928	6742	6645	55.15	0.963	5644
1929	6820	6669	54.75	0.997	5680
1930	6901	6709	54.50	0.851	5695
1931	7087	6821	55.01	0.626	5832
1932	7127	6760	54.17	0.453	6061
1933	7274	6916	55.07	0.444	6349
1934	6612	7099	56.17	0.519	6349
1935	6334	6219	48.85	0.721	5548
1936	6776	6757	52.74	0.755	5692
1937	6665	6620	51.39	0.794	5566
1938	7025	6845	52.73	0.709	5575
1939	7539	7173	54.80	0.664	5934
1940	7654	7581	57.43	0.669	6186
1941	7896	7602	57.07	0.873	6152
1942	9036	8644	64.18	1.098	6683
1943	9803	9469	69.37	1.277	7345
1944	9326	9706	70.28	1.269	7328
1945	9331	9489	67.97	1.354	6790
1946	9012	9144	64.76	1.562	6627
1947	8955	9215	63.99	1.842	6465
1948	8844	8873	60.52	1.998	6197
1949	9356	9211	61.74	1.685	6310
1950	9507	9328	62.84	1.736	6508

$l_t, y_{1t}$  = production of livestock and livestock products in million dollars at average prices

$l_t^*, y_{4t}$  = sales of livestock and livestock products in million dollars at average prices

$\frac{l_t^*}{n_t}, \frac{y_{4t}}{z_{6t}}$  = per capita sales of livestock and livestock products in dollars

$p_t, y_{6t}$  = index of the price of livestock and livestock products, equal to the ratio of value at current prices to value at average prices

$h_t, z_{1t}$  = January 1 inventory of livestock in million dollars of estimated potential production

$t, z_{2t}$  = time

TABLE IV  
FEED AGGREGATES CONSTRUCTED FOR THE COMPUTATIONS  
Part A

Year	$a_t, y_{8t}$	$b_t, y_{7t}$	$c_t, z_{3t}$	$f_t, y_{8t}$	$a_t^*, z_{9t}$
1920	117,055	7,183	239,414	363,653	141,565
1921	127,629	6,645	231,174	365,448	165,800
1922	126,584	6,830	241,362	374,775	143,483
1923	130,288	7,433	241,618	379,339	139,242
1924	125,451	7,998	235,721	369,170	134,097
1925	118,296	8,497	226,006	352,800	126,686
1926	125,386	9,070	227,276	361,732	146,844
1927	128,775	9,223	234,593	372,591	137,646
1928	127,548	9,050	237,586	374,184	140,664
1929	125,949	9,461	234,064	369,474	139,930
1930	123,316	9,528	216,503	349,346	130,351
1931	120,978	8,975	217,308	347,261	123,851
1932	135,156	8,582	225,410	369,148	149,590
1933	134,016	9,097	225,975	369,088	148,445
1934	103,011	8,773	219,872	331,655	106,357
1935	93,547	8,970	228,056	330,574	99,580
1936	116,040	10,221	216,842	343,103	118,835
1937	99,202	10,628	218,588	328,418	107,116
1938	124,410	10,622	237,998	373,029	149,965
1939	131,380	11,301	233,547	376,228	157,889
1940	133,220	12,097	253,003	398,320	161,051
1941	138,898	13,292	272,816	425,006	167,323
1942	165,464	15,444	287,255	468,163	184,508
1943	197,411	16,549	285,276	499,235	191,472
1944	172,008	16,553	283,252	471,813	173,771
1945	175,694	16,493	293,261	485,447	180,134
1946	171,223	15,321	276,620	463,164	181,722
1947	158,966	15,744	274,121	448,830	174,244
1948	148,437	16,347	265,504	430,288	169,556
1949	172,346	17,751	266,414	456,511	211,394
1950	170,641	18,811	271,352	456,605	200,059

$a_t, y_{8t}$  = total quantity of feed grains fed in million pounds TDN

$b_t, y_{7t}$  = total quantity of protein feeds fed in million pounds TDN

$c_t, z_{3t}$  = quantity of roughage fed in million pounds TDN

$f_t, y_{8t}$  = total quantity of feed fed in million pounds TDN

$a_t^*, z_{9t}$  = supply of feed grains in million pounds TDN

## THE OBSERVATIONS

TABLE IV—Continued  
Part B

Year	$d_t, y_{2t}$	$g_t, y_{3t}$	$b_t^*, z_{10t}$	$y_{9t}$	$y_{10t}$
1920	28.21	70.23	7,708	52,585	311,067
1921	12.15	45.49	7,556	52,345	313,103
1922	13.24	46.58	7,427	52,774	322,002
1923	17.30	51.46	8,000	54,099	325,240
1924	19.61	46.47	8,899	52,425	316,745
1925	20.28	47.46	9,733	50,563	302,237
1926	15.54	46.10	10,414	51,923	309,809
1927	17.95	47.75	10,562	53,553	319,038
1928	18.89	50.81	9,783	53,483	320,701
1929	18.74	50.22	10,191	53,294	316,179
1930	16.18	43.48	9,867	50,911	298,435
1931	9.95	32.30	9,508	50,667	296,594
1932	6.15	24.04	9,456	53,405	315,743
1933	8.61	25.88	9,550	53,480	315,609
1934	14.32	31.96	9,068	47,961	283,694
1935	15.45	33.29	9,126	50,115	280,459
1936	16.81	34.09	10,286	50,979	292,124
1937	17.64	36.79	10,844	50,227	278,191
1938	11.57	30.85	11,133	55,975	317,055
1939	11.62	33.78	11,335	56,000	320,228
1940	13.60	32.00	11,614	59,403	338,917
1941	15.07	38.22	13,230	64,100	360,907
1942	18.84	45.07	14,486	70,413	397,750
1943	24.97	46.49	16,418	73,996	425,240
1944	27.47	48.39	16,408	71,080	400,734
1945	26.70	47.59	16,290	72,880	412,567
1946	32.80	58.93	15,523	69,107	394,057
1947	42.87	63.77	16,201	67,575	381,255
1948	42.23	70.13	16,715	65,653	364,635
1949	28.22	59.91	18,031	68,830	387,681
1950	31.63	58.38			

$d_t, y_{2t}$  = price of feed grains in dollars per 1000 lb TDN

$g_t, y_{3t}$  = price of protein feeds in dollars per 1000 lb TDN

$b_t^*, z_{10t}$  = supply of protein feeds in million pounds TDN

$y_{9t}$  = total quantity of TDP fed to livestock in million pounds

$y_{10t}$  = total quantity of ODN fed to livestock in million pounds

In carrying out the computations, more significant figures were used than are reported in the above tables.

TABLE V  
OTHER VARIABLES USED IN COMPUTATIONS

Year	$y_t, z_{5t}$	$n_t, z_{6t}$	$r_t, z_{7t}$	$k_t, z_{8t}$
1920	69.2	106.5	154.8	32.2
1921	55.0	108.5	100.1	20.6
1922	59.1	110.1	97.3	20.1
1923	68.3	111.9	100.9	22.5
1924	69.1	114.1	97.1	23.0
1925	73.1	115.8	101.4	23.4
1926	75.8	117.4	100.0	23.8
1927	76.1	119.0	94.6	23.8
1928	78.1	120.5	94.8	23.8
1929	82.5	121.8	93.3	23.9
1930	73.7	123.1	85.9	22.2
1931	63.0	124.0	74.6	17.3
1932	47.8	124.8	68.3	12.8
1933	45.2	125.6	69.0	11.3
1934	51.6	126.4	76.9	12.6
1935	58.0	127.3	80.2	13.7
1936	66.1	128.1	80.7	14.8
1937	71.1	128.8	86.2	16.8
1938	65.5	129.8	80.6	16.6
1939	70.2	130.9	79.5	16.4
1940	75.7	132.0	80.8	16.8
1941	92.0	133.2	88.3	20.5
1942	116.7	134.7	97.0	26.7
1943	132.4	136.5	98.7	35.1
1944	147.0	138.1	99.6	41.9
1945	151.1	139.6	100.8	46.6
1946	158.9	141.2	114.9	50.3
1947	169.5	144.0	145.5	54.3
1948	188.4	146.6	159.8	57.5
1949	187.2	149.2	152.4	55.3
1950	205.5	151.8	159.2	55.3

$y_t, z_{5t}$  = consumer disposable income in billion dollars

$n_t, z_{6t}$  = U. S. population in millions

$r_t, z_{7t}$  = index of wholesale prices of commodities other than farm products  
(1926 = 100)

$k_t, z_{8t}$  = cash farm wage in cents per hour

## CHAPTER IV

### ESTIMATED RELATIONS

#### REVISED NOTATION

In this chapter the economic model of Chapter II is assumed to have a particular parametric form, and estimates of the parameters are obtained by alternative statistical procedures. To facilitate the application of certain theoretical propositions to the present model, a notation similar to one that has been used in various theoretical discussions is adopted. The value of a current endogenous variable for a particular time period is denoted by a  $y$  with two subscripts. The first subscript identifies the particular variable, and the second subscript refers to the time period. Similarly a  $z$  with two subscripts represents the value of a predetermined variable for a given time period. The new symbols are defined below. The symbol used to represent a particular variable in the notation of (2.19)-(2.26) is listed inside the parenthesis following each new symbol.

- $y_{1t}(l_t)$  = quantity of livestock and livestock products produced
- $y_{2t}(d_t)$  = price of feed grains
- $y_{3t}(g_t)$  = price of protein feed
- $y_{4t}(l_t^*)$  = quantity of livestock and livestock products sold
- $y_{5t}(p_t)$  = price of livestock and livestock products
- $y_{6t}(a_t)$  = amount of feed grains fed to livestock
- $y_{7t}(b_t)$  = amount of protein feed fed to livestock
- $y_{8t}(f_t)$  = total feed fed to livestock
- $z_{1t}(h_t)$  = beginning inventory of livestock on farms
- $z_{2t}(t)$  = time
- $z_{3t}(c_t)$  = quantity of roughage fed to livestock
- $z_{4t}(l_{t-1}^*)$  = quantity of livestock and livestock products sold in previous year

- $z_{6t}(y_t)$  = disposable personal income  
 $z_{6t}(n_t)$  = population  
 $z_{7t}(r_r)$  = general price index (excluding livestock and livestock products)  
 $z_{8t}(k_t)$  = price of farm labor  
 $z_{9t}(a_t^*)$  = available quantity of feed grains  
 $z_{10t}(b_t^*)$  = production of protein feeds

Capital letters are used to denote logarithms of the above variables; i.e.,  $Y_{it} = \log y_{it}$ ,  $Z_{it} = \log z_{it}$ . We now consider a statistical model in which the first seven relations of the economic model of Chapter II are represented by equations that are linear in the logarithms of the observed variables. Though there are certain conveniences in using equations of this form (computational simplicity, direct interpretation of coefficients as elasticities) and such usage is usually consistent with our qualitative presumptions about the form of our relations, it should still be recognized that their use involves a highly special and incompletely justified assumption. Statistical results utilizing other algebraic forms are reported briefly in later chapters. The results given in this chapter are based on the model given by (4.1) to (4.7) below, which is expressed in terms of the logarithms of observed variables, except for  $z_{2t}$  which represents time.

### A STATISTICAL MODEL

If  $\log z_{2t}$  had been inserted in the production relation (4.1 below), both the interpretation of the relation and the values obtained for estimates of the parameter of the variable would depend on an arbitrary choice of the origin from which time is measured. Inserting  $z_{2t}$  along with logarithms of the other variables implies a constant rate of increase in the quantity of livestock products obtained from given feed and herds. If the secular increase in feeding efficiency is to be represented by a variable representing time, this seems as reasonable as any alternative assumption about the way in which it enters.<sup>1</sup>

$$(4.1) \quad Y_{1t} + \beta_{16}Y_{6t} + \beta_{17}Y_{7t} + \gamma_{11}Z_{1t} + \gamma_{12}z_{2t} + \gamma_{13}Z_{3t} + \gamma_{10} = U_{1t}$$

(production relation)

<sup>1</sup> This agrees with Lorie's treatment of time in the production relation. See James H. Lorie, *op. cit.*, pp. 83-96.

$$(4.2) \quad Y_{2t} + \beta_{23}Y_{3t} + \beta_{25}Y_{5t} + \beta_{26}Y_{6t} + \gamma_{21}Z_{1t} + \gamma_{23}Z_{3t} + \gamma_{20} = U_{2t}$$

(demand for feed grain)

$$(4.3) \quad \beta_{32}Y_{2t} + Y_{3t} + \beta_{35}Y_{5t} + \beta_{37}Y_{7t} + \gamma_{31}Z_{1t} + \gamma_{33}Z_{3t} + \gamma_{30} = U$$

(demand for protein feed)

$$(4.4) \quad \beta_{41}Y_{1t} + \beta_{42}Y_{2t} + \beta_{43}Y_{3t} + Y_{4t} + \beta_{45}Y_{5t} + \gamma_{41}Z_{1t} + \gamma_{43}Z_{3t} + \gamma_{40}$$

$= U_{4t}$  (supply of livestock products)

$$(4.5) \quad \beta_{45}Y_{4t} + Y_{5t} + \gamma_{54}Z_{4t} + \gamma_{55}Z_{5t} + \gamma_{56}Z_{6t} + \gamma_{57}Z_{7t} + \gamma_{50} = U_{5t}$$

(demand for livestock products)

$$(4.6) \quad \beta_{62}Y_{2t} + Y_{6t} + \gamma_{69}Z_{9t} + \theta' s'_t = U_{6t}$$

(supply of feed grains)

$$(4.7) \quad \beta_{73}Y_{3t} + Y_{7t} + \gamma_{7,10}Z_{10t} + \theta'' s''_t = U_{7t}$$

(supply of protein feeds)

The symbols  $\beta_{ij}$ ,  $\gamma_{ij}$  in the above model represent unknown constant coefficients to be estimated. The  $U_{it}$  represent random disturbances.  $\theta'$ ,  $\theta''$  stand for coefficients of the unspecified variables  $s'_t$ ,  $s''_t$ . The fact that these variables are not known or utilized means that our model is incomplete and makes some of the estimates described below less efficient than the estimates that would be obtained if these variables were specified and observed. Each of the symbols  $s'_t$ ,  $s''_t$  should be thought of as representing an unknown number of important omitted variables with  $\theta'$ ,  $\theta''$  representing corresponding numbers of coefficients. If none of the unknown variables are endogenous, the model could be completed without additional equations. However, if some of the unknown variables are endogenous, additional relations would have to be specified to complete the model.<sup>2</sup>

The accounting relation given by (2.26) has been omitted since it is of secondary interest. If the disturbance in this relation is assumed to be independent of the other disturbances, then, for statistical purposes, the accounting relation may be regarded as a separate one-equation model.<sup>3</sup> An estimate of the parameter of this equation will be given in

<sup>2</sup> For a discussion on the concept of a complete model, see T. C. Koopmans, *When Is an Equation Complete for Statistical Purposes?* Cowles Commission Monograph 10, *op. cit.*, and T. C. Koopmans and William C. Hood *The Estimation of Simultaneous Linear Economic Relationships*, Cowles Commission Monograph 14, *op. cit.*, pp. 113-127.

<sup>3</sup> See pp. 13, 14.



Chapter VI. The unobserved random disturbances  $U_{it}$  are assumed to have a multivariate normal<sup>4</sup> distribution with zero means and a finite covariance matrix. For a given time period, the values taken by the disturbances may be correlated with each other, but they are assumed to be independent of the values of disturbances for other time periods. Their distribution is assumed to remain constant over the observation period. The meaning of any equation is not changed if each term is multiplied by a given constant. This makes it possible to choose arbitrarily a convenient value for one coefficient in each equation, provided a coefficient known to be nonzero is chosen. In the above equations, the coefficients  $\beta_{ii}$ ,  $i = 1, 2, \dots, 7$ , have been set equal to unity.

#### IDENTIFICATION OF RELATIONS

Estimates of the other parameters in (4.1) to (4.5) are desired. A question that logically arises prior to estimation of parameters is that of identification, because, in general, different sets of structural equations can lead to the same probability distribution of the current endogenous variables for given values of the predetermined variables. Our observations generally enable us to draw statistical inferences about the conditional distribution of the endogenous variables, but our ability to draw inferences about the structural relations depends on establishing logical connections between the structural equations and the conditional distribution of the endogenous variables.

An equation is said to be identified (or identifiable) if its parameters (coefficients of observed variables and parameters of the distribution of the disturbance) could be uniquely determined from the specification of the model and the conditional distribution of the endogenous variables. Unless a relation is identified, we cannot expect to obtain point estimates of its parameters, though we could typically estimate certain functions of the parameters. Koopmans and others have derived conditions for the identification of equations in models like the one presented above.<sup>5</sup> It

<sup>4</sup> The estimates to be presented remain consistent if the normality assumption is relaxed. See H. Chernoff and H. Rubin, *Asymptotic Properties of Limited-Information Estimates under Relaxed Conditions*, Cowles Commission Monograph 14, *op. cit.*, pp. 200-212.

<sup>5</sup> T. C. Koopmans, *Identification Problems in Economic Model Construction*, *Econometrica*, Vol. 17, 1949. Also Olav Reiersol and T. C. Koopmans, *The Identification of Structural Characteristics*, *Annals of Mathematical Statistics*, Vol. 21, 1950. For the necessary and sufficient conditions for identification in linear models see the following: T. C. Koopmans and William C. Hood, *op. cit.*, pp. 135-139, and also pp. 185-186; T. C. Koopmans, H. Rubin and R. B. Leipnik, *Measuring the Equation Systems of Dynamic Economics in Cowles Commission Monograph 10*, *op. cit.*, pp. 69-85. Interesting earlier discussions of identification and related problems in somewhat different language may be found in Henry Schultz, *Theory*

has been shown that, if there are  $H$  current endogenous variables that may enter a given equation, then a necessary condition for the equation to be identified is that there be at least  $H - 1$  observable predetermined variables in the system that are known not to enter the equation being considered. If the number of excluded predetermined variables is exactly  $H - 1$ , the equation is said to be just identified; if the number of excluded predetermined variables exceeds  $H - 1$ , the equation is said to be overidentified. With respect to equation 4.1, for example,  $H = 3$ , and the number of excluded predetermined variables is 7, so that this equation is overidentified. (4.2) to (4.5) can also be seen to be overidentified.

#### ESTIMATION OF PARAMETERS

Methods that have thus far been developed for obtaining estimates of parameters of simultaneous equations systems are based on the principle of maximum likelihood. If our model were complete, it would be possible to form the likelihood function for our sample of observations and to maximize this function with respect to all the unknown parameters. Estimates obtained in this way would be called full-information maximum-likelihood estimates and would have the usual maximum-likelihood properties.<sup>6</sup> Even if our model were complete, we would find this computation process exceedingly expensive for a model that contains overidentified equations. Partly to provide less expensive (though less efficient) procedures and partly to anticipate circumstances in which the investigator might not feel justified in making all the specifications necessary for the application of full-information maximum likelihood, an alternative procedure called the limited-information method has been developed.<sup>7</sup>

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*and Measurement of Demand*, University of Chicago Press, Chicago, 1938, especially Ch. II, Sec. IIIc; A. C. Pigou, *A Method of Determining the Numerical Values of Elasticities of Demand*, *Economic Journal*, Vol. 20, 1910, pp. 636-640; R. Frisch, *Pitfalls in the Statistical Construction of Demand and Supply Curves*, *Veroffentlichungen der Frankfurter Gesellschaft fur Konjunkturforschung*, Neue Folge, Heft 5, Leipzig, 1933; E. J. Working, *What Do Statistical Demand Curves Show?*, *Quarterly Journal of Economics*, Feb. 1929, pp. 212-235.

<sup>6</sup> The full-information maximum-likelihood procedure is given in T. C. Koopmans, H. Rubin, and R. B. Leipnik, *op. cit.*, pp. 110-183. See also T. C. Koopmans and William C. Hood, *op. cit.*, pp. 143-162.

<sup>7</sup> The limited-information procedure is developed in T. W. Anderson and H. Rubin, *Estimation of the Parameters of a Single Equation in a Complete System of Stochastic Equations*, *Annals of Mathematical Statistics*, Vol. 20, 1950, and *The Asymptotic Properties of Estimates of the Parameters of a Single Equation in a Complete System of Stochastic Equations*, *Annals of Mathematical Statistics*, Vol. 21, 1950. See also T. C. Koopmans and William C. Hood, *op. cit.*, pp. 162-185, and T. C. Koopmans, H. Rubin, and R. B. Leipnik, *op. cit.*, pp. 110-153. The method was referred to earlier as the reduced-form method. See M. A. Girshick and Trygve Haavelmo, *op. cit.*, pp. 93-95.

This method yields estimates for a selected subset of the equations of a system, neglecting information about which variables appear in particular equations in the remainder of the system. In particular, it can be used to estimate parameters of one equation at a time and can be applied when the equations not being estimated are incompletely specified. The estimates so obtained can be shown to be consistent,<sup>8</sup> but, unfortunately, little is known about their small-sample properties. Since economists often have only small samples to work with, this is a serious limitation.

However, pending further developments in the relevant statistical theory, it seems useful to try this procedure and to make such judgments as are possible of the results. When the limited-information method is applied to a single equation, the variables are, in effect, divided into four classes—current endogenous variables that enter the equation (sometimes denoted by  $y_*$ ), current endogenous variables excluded from the equation ( $y_{**}$ ), predetermined variables that enter the equation ( $z_*$ ), and predetermined variables excluded from the equation ( $z_{**}$ ). Except for the normalizing assumption (arbitrarily giving one of the nonzero coefficients a chosen value), the variables within a class are treated symmetrically in the calculation of estimates. The  $y_{**}$  do not enter the computations. Estimation of coefficients of the  $y_*$  may be interpreted as estimating a linear combination of the  $y_*$ ; for example, in estimating coefficients of the  $y_*$  in (4.1), we are estimating the linear combination,  $Y_{1t} + \beta_{16}Y_{6t} + \beta_{17}Y_{7t}$ .

Let the linear combination of the  $y_*$  be represented by  $y^0$ . Consider the linear regression of  $y^0$  on the  $z_*$  and the  $z_{**}$ . The limited-information estimates of the coefficients in  $y^0$  are those values that minimize the relative contribution of the  $z_{**}$  to the explanation of the variance of  $y^0$ . Since this contribution would be zero in the population regression of the true  $y^0$  on the  $z_*$  and  $z_{**}$ , the estimation procedure does not seem unreasonable under this interpretation. Estimates of the coefficients of the  $z_*$  ( $\gamma_{11}$ ,  $\gamma_{12}$ ,  $\gamma_{13}$ ,  $\gamma_{10}$  in 4.1) are the least-squares estimates of coefficients of the regression of the estimated  $y^0$  on the  $z_*$ .<sup>9</sup>

If some of the  $z_{**}$  are omitted in the calculation, the procedure still gives consistent (though less efficient) estimates of coefficients, provided at least  $H - 1$  of the  $z_{**}$  are utilized. This fact, together with the fact that the  $y_{**}$  do not enter the calculations at all, helps explain why the

<sup>8</sup> An estimate is consistent if it approaches, in a probability sense, the true parameter value, as the number of observations increases. See A. M. Mood, *Introduction to the Theory of Statistics*, p. 149, McGraw-Hill Book Co., New York, 1950. Also S. S. Wilks, *Mathematical Statistics*, Princeton University Press, Princeton, 1947, p. 133.

<sup>9</sup> For a more complete account of this interpretation, see T. C. Koopmans and William C. Hood, *op. cit.*, pp. 166-177.

method can be applied to incomplete systems. In an application to an incomplete system, certain  $y_{**}$  and  $z_{**}$  are ignored. Omitting the former does not affect the computations; omitting the latter sacrifices some efficiency but does not destroy consistency of the estimates. In view of this, one may sometimes consider using only part of the available  $z_{**}$  in order to simplify computations. In such a case it seems reasonable to use those  $z_{**}$  that might be expected to minimize the variances of residuals in the regressions of the  $y_*$  on the  $z_*$  and  $z_{**}$ .<sup>10</sup>

#### PARAMETERS OF PRODUCTION RELATION

Limited-information estimates for the coefficients in equation 4.1 are given in the first row of Table VI. These were computed, using  $Z_{4t}$ ,  $Z_{5t}$ ,  $Z_{7t}$ ,  $Z_{8t}$ ,  $Z_{9t}$ ,  $Z_{10t}$  as the  $z_{**}$ .  $Z_{6t}$ , population, was omitted because

TABLE VI  
RESULTS FOR PRODUCTION RELATION

Method	Estimates of Coefficients					
	$\beta_{16}$	$\beta_{17}$	$\gamma_{11}$	$\gamma_{12}$	$\gamma_1$ <sup>e</sup>	$\gamma_{10}$
Limited in- formation	-0.222(0.078)	-0.349(0.093)	-1362(0.119)	-0.0009(0.001)	+0.221(0.122)	-3.911(0.931)
Least squares	-0.350(0.061)	-0.222(0.075)	-0.252(0.103)	-0.0023(0.0009)	+0.229(0.110)	-4.885(0.927)

$$R^2 = 0.982$$

it was believed to be too closely related to  $z_{2t}$  and other predetermined variables having a significant trend to contribute much to the accuracy of the estimates. The second row of Table VI contains least-squares estimates of the coefficients in (4.1). The computations relating to this equation are given in detail in the appendix. The numbers in parentheses beside the estimates are the calculated standard errors of the estimates.  $R^2$  is the coefficient of determination for the least-squares computation.

To obtain least-squares estimates of parameters of a given equation, it is necessary to designate one of the observed variables as dependent in that equation. The other observed variables are called independent. Least-squares procedures are derived under the assumption that the random disturbance in the equation is distributed independently of the independent variables. If an equation contains one current endogenous variable and several predetermined variables and if the current endoge-

<sup>10</sup> For a fuller account of the properties of limited-information estimates, see T. W. Anderson and H. Rubin, *The Asymptotic Properties of Estimates of the Parameters of a Single Equation in a Complete System of Stochastic Equations*, *op. cit.* See also H. Chernoff and H. Rubin, *op. cit.*

nous is regarded as dependent in applying least squares, then least-squares and limited-information procedures are identical, as are the assumptions under which they are derived. If an equation contains several current endogenous variables, then all but one of these have to be regarded as independent in the application of least squares.

This involves an assumption in addition to those underlying the limited-information calculations. To the extent that this assumption is not justified, the least-squares estimates of coefficients and their calculated standard errors contain additional biases. However, this could be outweighed by the greater efficiency of least-squares estimates if the least-squares assumptions were approximately true. As was indicated in Chapter II, it has often been contended that, for many relations involving agricultural commodities, least-squares assumptions are sufficiently realistic. For this reason and because little is known about the small-sample properties of limited-information estimates, it has seemed desirable in this study to estimate coefficients using both procedures.

Intuitive considerations, which are in a general way supported by Bronfenbrenner's results,<sup>11</sup> suggest that least-squares bias might be minimized by treating as independent those current endogenous variables that are most strongly influenced by predetermined variables not appearing in the equation being estimated (i.e. the  $z_{**}$ ). In equation 4.1, we would expect  $Y_6$  and  $Y_7$  to depend very strongly on  $Z_9$  and  $Z_{10}$ , respectively, so  $Y_1$  was treated as dependent in obtaining the least-squares estimates given in Table VI. This is also consistent with the reasoning underlying our first-approximation model of Chapter II, in which feed fed to livestock was regarded as predetermined.

#### RESULTS FOR FARM DECISION RELATIONS

The choice of dependent variables for equations 4.2, 4.3, 4.4 is somewhat more arbitrary than for equation 4.1. The least-squares results given in Tables VII, VIII, IX were obtained, using  $Y_2$  as dependent in (4.2),  $Y_3$  in (4.3), and  $Y_4$  in (4.4). In computing limited-information estimates, the  $z_{**}$  used for (4.2) and (4.3) were  $Z_4, Z_5, Z_7, Z_8, Z_9, Z_{10}$ . The  $z_{**}$  used for (4.4) were the same, except that  $Z_3$  was included and  $Z_8$  excluded.

The choice of a dependent variable in (4.5) is fairly clear from the fact that we expect  $Y_4$  to be strongly influenced by  $Z_1, Z_3, Z_9, Z_{10}$

<sup>11</sup> Jean Bronfenbrenner, *Asymptotic Bias in Least-Squares Estimates of the Parameters of a Single Linear Stochastic Equation in a Complete System*, unpublished Ph.D. thesis, University of Chicago, 1950. See also Jean Bronfenbrenner, *Source and Size of Least-Squares Bias in a Two-Equation Model*, Cowles Commission Monograph 14, *op. cit.*

## ESTIMATED RELATIONS

TABLE VII  
DEMAND FOR FEED GRAINS

Method	Estimates of Coefficients					
	$\beta_{22}$	$\beta_{25}$	$\beta_{26}$	$\gamma_{21}$	$\gamma_{23}$	$\gamma_{26}$
Limited information	0.259(0.229)	-1.582(0.215)	1.472(0.205)	-2.760(0.572)	0.708(0.620)	7.369(7.359)
Least squares	-0.040(0.191)	-1.295(0.179)	1.258(0.261)	-2.430(0.528)	0.441(0.567)	8.619(3.541)

$$R^2 = 0.948$$

TABLE VIII  
DEMAND FOR PROTEIN FEEDS

Method	Estimates of Coefficients					
	$\beta_{32}$	$\beta_{35}$	$\beta_{37}$	$\gamma_{31}$	$\gamma_{33}$	$\gamma_{36}$
Limited information	0.049(0.108)	-0.982(0.146)	0.543(0.067)	-0.644(0.313)	0.278(0.356)	-1.554(2.380)
Least squares	-0.122(0.073)	-0.739(0.101)	0.533(0.059)	-0.342(0.262)	-0.083(0.298)	-1.213(1.604)

$$R^2 = 0.965$$

TABLE IX  
SUPPLY OF LIVESTOCK PRODUCTS

Method	Estimates of Coefficients						
	$\beta_{41}$	$\beta_{42}$	$\beta_{43}$	$\beta_{45}$	$\gamma_{41}$	$\gamma_{45}$	$\gamma_{46}$
Limited information	-0.908 (0.143)	-0.149 (0.032)	0.061 (0.111)	0.191 (0.121)	-0.0013 (0.136)	-0.009 (0.945)	-0.661 (2.839)
Least squares	-0.801 (0.066)	-0.135 (0.929)	0.133 (0.050)	0.136 (0.052)	-0.083 (0.074)	-0.120 (0.035)	-0.993 (0.568)

$$R^2 = 0.992$$

and that  $Y_4$  was treated as predetermined in our first model.  $Y_5$  is therefore treated as dependent in the application of least squares to (4.5).

#### DEFLATION OF VARIABLES IN DEMAND RELATION

Since the demand relation expresses the aggregate result of the behavior of a larger number of individual consumers, it seems reasonable to express the demand relation in per-capita terms. This involves deflating the original quantity variables ( $y_4$ ,  $z_4$ ,  $z_5$ ) by population. If we retain the assumption that the relation is linear in the logarithms of the observed variables, then our per-capita demand relation is given by

$$(4.5') \quad \beta'_{54} Y'_{4t} + Y_{5t} + \gamma'_{54} Z'_{4t} + \gamma'_{55} Z'_{5t} + \gamma'_{57} Z'_{7t} + \gamma'_{50} = U'_{5t}$$

TABLE X  
DEMAND FOR LIVESTOCK PRODUCTS

Method	Estimates of Coefficients				
	$\beta'_{44}$	$\gamma'_{44}$	$\gamma'_{55}$	$\gamma'_{57}$	$\gamma'_{59}$
Limited information	1.319(0.348)	0.041(0.344)	-0.986(0.100)	-0.492(0.110)	-6.744(4.022)
Least squares	1.307(0.295)	0.045(0.306)	-0.984(0.089)	-0.494(0.098)	-6.690(2.469)

$R^2 = 0.981$

where

$$(4.8) \quad Y'_{4t} = Y_{4t} - Z_{6t} = \log y_{4t}/z_{6t}$$

$$(4.9) \quad Z'_{4t} = Z_{4t} - Z_{6,t-1} = \log z_{4t}/z_{6,t-1} = Y'_{4,t-1}$$

$$(4.10) \quad Z'_{5t} = Z_{5t} - Z_{6t} = \log z_{5t}/z_{6t}$$

The least-squares results reported in Table X were obtained by using  $Y_5$  as the dependent variable and the other observed variables in (4.5') as independent.

To obtain limited-information estimates of (4.5'), we visualize a model in which  $Y'_{4t}$ ,  $Z'_{4t}$ , and  $Z'_{5t}$  have been substituted for  $Y_{4t}$ ,  $Z_{4t}$ , and  $Z_{5t}$ , respectively, in the system given by (4.1) to (4.7). The effect of this is to replace (4.5) by (4.5') and to replace (4.4) by

$$(4.4') \quad \beta_{41}Y_{1t} + \beta_{42}Y_{2t} + \beta_{43}Y_{3t} + Y'_{4t} + \beta_{45}Y_{5t}\gamma_{41}Z_{1t} + Z_{6t} + \gamma_{48}Z_{8t} + \gamma_{40} = U_{4t}$$

The significance of (4.4') is that the substitution of  $Y'_4$  for  $Y_4$  has introduced  $Z_6$  into the equation. In the estimation of (4.5'), therefore,  $Z_6$  may be used as a  $z_{**}$ . The  $z_{**}$  used in obtaining the limited-information estimates given below were  $Z_1, Z_3, Z_6, Z_8, Z_9, Z_{10}$ .

In estimating coefficients of (4.5'), observations relating to the years 1943 through 1946 were omitted because price control and rationing of livestock products were effective during this period.<sup>12</sup>

In all of the above tables, the calculated standard errors of the limited-

<sup>12</sup> There is a question of whether these observations should also have been omitted in the limited-information estimation of other relations. If we assume that the livestock demand relation is the only one appreciably affected and that the change in this relation can be adequately represented by the intervention of new exogenous factors, then the variables representing these exogenous factors would, if observed, enter as  $z_{**}$  in the estimation of (4.1) to (4.4). However, if they are unobserved and omitted from the calculations, the resulting estimates of (4.1) to (4.4) remain consistent.

information estimates are based on asymptotic formulas<sup>13</sup> and may be expected to be biased for small samples. In the absence of a small-sample theory, they are perhaps as good indicators as we have of the order of accuracy of the limited-information estimates. The choice of units of measurement for the variables affects only the constant term. In the calculation of the estimates presented here,  $y_1, y_4, z_1, z_4, z_6$  are in dollars;  $y_6, y_7, z_3, z_9, z_{10}$  are in thousands of pounds; the unit for  $y_2$  and  $y_3$  is dollars per thousand pounds.  $y_5$  is an index whose average level, 1920-49, is approximately 1;  $z_7$  is an index whose average value, 1935-39, is 100.  $z_2$  is in years,  $z_8$  in cents per hour, and  $z_6$  in millions of persons.

#### SOME ASPECTS OF THE INITIAL RESULTS

For the most part, differences between estimates obtained by the two methods are not striking when compared with the indicated magnitudes of sampling fluctuations. Except for equation 4.5, however, they are large enough to have important practical consequences if their reliability were firmly established. An interesting example concerns the differences in signs of the estimated coefficients of  $\beta_{23}$  in (4.2) and  $\beta_{32}$  in (4.3). The positive coefficients obtained by limited information imply that feed grains and protein feeds are technical complements, whereas the negative coefficients obtained by least squares imply that they are technical substitutes. Our a priori knowledge in this case is probably insufficient for us to regard either outcome as implausible. For the least-squares treatment, the outcomes of ordinary  $t$  tests of the hypotheses that particular coefficients are equal to zero can be told fairly well by inspection. Tests of coefficients in (4.1), (4.2), and (4.3) involve the distribution of  $t$  with 24 degrees of freedom; for coefficients in (4.4) and (4.5)  $t$  has 23 and 21 degrees of freedom, respectively. The value of  $t$  in a given case is simply the ratio of the estimated coefficient (in the least-squares row) to the estimated standard error. For 21 or more degrees of freedom, the distribution of  $t$  is almost normal, the 5% point being 1.72 for  $t$  with 21 degrees of freedom and 1.64 for the normal distribution.<sup>14</sup> Analogous tests based on limited-information estimates are not available, but the ratios

<sup>13</sup> Limited-information estimates of coefficients have been shown to have an asymptotic normal distribution, whose covariance matrix may be obtained from the expected values of the second partial derivatives of the logarithm of the likelihood function. The variances of the estimates are functions of the true values of the coefficients of an equation. The calculated standard errors given in the tables are obtained by replacing the true coefficients with their limited-information estimates. See T. W. Anderson and H. Rubin, *Estimation of the Parameters of a Single Equation in a Complete System of Stochastic Equations*, *op. cit.*, pp. 53-56. Also T. C. Koopmans, H. Rubin, and R. B. Leipnik, *op. cit.*, pp. 133-153, and T. C. Koopmans and William C. Hood, *op. cit.*, pp. 177-178.



of estimates to calculated standard errors should still give a rough idea of the reliability attached to the signs of the estimates.

The failure of one of the tests indicated above to reject significantly the null hypothesis could arise because the variable whose coefficient is tested is not important in the relation considered, or for a variety of other reasons. Variations in the observed values of the variable in our particular sample might be small or might be closely related to variations in some of the other variables. Use of an inappropriate algebraic form for the relation or other defects in the statistical specification could also bias the estimates and the associated tests. Therefore, we do not necessarily drop a variable from the analysis if our estimate of its coefficient does not appear significant. The relation between the estimates and their calculated standard errors is one type of evidence taken into account in considering possible revisions of the statistical model.

Another type of evidence taken into consideration is the comparison of the estimates with what we would regard as plausible from our a priori knowledge of the underlying relations of our model. The estimates of  $\gamma_{13}$  in Table VI appear quite implausible in that a negative coefficient for  $Z_{13}$  implies that increasing the amount of roughage fed to livestock, while holding other factors constant, would decrease the production of livestock products. The negative sign for  $\gamma_{33}$  in Table VIII obtained by applying least squares to equation 4.3 would also lead to a rather implausible interpretation: namely, that increased quantities of roughage fed to given herds of animals would lead producers to increase the quantities of protein feeds fed. It is worth noting, in this connection, that errors of observation in  $z_3$  may be unusually large, since the dominant component is pasture consumption, and this is particularly difficult to measure.<sup>14</sup> Other results that seem inconsistent with our a priori information are the positive estimates of  $\beta_{43}$  and the small absolute values of the estimates for  $\gamma_{41}$  in Table IX, and the positive estimates of  $\gamma'_{54}$  in Table X. These will be considered again in later chapters along with some results using alternative statistical specifications.

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<sup>14</sup> In considering these tests we are mainly interested in whether or not the data are consistent with the hypothesis that the true value of the coefficient is zero or of opposite sign to that of the estimate. Consequently the 5% points indicated are for one-tailed tests. See A. M. Mood, *op. cit.*, p. 425.

<sup>15</sup> For the procedure adopted in obtaining a measure of pasture consumption by livestock see pp. 47 to 49 in Chapter III.

## CHAPTER V

### THE PRODUCTION RELATION

#### DATA LIMITATIONS

Certain aspects of the aggregate production relation are considered in this chapter along with results of statistical analysis of equation 4.1 and some alternative equations which were also studied. In view of this examination none of the equations studied appear to give a satisfactory empirical representation of the underlying relation, and the need for further study, possibly with different sources of data and different techniques, is strongly indicated. The results are presented in some detail, to indicate to other research workers the sort of attempts that have been made and the nature of the difficulties that arose. It is hoped that this will assist others in developing more fruitful avenues of attack in future studies. In addition to uncertainties about the theoretical specification (form of equation, way in which stochastic elements enter, etc.) and limitations of existing estimation procedures, two special difficulties with respect to the available data have already been cited and need to be borne in mind in considering the production relation. One is the difficulty in accurately measuring pasture consumption which constitutes about 35% of the total feed consumed by livestock.<sup>1</sup> The other is the lack of any observations on variables other than feed and animal numbers that might be expected to influence the relation significantly.

Pasture seemed too important to be omitted, and so such observations as could be obtained were used throughout the study. In Chapter II it was provisionally assumed that important unobserved factors had increased fairly smoothly over time and could be approximately taken into account by introducing a time variable in the production equation. One attempt to improve on this assumption is reported below (see pp. 85 to 87).

<sup>1</sup> There are, of course, serious difficulties in the measurement of other variables. However, the importance of pasture in this relation and the likelihood that the observations of pasture consumption are still less accurate than observations of other variables seem to justify citing it as a special problem.

## STATISTICAL TESTS EMPLOYED

Fitted equations have been examined by considering the plausibility of estimates of coefficients, by applying statistical tests of some of the assumptions on which the estimates are based, and by checking the fitted equations against data for 1950, the only year for which observations were available that was not included in the sample. Where estimated coefficients seemed sufficiently implausible, other steps in the examination were omitted. Checks against 1950 data are reported in Chapter VIII. The two statistical tests applied in this chapter are the Durbin-Watson test<sup>2</sup> for serial independence of disturbances and the Rubin-Anderson test<sup>3</sup> of overidentifying restrictions. The latter is only appropriate when the equation in question is viewed as a member of a system of equations; hence, it is applied only to equations fitted by the limited-information method.

## DURBIN-WATSON TEST OF SERIAL INDEPENDENCE

The Durbin-Watson test is based on the statistic  $d$  given by

$$(5.1) \quad d = \frac{\Delta^2}{S^2} \quad \text{where}$$

$$(5.2) \quad \Delta^2 = \sum_{t=2}^T (\tilde{u}_t - \tilde{u}_{t-1})^2$$

$$S^2 = \sum_{t=1}^T \tilde{u}_t^2$$

$\tilde{u}_t$  is the residual of the fitted relation for time  $t$ . Durbin and Watson have shown that no exact critical region corresponding to a given level of significance can be found for  $d$ , but that upper and lower limits for each boundary of the critical region can be calculated. They have tabulated these limits, and their tables are used in the tests reported here. All of the serial-correlation tests in the present study are two-tailed tests (tests against both positive and negative serial correlation) at the 0.05 significance level (the relevant limits are thus contained in Durbin and Watson II, Table V, p. 174). If the calculated value of  $d$  for a particular equation is either less than the lower limit ( $d_L$ ) for the boundary of the lower part of the critical region or greater than the upper limit ( $4 - d_L$ ) for the

<sup>2</sup> J. Durbin and G. S. Watson, Testing for Serial Correlation in Least-Squares Regression I, *Biometrika*, Vol. 37, p. 409, and, Testing for Serial Correlation in Least-Squares Regression II, *Biometrika*, Vol. 38, p. 159.

<sup>3</sup> T. W. Anderson and H. Rubin, Estimation of the Parameters of a Single Equation in a Complete System of Stochastic Equation., *op. cit.*, p. 56.

boundary of the upper part of the critical region, the null hypothesis of serially independent disturbances is rejected. If calculated  $d$  lies between the upper limit ( $d_U$ ) for the boundary of the lower part of the critical region and the lower limit ( $4 - d_U$ ) for the boundary of the upper part of the critical region, the hypothesis of serial independence is not rejected. If the calculated statistic falls between  $d_U$  and  $d_L$  or between  $4 - d_U$  and  $4 - d_L$ , the result of the test is inconclusive. Durbin and Watson have presented an approximate test to be applied to inconclusive cases, but, since it would involve substantial extra calculations in our cases, and since they only assert that it is a good approximation when one has more than 40 observations, this has not been applied.

The test has been developed for the single-equation case with the independent variables regarded as fixed. Thus the Durbin-Watson limits are inexact when the test is applied to a least-squares equation containing a lagged dependent variable or when it is applied to an equation fitted by limited information. None of our least-squares equations contains the lagged value of the dependent variable, and so there is no difficulty as far as our least-squares equations are concerned. The use of the tabulated limits must be regarded as approximate when applied to an equation fitted by limited information. It was pointed out in Chapter IV that limited-information estimates of coefficients of the predetermined variables in an equation (the  $z_*$ ) could be regarded as least-squares estimates of coefficients appearing in the regression of a linear combination ( $y^0$ ) of the current endogenous variables ( $y_*$ ) on the  $z_*$ . If the appropriate linear combination were exactly known, then no qualification would attach to the application of the Durbin-Watson test to residuals from limited-information equations. The fact that we only have estimates of coefficients of  $y^0$  makes the test inexact. However, since we have no exact test, it was applied. The tabulated limits depend on the number ( $k'$ ) of independent variables in the equation. Because of the above least-squares interpretation of limited-information estimates,  $k'$  was taken to be equal to the number of predetermined variables in the equation when the test was applied to limited-information residuals.

For the limited-information version of (4.1), the calculated value of  $d$  is 1.11. The limits of the lower bound of the critical region for 30 observations and  $k' = 3$  are  $d_L = 1.12$ ,  $d_U = 1.54$ . Hence, the hypothesis of serial independence is rejected by this test. For the least-squares version of (4.1), the calculated value of  $d$  is 1.28. The limits for 30 observations and  $k' = 5$  are  $d_L = 0.98$ ,  $d_U = 1.73$ . The test is inconclusive in this case. However, in both cases, some doubt is cast on the validity of the assumption of serial independence of disturbances. The calculation of the statistic for both versions of (4.1) is shown in the appendix (pp.

146 and 149). For other equations the same procedure is used, but the details are not shown.

#### RUBIN-ANDERSON TEST OF OVERIDENTIFYING RESTRICTIONS

The Rubin-Anderson test was designed to test the assumption that certain coefficients are zero (certain variables do not appear) in a particular equation, given the validity of the remainder of the specification. It was observed in Chapter I that, in many applications to economics, the investigator is likely to have better grounds for the specification of which variables enter particular relations than for other aspects of his specification. In such cases the interpretation of the test is subject to doubt, but it remains true that an extreme value for the test statistic is an indication of difficulty somewhere in the statistical specification used.<sup>4</sup> The test statistic is  $T \log (1 + 1/\lambda)$  where  $T$  is the number of observations and  $\lambda$  is the largest characteristic root of a determinantal equation used in the calculation of limited-information estimates.<sup>5</sup> The statistic has been shown to have a limiting  $\chi^2$  distribution, with degrees of freedom equal to the number of overidentifying restrictions used in the calculation of the estimates. In the notation of Chapter IV, the number of overidentifying restrictions used is the number of  $z_{**}$  employed in computing the estimates less  $(H - 1)$ , the number of  $z_{**}$  necessary for identification. Six  $z_{**}$  were used in calculating limited-information estimates of (4.1), and two were necessary for identification (the equation contained three current endogenous variables); hence the statistic has 4 degrees of freedom in this case. The calculated value for  $T \log (1 + 1/\lambda)$  was 18.9. Since values greater than 13.3 would arise by chance only one time in one hundred, this is a strong rejection of the set of specifications on which limited-information estimates are based.<sup>6</sup>

<sup>4</sup> See Carl Christ, *A Test of an Econometric Model for the United States, 1921-47*, Conference on Business Cycles, National Bureau of Economic Research, 1951, pp. 35-129. Similar observations could be made about the Durbin-Watson test in the applications considered here. Apparent serial correlation of disturbances could, for example, be caused by the use of an inappropriate algebraic form or the omission of an important variable.

<sup>5</sup> See appendix, pp. 141-142, for an example of the determinantal equation and the calculation of the largest root.

<sup>6</sup> One of the assumptions on which the test is based is that the moment matrix of the predetermined variables converges to a fixed limit as the number of observations increases indefinitely. This is violated by our inclusion of time as a variable unless we assume that, beyond some point, the factors associated with time cease to increase linearly and approach a finite limit. Though this does not seem unreasonable, the questions raised emphasize both the undesirability of applying asymptotic theory to a fairly small sample and the complications that arise with the use of time as a variable.

The implausibility of the estimates of the coefficient of  $Z_{13}$  and the implications of statistical tests that have been performed indicate a need for other formulations of this relationship but do not indicate very precisely the directions in which one might usefully look for alternatives. In selecting alternatives to be examined in the present study, the authors limited themselves to equations that could be analyzed with the time-series data readily available and that could be readily incorporated into the economic model developed in Chapter II. More substantial revisions would have demanded more time and resources than could be allocated but may be necessary if real progress is to be made.

#### ESSENTIAL INPUTS PROPERTY OF LOGARITHMIC RELATION

One feature of equations linear in the logarithms of observed variables that is of interest in connection with equation 4.1 is the following. If we imagine that all of the variables except two are held constant (at finite positive values) and one of these is arbitrarily set equal to zero, the other will be zero or infinite, depending on whether the signs of the coefficients of the logarithms of the two variables are different or the same. If we consider a production relation of this form, relating one output to several inputs, and if the coefficient of the log of output is positive and coefficients of the logs of inputs are negative, then, for output to be nonzero, all inputs must be positive. That is to say, at least some of every input is essential if any output at all is to be produced.

Clearly the appropriateness of assuming this property depends on the definitions of inputs employed. If the resources that may be used in production are classified into a large number of highly special inputs, it is likely that only a few will be absolutely essential to the production of some output. On the other hand, if only a few categories of inputs are set up, it is more likely that some amount from each category will be necessary for any production to be realized.

#### REAGGREGATION OF FEED INPUTS

In reference to the variables of equation 4.1, there is no question but that some output of livestock and livestock products could be produced with zero input of any of the types of feed, provided all three were not simultaneously zero. Thus there exists a discrepancy between our a priori knowledge and the implications of the form of equation adopted in (4.1). This discrepancy concerns a property of the equation in the large (i.e., it arises only if large variations in the variables are considered), and we are ordinarily more interested in the local properties of our equations. However, the existence of the discrepancy, combined with other evidence that (4.1) is unsatisfactory, gives some inducement to consider

TABLE XI  
ALTERNATIVE PRODUCTION RELATION

Method	Estimates of Coefficients			
	$\beta_{13}$	$\gamma'_{11}$	$\gamma'_{12}$	$\gamma'_{10}$
Limited information	-0.463(0.111)	-0.274(0.152)	-0.0045(0.0004)	-3.132(1.009)
Least squares	-0.565(0.107)	-0.170(0.148)	-0.0043(0.0004)	-3.274(0.987)

$$R^2 = 0.978$$

alternatives in which this property is modified. Since the feed variables are measured in common units (pounds of digestible nutrients), one way to remove this inconsistency between our a priori knowledge and the form of equation used is to introduce a variable representing aggregate feed fed to livestock into the production relation instead of the three separate feed variables of (4.1). In the notation of Chapter IV, we then have

$$(5.3) \quad y_{8t} = y_{6t} + y_{7t} + z_{3t}$$

$$(5.4) \quad Y_{1t} + \beta_{13}Y_{8t} + \gamma'_{11}Z_{1t} + \gamma'_{12}z_{2t} + \gamma'_{10} = U'_{1t}$$

where  $Y_8 = \log y_{8t}$ . Whether or not (5.4) permits a better approximation to the underlying relation than (4.1) can hardly be answered a priori. Whereas (4.1) did not seem to allow sufficiently for the substitutability of feeds, (5.4) treats them as perfect substitutes. Estimates of the coefficients of (5.4) are given in Table XI.<sup>7</sup>

In the calculation of the least-squares estimates,  $Y_1$  was again treated as the dependent variable. To obtain the limited-information estimates the model given by equations 5.3, 5.4, and 4.2 to 4.7 was considered. Because (5.3) is not linear in the logarithms of the observed variables, this is a mixed linear, nonlinear system. Rubin and Anderson have shown<sup>8</sup> that the limited-information method yields consistent estimates of a linear subset (in this case 5.4 alone) of such a system. The estimates in the first row of Table XI were obtained, using  $Z_3, Z_4, Z_5, Z_7, Z_8, Z_9, Z_{10}$  as predetermined variables not appearing in the relation being estimated.

Both sets of estimates in Table XI appear plausible on first inspection.

<sup>7</sup> The production equation fitted by Lorie, *op. cit.*, Ch. IV, was of the same form as (5.4), with some differences in definitions of variables. In our notation, his estimates of coefficients, obtained by least squares, using data for the period 1910-44, were  $\beta_{13} = -0.330, \hat{\gamma}'_{11} = -0.523, \hat{\gamma}'_{12} = -0.0021$ .

<sup>8</sup> See T. W. Anderson and H. Rubin. The Asymptotic Properties of Estimates of the Parameters of a Single Equation in a Complete System of Stochastic Equations, *op. cit.* pp. 200-212.

The estimates of  $\gamma'_{12}$  imply an annual increase of about 1% per year in product obtained from given quantities of feed and animals. The sum of estimates of  $\beta_{18}$  and  $\gamma'_{11}$  is  $-0.74$ , using either limited-information or least-squares estimates. In the initial stages of the study, when the a priori impressions of a number of informed persons were sought on various aspects of livestock production and marketing, it was quite generally conjectured that, at levels of output that have prevailed in the past thirty years, livestock production is characterized by nearly constant returns to scale. It was somewhat less generally stated that numbers of animals fed and amounts of feed used probably dominated production to such an extent that output would be almost proportional to changes in these two inputs (it being recognized that some observed inputs would tend to change along with animal numbers and feed).

#### TEST OF IMPORTANCE OF OTHER INPUTS

If both these conjectures are true, then, except for sampling fluctuations, the sum of the estimates of  $\beta_{18}$  and  $\gamma'_{11}$  should be  $-1$ . If the first conjecture is accepted a priori but not the second, a test of the hypothesis that  $\beta_{18} + \gamma'_{11} = -1$  may be regarded as a test of the importance of unobserved factors that did not consistently vary with animal numbers or feed. We are inclined to interpret the test in this way, but we recognize that other a priori assumptions would lead to other interpretations. Since our limited-information procedures do not include tests of linear hypotheses, the test is based on the least-squares calculations. The test statistic has the  $F$  distribution with 1 and  $T - k' - 1$  degrees of freedom where  $k'$ , as before, is the number of independent variables in the equation.<sup>9</sup> The calculated value of  $F$  in this case is 6.97, whereas the value corresponding to the 0.05 significance level is 4.22. Thus, by our interpretation, the test offers substantial evidence of the importance of omitted variables.

#### PLAUSIBILITY OF MARGINAL RESPONSES

Marginal response of output to small increments of feed or animals can be calculated for particular values of the inputs by solving the equa-

<sup>9</sup> Let the regression equation be given by  $y_t + \alpha x'_t = u_t$  where  $y$  = dependent variable,  $x_t$  = vector of independent variables,  $\alpha$  = vector of coefficients. Let the null hypothesis be given by  $b\alpha' = \beta_0$ . Then

$$F = \frac{(T - k' - 1)(b\hat{\alpha}' - \beta_0)^2}{T\hat{\sigma}^2bM_{xx}^{-1}b'}$$

has the  $F$  distribution with 1 and  $T - k' - 1$  degrees of freedom.  $M_{xx} = T\sum_{t=1}^T x_t x'_t$ ,  $\hat{\alpha}$  is the least-squares estimate of  $\alpha$ ,  $\hat{\sigma}^2 = \frac{1}{T}\sum_{t=1}^T (y_t + \hat{\alpha}x'_t)$ . For a general discussion of tests of linear hypotheses see A. M. Mood, *op. cit.*, p. 305.



tions for  $y_1$  and differentiating with respect to each input. This yields

$$(5.5) \quad \frac{\partial y_1}{\partial y_8} = -\beta_{18}(\text{antilog} - \gamma'_{10})y_8^{-\beta_{18}-1}z_1^{-\gamma'_{11}}10^{-\gamma'_{12}}z_2$$

$$(5.6) \quad \frac{\partial y_1}{\partial z_1} = -\gamma'_{11}(\text{antilog} - \gamma'_{10})y_8^{-\beta_{18}}z_1^{-\gamma'_{11}-1}10^{-\gamma'_{12}}z_2.$$

Whichever set of coefficients is used, marginal response to one input tends to decrease with increases in that input and to increase with increases in the other.

Marginal returns to feed and inventory have been calculated for an "average," year considered for this purpose to be a year in which the variables  $y_8$ ,  $z_1$ ,  $z_2$  take on their average values over the period of the observations. These average values are 392 million for  $y_8$ , 6.09 billion for  $z_1$ , and 15.5 for  $z_2$ . The units are those used in obtaining the estimates given in Table XI, namely thousands of pounds of TDN for  $y_8$ , dollars worth of estimated potential production for  $z_1$ , and years for  $z_2$ . If we use these average values for the variables and the limited-information coefficients given in Table XI, the calculated marginal return for feed is 8.67, and the marginal return to livestock inventory is 0.32. If we use least-squares estimates of coefficients, the calculated marginal return to feed is 10.47 and to inventory 0.20.

The aggregate production relation of our model depends on the production functions of individual producers and on relations determining the distribution of inputs among individual producers. For a small increase in the aggregate of one input, the rate of response of aggregate production will be a weighted average of the marginal productivities of the input for individual firms, and the weights will be proportional to the amounts by which the individual firms increase their use of the input in question.<sup>10</sup> To the extent that individual producers succeed in equating marginal productivity of an input to the input-output price ratio, we should expect the marginal response of aggregate output also to approximate this ratio. In general, we would expect the approximation to

<sup>10</sup> Let  $y = f(a, b, c)$  be the aggregate production relation with  $y$  as aggregate output and  $a, b, c$  as aggregates of inputs. Let  $y_i, a_i, b_i, c_i$  be the output and inputs of an individual firm with production function  $y_i = f_i(a_i, b_i, c_i)$ . We have  $y = \sum_i y_i, a = \sum_i a_i$ , etc. Suppose the process by which the input  $a$  is allocated among firms can be summarized in a set of relations  $a_i = g_i(a)$ . For a small increase in  $a$ , with  $b$  and  $c$  fixed, we have  $\frac{\partial y}{\partial a} = \sum_i \frac{\partial y_i}{\partial a_i} \frac{\partial a_i}{\partial a}$ . It would be useful to consider in detail the implications of various assumptions about the distribution relations and the process underlying them. This, however, is outside the scope of the present study.

be closer for current endogenous inputs than for predetermined inputs, since the latter cannot be adjusted in response to current information.

In any case we do not have data on the cost of increases in inventory. We can verify that the general order of magnitude of the calculated marginal returns to the inventory variable are reasonable, by noting that average returns are about 1.2 and that average returns should decline if the size of the herd is increased while feed consumption remains constant. This implies that marginal returns are below average returns.

Data are also lacking on the cost of roughage. However, since roughage is predetermined, marginal adjustments in quantity of feed consumed take place largely through variations in grains fed. We might expect, therefore, a rough correspondence between marginal returns to grain and price of grain. Our calculated marginal returns above, 8.67 using limited-information coefficients and 10.47 using least-squares, are in dollars worth of product per thousand pounds of TDN. The average price of grain per thousand pounds of TDN during our sample period was \$19.40.<sup>11</sup> Thus there is a substantial discrepancy in this comparison.<sup>12</sup>

An explanation that readily suggests itself is that digestible nutrients obtained through roughage may cost substantially less than digestible nutrients obtained through grains. This is confirmed if we compare average production with average quantity of feed fed. Average production was \$7.4 billion at average prices. If the average quantity of feed, 392 billion lb of TDN, is evaluated at \$19.40 per 1000 lb, then the feed fed to animals was worth about \$7.6 billion. At the same time that this provides an explanation for the discrepancy noted above, it also makes highly questionable the assumption (utilized in 5.4) that digestible nutrients in one type of feed are perfect substitutes for digestible nutrients in another type of feed.

#### TESTS OF PRODUCTION RELATION WITH AGGREGATE FEED VARIABLE

The statistical tests described earlier in the chapter and applied to equation 4.1 also tend to cast doubt on the appropriateness of (5.4) as a formulation of the production relation. The Durbin-Watson statistic calculated for the residuals from the least-squares version of (5.4) is 0.634, whereas the lower limit ( $d_L$ ) of the lower boundary of the critical region for 30 observations and 3 independent variables is 1.12. For the limited-information equation, the calculated statistic  $d$  is 0.596 and  $d_L$  for  $k' = 2$  is 1.18. In both cases the test significantly rejects the assumption of serially uncorrelated disturbances. The value of  $T \log (1 + 1/\lambda)$

<sup>11</sup> This is a simple average of the figures in col. 1, Table V, on p. 63.

<sup>12</sup> Comparisons, using data for scattered individual years instead of averages, are similar.

calculated for this relation is 41.7, which is also highly significant, since a value as large as 16.8 would occur only one time in 100 if all of the assumptions on which estimation was based were valid.

#### ASSUMPTIONS UNDERLYING THIRD FORM OF PRODUCTION RELATION

Another form of the production relation for which estimates were obtained involved different treatment of the feed variables and omission of the time variable. An attempt was made to learn of possible sources of the secular increase in feeding efficiency in livestock enterprises and to consider to what extent these might be related to observable variables that could be utilized in studying the relation. The principal explanatory factors that seemed to the authors and others to be of importance were improved breeds, improved feeding practices, and improved care of animals. If observations were available on all the inputs used in livestock production, it would seem that improved care and improved feeding practices could be related, to a considerable extent, to increased use of labor and/or equipment, and to changes in the composition of livestock rations. Different breeds of animals could be regarded as different inputs if sufficient data were available. Although the data necessary to study various of these factors are sometimes available in the analysis of feeding experiments and in intensive farm-management studies of a few farms, the only aggregate data that seemed promising for incorporation in the present study related to the amounts of particular feeds fed to livestock during the sample period.

Several advisers who were consulted suggested that one of the important historical changes during the period of observation was the increased recognition of the value of protein in animal diets and the increased use of high-protein feeds. Although the advisers disagreed somewhat in their speculation about the relative importance of this and other factors, it did seem worth while to consider a form of the production relation in which protein was distinguished from other digestible nutrients. It was possible to estimate the number of pounds of protein (DP) and the number of pounds of other digestible nutrients (ODN) fed to animals from the estimates of protein and total nutrient content of individual feeds given by Morrison and Jennings. The use of their estimates is explained in Chapter III (see especially footnote 43), and the composition figures actually used in this study are summarized in Table II, pp. 51-52.

#### ESTIMATED COEFFICIENTS FOR REVISED RELATION

Let  $y_{9t}$  be the total number of pounds of digestible protein contained in feed fed to livestock in period  $t$ , and let  $y_{10t}$  be the total number of

pounds of other digestible nutrients.  $Y_{9t} = \log y_{9t}$ ,  $Y_{10t} = \log y_{10t}$ . (5.4) was modified by taking out  $z_2$  and by substituting  $Y_9$ ,  $Y_{10}$  for  $Y_8$ . Coefficients in the resulting equation were estimated by limited information and by least squares with these results:

$$(5.7) \quad Y_1 - 3.98Y_9 + 3.65Y_{10} - 0.67Z_1 + 3.62 = 0$$

(0.91)                      (1.10)                      (0.36)                      (2.32)

$$(5.8) \quad Y_1 - 1.97Y_9 + 1.17Y_{10} - 0.28Z_1 - 1.80 = 0$$

(0.40)                      (0.48)                      (0.24)                      (0.77)                       $R^2 = 0.88.$

The limited-information coefficients are given in (5.7), the least squares in (5.8). The time subscript has been omitted from each variable. Calculated standard errors of the estimated coefficients are given in parentheses below their respective coefficients.

#### HISTORICAL CHANGES IN PROTEINS AND OTHER NUTRIENTS

The result that output is decreased by increases in other digestible nutrients is quite implausible, as is the implication that production could be so greatly increased by small increases in protein fed. Several possible explanations for the implausibility of the results were considered. It was noted with mild surprise that the simple correlation coefficient between  $Y_9$  and  $Y_{10}$  was 0.98 for our sample period. The ratio of protein to other digestible nutrients fed was 0.166 for the first five years of observation (1920-24), compared with 0.177 for the last five years (1945-49). The prior impressions of the investigators had led them to expect a fairly substantial increase in protein, relative to other nutrients. Because others may have similar impressions, Table XII, accounting for protein and other nutrients by sources, may be of some interest.

Though the use of protein concentrates did expand greatly during our period of observation, a fairly modest expansion in the feeding of grains relative to forage was sufficient to keep the ratio of digestible protein to other digestible nutrients almost constant. To the extent that high correlation between  $Y_9$  and  $Y_{10}$  contributed to the inaccuracy of the estimates of coefficients in (5.7) and (5.8), we should expect the calculated standard errors of the estimated coefficients to be large. Though the calculated standard errors are, in fact, large, they are small enough to suggest that this was not the only source of difficulty with this equation.

#### OTHER FORMS OF RELATION

Two other production equations were fitted. An alternative algebraic form involving the same variables as (5.7), (5.8) is given in (5.9).



$$(5.11) \quad Y_1 + 1.32Y_9 - 1.84Y_{10} - 0.09Z_1 - 0.006z_2 - 3.54 = 0$$

(0.47)            (0.46)            (0.13)            (0.008)            (0.84)       $R^2 = 0.91$

In these cases the implausibility of the coefficients of  $Y_{10}$  made it seem not worth while to apply further analysis.

#### POSSIBLE MODIFICATIONS IN APPROACH

Reconsideration of our approach to the production relation is clearly in order. A variety of alternative approaches are in principle possible, and we have not investigated them thoroughly enough to try to make strong recommendations at this time. By way of illustration, two possible ways of proceeding are briefly outlined.

On the assumption that an important weakness in the approaches already tried is the lack of observations on inputs other than feed and animals, one might consider trying to specify the missing inputs and the relations that determine their values, in the hope that the other variables in these relations would be observable. If so, one could eliminate the unobservable inputs from these equations, thus obtaining a smaller number of equations that would not contain the unobserved variables. The new equations would be called partially reduced form equations and would have to be reinterpreted in light of the way in which they were derived. Consider the following hypothetical example. Let (5.12), (5.13) be two equations of a larger structural system.

$$(5.12) \quad y_1 + \beta_{12}y_2 + \gamma_{11}z_1 + \gamma_{12}z_2 + \gamma_{10} = u_1$$

$$(5.13) \quad y_2 + \beta_{23}y_3 + \gamma_{22}z_2 + \gamma_{23}z_3 + \gamma_{20} = u_2$$

Suppose that  $y_2$  is not observable. By solving (5.13) for  $y_2$  and substituting for it in (5.12), the following relation is obtained.

$$(5.14) \quad y_1 - \beta_{12}\beta_{23}y_3 + \gamma_{11}z_1 + (\gamma_{12} - \beta_{12}\gamma_{22})z_2 - \beta_{12}\gamma_{23}z_3 = (u_1 + \beta_{12}u_2)$$

In general, statistical specifications commonly made in analyzing linear equations will hold for (5.14) if they are valid for (5.12), (5.13). If  $u_1$ ,  $u_2$  are normal, independent of the  $z$ 's and serially independent, for example, the new disturbance ( $u_1 + \beta_{12}u_2$ ) will also have these properties. (5.14) will be less autonomous<sup>13</sup> than (5.12), (5.13) and typically more difficult to interpret, but its substitution for (5.12), (5.13) does not affect the validity of the model.

Another possibility is to seek sampling data on the operations of a number of individual producers and to use these as the basis for the analysis. To draw reliable inferences about the relation among aggregate

<sup>13</sup> For a discussion of the autonomy of economic relations see Trygve Haavelmo. *The probability Approach in Econometrics*, *op. cit.* pp. 26-39.

variables, the sample data would have to be rather comprehensive, and the model would have to specify (perhaps only in part) relations between individual and aggregate observable variables and between individual and aggregate parameters. Tobin's use of sample data to estimate income elasticity in an aggregate demand relation is an example of this approach.<sup>14</sup> As sampling data continue to become more available the possibilities for successfully developing this approach should improve.<sup>15</sup> The possibility of making use of data provided by technical experiments in this connection is also worth exploring. It is possible that models like those used in *Activity Analysis*<sup>16</sup> could be developed to provide a basis for combining experimental data with sampling data and time series.

<sup>14</sup> See James Tobin., *op. cit.*

<sup>15</sup> Some preliminary speculation about problems that might be encountered in this development are given in Clifford Hildreth, *Combining Cross Section Data and Time Series*, *Cowles Commission Discussion Paper, Statistics 347*. Cowles Commission for Research in Economics, University of Chicago, Chicago, May 15, 1950. See also

Stephen G. Allen Jr., *Estimation of a Single Equation in a Complete System of Stochastic Equations with Cross-Section, Time-Series Data*, *Cowles Commission Discussion Paper, Statistics 366*, Cowles Commission for Research in Economics, University of Chicago, Chicago, Oct. 8, 1951.

Lawrence R. Klein. *Sample Surveys of Households: a New Tool in Econometrics* *Econometrica*, Vol. 19, p. 345, 1951.

<sup>16</sup> See *Activity Analysis of Production and Allocation*, T. C. Koopmans, editor, Cowles Commission Monograph 13, John Wiley & Sons, New York, 1951.

## CHAPTER VI

### THE FARM DECISION RELATIONS

#### PLAN OF CHAPTER

The farm decision relations reflect the economic behavior of the producers of livestock products. They represent decisions with respect to the use of feed grains (equation 4.2) and protein feeds (equation 4.3) and the sale of livestock and livestock products (equation 4.4). For reference they are rewritten here.

$$(4.2) \quad Y_2 + \beta_{23}Y_3 + \beta_{25}Y_5 + \beta_{26}Y_6 + \gamma_{21}Z_1 + \gamma_{23}Z_3 + \gamma_{20} = U_2$$

$$(4.3) \quad \beta_{32}Y_2 + Y_3 + \beta_{35}Y_5 + \beta_{37}Y_7 + \gamma_{31}Z_1 + \gamma_{33}Z_3 + \gamma_{30} = U_3$$

$$(4.4) \quad \beta_{41}Y_1 + \beta_{42}Y_2 + \beta_{43}Y_3 + Y_4 + \beta_{45}Y_5 + \gamma_{41}Z_1 + \gamma_{43}Z_3 + \gamma_{40} = U_4$$

Estimates of coefficients in the equations were given in Tables VII, VIII, IX of Chapter IV. To supplement the rationalization offered in Chapter II for the choice of variables in these relations, a formal theory of decision making is developed in the latter part of this chapter, and the theoretical results are related to the empirical relations used in the study. The theory will also serve the purpose of calling attention to certain ways in which the empirical approach can be improved if new data can be found or new a priori information brought to bear. Before this theory is developed, the results of applying the statistical tests described in Chapter V to the decision relations and some results of experiments with alternative forms of the relation expressing demand for feed grains are presented.

#### LINEAR FORM OF DEMAND FOR FEED GRAINS

It was desired throughout the study to experiment as much as possible with different formulations of the various relations of the model to see whether indications of the appropriateness of alternative specifications could be found. For lack of time, the demand for feed grains was the only one of the decision relations on which alternatives were tried. A version of this relation that was linear in the observed variables was fitted both by limited-information and least-squares methods. In the



notation adopted in Chapter IV, the relation is given by

$$(6.1) \quad y_{2t} + \beta'_{23}y_{3t} + \beta'_{25}y_{5t} + \beta'_{26}y_{6t} + \gamma'_{21}z_{1t} + \gamma'_{23}z_{3t} + \gamma'_{20} = u_{2t}$$

where  $y_2$  represents price of feed grains measured in dollars per thousand pounds of TDN and  $y_3$  stands for price of protein feeds, also in dollars per thousand pounds of TDN.  $y_6$  is an index of the price of livestock products, such that unity corresponds to average prices during the period of observation.  $y_6$  represents the amount of feed grain fed in billions of pounds of TDN,  $z_1$  is beginning inventory of animals measured in billions of dollars worth of estimated potential production, and  $z_3$  denotes roughage consumed by livestock, measured in billions of pounds of TDN. The units for  $y_6$ ,  $z_1$ ,  $z_3$  have been changed from those used in Chapter IV, to keep their coefficients in the linear version from being inconveniently small. Estimates of coefficients of (6.1) are given in Table XIII.

#### COMPARISON OF RESULTS

Predetermined variables not appearing in (6.1) that were used (as  $z_{**}$ ) in the limited-information computation were  $z_4$ ,  $z_5$ ,  $z_7$ ,  $z_8$ ,  $z_9$ ,  $z_{10}$ . Equation 4.2, which was discussed in Chapter IV, contains the same observed variables as (6.1), but is expressed as a linear function of the logarithms of the observed variables. In (4.2) the coefficients represent elasticities, whereas in (6.1) they represent slopes or marginal rates of response. To compare the results, it is convenient to solve each equation for  $y_6$  (after setting the disturbance equal to zero), since  $y_6$  is regarded as a decision variable by the individual producer, and it seems more natural, therefore, to make conjectures about the response of quantity fed to changes in the other variables. Equation 4.2A below was obtained by solving (4.2) for  $y_6$  and using as coefficients the limited-information estimates reported in Table VII, p. 72. The least-squares estimates were used to obtain (4.2B). (6.1A) and (6.1B) result from expressing  $y_6$  as a function of the other variables in (6.1) and using,

TABLE XIII  
DEMAND FOR FEED GRAINS IN LINEAR FORM

Method	Estimates of Coefficients in (6.1)					
	$\beta'_{23}$	$\beta'_{25}$	$\beta'_{26}$	$\gamma'_{21}$	$\gamma'_{23}$	$\gamma'_{20}$
Limited in-formation	0.026(0.076)	-26.53(2.92)	0.202(0.042)	-7.26(1.56)	0.004(0.039)	21.34(16.127)
Least squares	-0.011(0.067)	-25.22(2.61)	0.167(0.038)	-6.67(1.50)	0.019(0.037)	19.14(6.585)

$$R^2 = 0.980$$

respectively, as coefficients, the limited-information and the least-squares estimates from Table XIII.

$$(4.2A) \quad y_6 = (961.6)y_2^{-0.679} y_3^{-0.176} y_5^{1.075} z_1^{1.875} z_3^{-0.481}$$

$$(4.2B) \quad y_6 = (269.8)y_2^{-0.795} y_3^{0.032} y_5^{1.029} z_1^{1.932} z_3^{-0.351}$$

$$(6.1A) \quad y_6 = -4.951y_2 - 0.129y_3 + 131.3y_5 \\ + 35.94z_1 - 0.020z_3 - 105.6$$

$$(6.1B) \quad y_6 = -5.988y_2 + 0.066y_3 + 151.0y_5 \\ + 39.94z_1 - 0.114z_3 - 114.6$$

For convenience, the first factors on the right in (4.2A) and (4.2B) have been adjusted so that the units in which variables are measured in these equations are the same as the units in (6.1). In Table XIV the slopes of the equations in directions parallel to the various input axes are listed. The slopes of (4.2A) and (4.2B) depend on where the slopes are measured. For the third and fourth rows of the table, average values of the variables on the right-hand side of (4.2A) and (4.2B) were used. For the years 1920-49, these were  $y_2 = 19.43$ ,  $y_3 = 44.65$ ,  $y_5 = 0.981$ ,  $z_1 = 6.093$ ,  $z_3 = 244.9$ . The average value of  $y_6$  over this period was 135.6. Partial derivatives in the last two columns correspond to the 1950 values of the variables on the right. For 1950,  $y_2 = 31.63$ ,  $y_3 = 58.38$ ,  $y_5 = 1.736$ ,  $z_1 = 6.508$ ,  $z_3 = 271.4$ ,  $y_6 = 170.6$ . Table XV contains elasticities with respect to the indicated variables. For equations 6.1A and B, elasticities have been calculated for both average and 1950 values of these variables.

The larger responses of grain consumption ( $z_6$ ) to variations in roughage consumption ( $z_3$ ) indicated by equations 4.2A and B seem more plausible than the smaller responses indicated by equations 6.1A and B.

TABLE XIV  
ESTIMATED SLOPES FOR FEED-GRAINS DEMAND EQUATIONS

Equation	Assumed Values of Variables	Partial Derivative of $y_6$ with Respect to:				
		$y_2$	$y_3$	$y_5$	$z_1$	$z_3$
6.1A	....	-4.95	-0.13	131	35.9	-0.02
6.1B	....	-5.99	0.07	151	39.9	-0.11
4.2A	Average	-4.73	-0.53	148	41.6	-0.27
4.2B	Average	-5.50	0.10	141	42.6	-0.19
4.2A	1950	-3.96	-0.56	114	53.1	-0.33
4.2B	1950	-4.56	0.10	108	53.9	-0.23

TABLE XV  
ESTIMATED ELASTICITIES FOR FEED-GRAINS  
DEMAND EQUATIONS

Equation	Assumed Values of Variables	Elasticity of $y_6$ with Respect to:				
		$y_2$	$y_3$	$y_5$	$z_1$	$z_2$
4.2A	.....	-0.68	-0.18	1.07	1.88	-0.48
4.2B	.....	-0.80	0.03	1.03	1.93	-0.35
6.1A	Average	-0.71	-0.04	0.95	1.62	-0.04
6.1B	Average	-0.86	0.02	1.09	1.80	-0.21
6.1A	1950	-0.84	-0.04	1.22	1.25	-0.03
6.1B	1950	-0.99	0.02	1.37	1.36	-0.16

However, the differences in slopes and elasticities among the four equations are, in general, sufficiently small to be explained by modest allowances for sampling fluctuations. Therefore the comparisons made do not give any strong indications regarding the appropriateness of the alternative forms of equations.

#### RESULTS OF STATISTICAL TESTS

Results of applying the Durbin-Watson test to residuals from the limited-information and least-squares versions of (6.1), (4.2), (4.3), and (4.4) are given in Table XVI. Table XVII shows the results of the Rubin-Anderson test applied to the limited-information fittings. Both tests tend to cast doubt on the assumptions underlying equation 4.4. Although the Rubin-Anderson test does not reject (6.1), the calculated statistic is sufficiently close to the critical value so that changing the significance level to about 6% would result in rejection of the null hypothesis. As was indicated in Chapter V, when one is uncertain about several aspects of the specification, the tests may be interpreted as general tests of the assumptions on which estimates are based. From this point of view, it would be desirable to know something about the power of each test against various alternatives. Unfortunately this has not been investigated.

#### RELATIONS INCLUDING PHYSICAL SUPPLY OF GRAINS

Most livestock producers also produce feed grains. A substantial number of these plan as a regular practice to "market their grain in the form of livestock." Among such producers, it seemed that there might be some tendency for their use of grain to depend partly on their own physical supply, as well as on price and the other variables indicated. It

TABLE XVI  
APPLICATION OF DURBIN-WATSON TEST TO FARM DECISION  
EQUATIONS  
5% Level of Significance

Equation No.	Method of Estimation	Calculated Statistic, $d$	No. of Predetermined or dependent Variables, $k'$	Limits of Relevant Boundary		Results
				Lower limit, $d_L$	Upper limit, $d_U$	
6.1	L. I.	1.62	2	1.18	1.46	N
6.1	L. S.	1.53	5	.98	1.73	I
4.2	L. I.	1.64	2	1.18	1.46	N
4.2	L. S.	1.56	5	.98	1.73	I
4.3	L. I.	1.69	2	1.18	1.46	N
4.3	L. S.	1.89	5	.98	1.73	N
4.4	L. I.	2.85	2	2.54*	2.82*	R
4.4	L. S.	2.86	6	2.17*	3.08*	I

Abbreviations: L. I.—Limited information; L. S.—Least squares; N—Null hypothesis not rejected; I—Inconclusive; R—Null hypothesis rejected.

\* Since we are concerned with the boundary of the upper part of the critical region in this case, its lower limit is given by  $(4 - d_U)$  and its upper limit by  $(4 - d_L)$ .

TABLE XVII  
APPLICATION OF RUBIN-ANDERSON TEST TO FARM  
DECISION EQUATIONS  
5% Level of Significance

Equation No.	Calculated Statistic, $T$ $\log(1 + 1/\lambda)$	Degrees of Freedom	Critical Values of Statistics	Test Result
				N = Not Rejected R = Rejected
6.1	9.16	3	0.216-9.35	N
4.2	2.03	3	0.216-9.35	N
4.3	4.45	3	0.216-9.35	N
4.4	0.051	2	0.051-7.38	R

seemed difficult to assess, on a priori grounds, the possible importance of this tendency in the aggregate, and so a version of the demand for feed grains relation was considered, in which physical supply of grain ( $z_2$ ) was included, along with the variables that appear in (4.2) and (6.1). If the tendency cited above were important, we should expect this relation to

show that, for given values of the other variables, an increase in physical supply of feed grain should be associated with an increase in quantity fed.

Estimates of parameters were obtained for demand equations that included  $z_9$ , the physical supply of feed grain. (6.2) was obtained by limited information, it being assumed that the relation was linear in the logs of the observed variables. (6.3) and (6.4) were obtained by least squares, the equation being treated first as linear in the logs and then (in 6.4) as linear in the observed variables.

$$(6.2) \quad Y_2 + \underset{(0.230)}{0.138}Y_3 - \underset{(0.225)}{1.446}Y_5 + \underset{(0.597)}{0.856}Y_6 - \underset{(0.694)}{2.230}Z_1 \\ + \underset{(0.636)}{0.380}Z_2 + \underset{(0.420)}{0.477}Z_9 + \underset{(5.23)}{6.241} = U_2$$

$$(6.3) \quad Y_2 - \underset{(0.184)}{0.153}Y_3 - \underset{(0.176)}{1.161}Y_5 + \underset{(0.440)}{0.433}Y_6 - \underset{(0.573)}{1.756}Z_1 \\ + \underset{(0.571)}{0.064}Z_2 + \underset{(0.330)}{0.741}Z_9 + \underset{(3.18)}{7.098} = U_2$$

$$R^2 = 0.957$$

$$(6.4) \quad y_2 - \underset{(0.060)}{0.033}y_3 - \underset{(2.34)}{24.22}y_5 + \underset{(0.513)}{0.595}y_6 - \underset{(0.149)}{0.481}z_1 \\ - \underset{(0.341)}{0.082}z_2 + \underset{(0.340)}{0.940}z_9 + \underset{(9.64)}{14.90} = u_2$$

$$R^2 = 0.970$$

In all cases the estimated coefficient of  $z_9$  implied a tendency to decrease consumption with increases in the physical supply (other variables held constant). This seemed sufficiently implausible that further analysis of these equations was not undertaken.

### THE INVENTORY RELATION

The inventory relation 2.26 is closely associated with the farm decision relations, expressing the necessary condition that the inventory of animals available for future production can be increased only to the extent that current production exceeds current sales. In the revised notation it is written

$$(6.5A) \quad (z_{1t+1} - z_{1t}) = \alpha(y_{4t} - y_{1t}) + v_t$$

If  $v_t$  were independent of the disturbances in the rest of the model,  $y_{4t}$  and  $y_{1t}$  could properly be regarded as predetermined in this relation, and the least-squares estimate of  $\alpha$  would be unbiased. That this is strictly

true seems doubtful, since producers must plan their sales and inventories simultaneously and might be expected to recognize and be influenced by factors included in  $v_t$ . However, since this relation was thought to be of less interest in the present study, and since computing facilities were fully employed with the other analyses, this relation was estimated only by least squares. The least-squares estimate of  $\alpha$  in (6.5A) is 1.056, and the simple correlation between  $(z_{1t+1} - z_{1t})$  and  $(y_{4t} - y_{1t})$  is 0.88. If we do not constrain the relation to pass through the origin, i.e. if we consider

$$(6.5B) \quad (z_{1t+1} - z_{1t}) = \alpha_0 + \alpha_1(y_{4t} - y_{1t}) + v$$

then the least-squares estimates of  $\alpha_0$  and  $\alpha_1$  are, respectively, 0.0609 and 1.140. An ordinary  $t$  test of the hypothesis that  $\alpha_0 = 0$  rejects the hypothesis at the 0.05 level of significance. The calculated value for  $t$  is 2.8; the upper critical value for  $t$  with 28 degrees of freedom is 2.05. If we were dealing with a homogeneous commodity,  $\alpha_0$  would, of necessity, be equal to zero. The discrepancy in our sample could be caused by a systematic change in the composition of our aggregates over the sample period or by a tendency to overstate sales and/or understate production in our observations.

#### THEORY AS AN AID TO MODEL CONSTRUCTION

Problems encountered in empirical research seem to fall naturally into two classes. These are problems of analyzing models and problems of constructing models. The separation is basically the same as the distinction often drawn between testing hypotheses and formulating hypotheses. Though much useful work remains to be done in both areas, problems of analyzing models seem to be, in one sense, less baffling, in that logic, mathematics, and especially mathematical statistics seem to provide relatively well-defined approaches to these problems. Given a well-defined model or class of models and associated observations, we know something about how to check their implications. Mathematical problems beyond our abilities and computational problems beyond our resources may arise, but we are seldom in the position of not knowing, in principle, a reasonable way to proceed.

Approaches to problems of model construction are much more difficult to describe or discuss. Certainly some familiarity with the economic reality to be represented by a model is necessary, and close familiarity is an advantage. It has often been asserted that model construction partakes more of art than of science. Without trying to develop a position on fundamental questions of this sort, we should like to observe that practice and communication of experiences and reactions seem to play

important roles, both in art and in science. In the present stage of econometrics, there may be more to be gained by developing and explaining examples of model construction than by trying to formulate general rules or to systematize particular approaches. To make the example furnished by our study a little more complete, we shall try to illustrate briefly the possibility of developing formal theories, as an aid to other processes of model construction.

In most contexts in which theories are constructed, strong motives exist for making the theories as general as possible. The theorist tries to abstract from the details that may apply to particular persons, markets, industries, or institutional settings, and to make clear the nature of the more general and fundamental aspects of his problem. In applying theory to a particular economic situation, it may be useful to reintroduce some of the details that characterize the chosen situation and to rework the theory with these elements present. Of course, if all the special features were introduced, the amount of logical or mathematical detail to be worked through in order to learn the implications of the theory would be overwhelming. There is still the problem of selecting those properties that are important for the purposes at hand, but these purposes differ when one constructs a theory as an aid to development and exposition of general principles and when one empirically studies a chosen realm of economic behavior.

#### A THEORETICAL MODEL OF LIVESTOCK DECISIONS

An effort has been made to include important features of the economic problems faced by livestock producers in the theoretical model developed below. The model is first developed in terms of abstract inputs, outputs, prices, and other variables, and these are then related to variables that appear in the decision relations of our economic model. This makes it convenient to use a different notation for the abstract model, and so the notation is explained first. Of course, many simplifying assumptions have been made.

#### NOTATION AND ASSUMPTIONS

Capital letters denote stocks of a commodity at a given date, small letters denote flows during a given interval. As before, the subscript  $t$  identifies a time interval. A capital letter with subscript  $t$  is a stock at the beginning of the  $t$ th period. We consider the decisions and plans<sup>1</sup> of an entrepreneur during the  $t$ th time period.

<sup>1</sup> In the terminology to be used in this section plans are regarded as conditional decisions. The term *anticipations* will be used to denote possible outcomes of future events that the entrepreneur regards as likely and uses as a basis for the forma-

$n_t$  represents the output produced during the  $t$ th period.  $x_{1t}$ ,  $x_{2t}$  are quantities of inputs used. They represent either materials that are used up (disappear) in the production process or services that cannot be stored.  $X_{3t}$  represents durable capital equipment on hand at the beginning of period  $t$ . It is assumed that equipment acquired during period  $t$  only affects the output of periods subsequent to  $t$ .

Further, it is assumed that the firm's own output is required in the production process.  $N_t$  is the quantity of past output on hand at the beginning of the period in question. The firm's production function is given by

$$(6.6) \quad n_t = \varphi(x_{1t}, x_{2t}, X_{3t}, N_t)$$

The amount of output sold during the period is denoted by  $s_t$ . It is related to output on hand at the beginning and end of the period and to production by the accounting relation

$$(6.7) \quad N_{t+1} = N_t + n_t - s_t$$

$p_t$ ,  $q_{1t}$ ,  $q_{2t}$ ,  $q_{3t}$  represent prices of  $s_t$ ,  $x_{1t}$ ,  $x_{2t}$ ,  $X_{3t}$ . Money withdrawn from the enterprise during the  $t$ th period  $w_t$  is given by

$$(6.8) \quad w_t = p_t s_t - q_{1t} x_{1t} - q_{2t} x_{2t} - q_{3t} (X_{3t+1} - \rho X_{3t})$$

where  $\rho$  represents the proportion of the fixed capital at the beginning of the period that is still available for use at the end of the period; i.e.,  $(1 - \rho)$  is the rate of physical depreciation, and for simplicity it is assumed to be independent of the intensity of use.

#### PLANS AND DECISIONS

The market for  $x_2$  is assumed to be such that the entrepreneur must choose the amount of this input to be used one period in advance; hence  $x_{2t}$  is known at the beginning of period  $t$ .  $N_t$ ,  $X_{3t}$  are, of course, also known at the beginning of the period. We regard prices for period  $t$  as determined in markets that operate independently of this producer's current decisions and treat them as known at the time the decisions and plans of the period are made by the producer.<sup>2</sup>

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tion of plans. Anticipations relate to outcomes of events that are determined independently of the entrepreneur's actions. Plans relate to events that the entrepreneur can control (within limits). In general, later decisions will coincide with plans if all the anticipations on which plans are based are realized. Occasionally both plans and anticipations will be referred to as expectations.

<sup>2</sup> In the classical theory of competitive markets, individual buyers and sellers regard price as a datum, even though their aggregate behavior determines price. Our further assumption, that prices are known when current decisions are made, is realistic for some decisions, less so for others. A livestock producer typically



During the period  $t$  the producer decides  $x_{1t}$ ,  $n_t$ ,  $s_t$ ,  $N_{t+1}$ ,  $x_{2t+1}$ ,  $X_{3t+1}$ ,  $w_t$  subject to the restrictions given by (6.6), (6.7), (6.8). He is assumed to make these decisions according to a preference function given by

$$(6.9) \quad P_t = \psi(w_t, \tilde{w}_{t+1}, \tilde{N}_{t+2}, \tilde{X}_{3t+2})$$

where  $\tilde{w}_{t+1}$  is the planned money withdrawal during period  $t + 1$  and  $\tilde{N}_{t+2}$ ,  $\tilde{X}_{3t+2}$  are planned holdings of product and capital equipment at the end of period  $t + 1$ . The producer's plans for future periods are relevant for his current decisions, because he always faces the choice of taking more current withdrawals at the cost of depleting the resources ( $N$ ,  $X_3$ ) available for future production. In our example, he must also choose the input  $x_2$  in advance of its use in production. Plans are exceedingly complex and subtle phenomena, and are hard to incorporate realistically in a formal theory. In general, plans for the immediate future tend to be more specific; plans for the distant future tend to be more vague.

In our model, we assume that specific plans are made one period ahead; variables that must be decided on in  $t + 1$  are conditionally decided on in period  $t$ . The plans made in period  $t$  are based on anticipations of prices for period  $t + 1$ . Anticipated prices are given by  $\tilde{p}_{t+1}$ ,  $\tilde{q}_{1t+1}$ ,  $\tilde{q}_{2t+1}$ ,  $\tilde{q}_{3t+1}$  in our notation. For plans to be consistent, the technical and accounting restrictions that will hold in period  $t + 1$  must be taken into account. These are stated below and are similar to the restrictions (6.6) to (6.8) that relate to period  $t$ .

$$(6.10) \quad \tilde{n}_{t+1} = \tilde{\varphi}(\tilde{x}_{1t+1}, x_{2t+1}, X_{3t+1}, N_{t+1})$$

$$(6.11) \quad \tilde{N}_{t+2} = N_{t+1} + \tilde{n}_{t+1} - \tilde{s}_{t+1}$$

$$(6.12) \quad \tilde{w}_{t+1} = \tilde{p}_{t+1}\tilde{s}_{t+1} - \tilde{q}_{1t+1}\tilde{x}_{1t+1} - \tilde{q}_{2t+2}x_{2t+1} - \tilde{q}_{3t}(\tilde{X}_{3t+2} - \rho X_{3t+1})$$

$\tilde{n}_{t+1}$  is planned production for  $t + 1$ .  $\tilde{x}_{1t}$  is planned input of  $x_1$ .  $\tilde{\varphi}$  is the producer's subjective idea of his production function.  $\tilde{s}_{t+1}$  is planned sales.  $x_{2t+1}$  appears in (6.5) and (6.7) as a decision rather than a plan, because of our assumption that this quantity must be decided one period ahead. To know how highly he values planned holdings of assets for production in periods beyond  $t + 1$ , the producer will have to have some notions about possible prices in subsequent periods and some ideas on the desirability of withdrawals in those periods compared with with-

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knows, within fairly close limits, the prices of commodities bought and sold at the time transactions are made. He does, however, have to make some current-input decisions while facing considerable uncertainty about price of output. To make the theory correspond to reality in this respect, we should have to recognize, in some fashion, time periods shorter than a year.

drawals in  $t$  and  $t + 1$ . We assume that explicit plans and anticipations for the later periods are not formulated, and that the relevance for decision making in period  $t$  of such vague plans and anticipations as do exist is that they help determine the form of the preference function 6.9. For example, if the entrepreneur has a vague notion that prices in periods  $t + 2$ ,  $t + 3$ , etc. will be highly favorable, he is likely to value  $\bar{N}_{t+2}$ ,  $\bar{X}_{3t+2}$  more highly relative to  $w_t$ ,  $\bar{w}_{t+1}$  than if he anticipates unfavorable prices.

A major simplification in the situation outlined is the neglect of aspects of the economic life of the entrepreneur other than his livestock enterprise.<sup>3</sup> His consumption, investments, cash balance, and other enterprises are neglected. For our present purposes these may also be regarded as influencing the form of the preference function. Another way of saying this is that, if we considered more aspects of the entrepreneur's life, the preference function would depend on additional variables, and the entrepreneur would have other decisions and plans to make. When we neglect these other decisions and plans, it is understood that his preferences among alternative combinations of present and prospective withdrawals from his livestock enterprise and livestock assets to be held depend on his opportunities for outside investment, consumption, and production in other enterprises.

#### THE MAXIMIZATION PROBLEM

To summarize the decision-making situation that is postulated: During a given time interval  $t$ , the entrepreneur knows  $x_{2t}$ ,  $X_{3t}$ ,  $N_t$ . He observes  $p_t$ ,  $q_{1t}$ ,  $q_{2t}$ ,  $q_{3t}$  and forms anticipations  $\bar{p}_{t+1}$ ,  $\bar{q}_{1t+1}$ ,  $\bar{q}_{2t+1}$ ,  $\bar{q}_{3t+1}$ . He then decides  $x_{1t}$ ,  $n_t$ ,  $s_t$ ,  $x_{2t+1}$ ,  $X_{3t+1}$ ,  $N_{t+1}$ ,  $w_t$  and forms plans  $\bar{x}_{1t+1}$ ,  $\bar{n}_{t+1}$ ,  $\bar{s}_{t+1}$ ,  $\bar{X}_{3t+2}$ ,  $\bar{N}_{t+2}$ ,  $\bar{w}_{t+1}$ , so as to maximize his preference function (6.9), subject to the constraints given by (6.6) to (6.8) and (6.10) to (6.12). To consider the implications of the situation, the maximization process is carried out symbolically. The Lagrangian function 6.13 is formed, and its partial derivatives with respect to variables to be decided and planned are equated to zero. It is assumed that the conditions so obtained define the restricted maximum.<sup>4</sup> The conditions are then interpreted.

<sup>3</sup> A somewhat more general theory of farm decision making was outlined by the senior author in, *A Model of Farm Production, Paper 15* (mimeographed), *Seminar on Economic Efficiency, Social Science Research Council Project*, University of Chicago, summer 1950.

<sup>4</sup> This involves implicit assumptions of continuous differentiability of  $\varphi$ ,  $\bar{\varphi}$ , and  $\psi$ ; absence of effective boundary conditions; and the existence of a unique maximal stationary point.

$$\begin{aligned}
 (6.13) \quad F = & \psi - \lambda_1(n_t - \varphi) - \lambda_2(N_{t+1} - N_t - n_t + s_t) \\
 & - \lambda_3(w_t - p_t s_t + q_{1t}x_{1t} + q_{2t}x_{2t} + q_{3t}X_{3t+1} - q_{3t\rho}X_{3t}) \\
 & - \lambda_4(\tilde{y}_{t+1} - \tilde{\varphi}) - \lambda_5(\tilde{N}_{t+2} - N_{t+1} - \tilde{n}_{t+1} + \tilde{s}_{t+1}) \\
 & - \lambda_6(\tilde{w}_{t+1} - \tilde{p}_{t+1}\tilde{s}_{t+1} + \tilde{q}_{1t+1}x_{1t+1} + \tilde{q}_{2t+1}x_{2t+1} \\
 & \qquad \qquad \qquad + \tilde{q}_{3t+1}\tilde{X}_{3t+2} - \tilde{q}_{3t+1\rho}X_{3t+1})
 \end{aligned}$$

Differentiation yields

$$(6.14) \quad \frac{\partial F}{\partial x_{1t}} = \lambda_1 \frac{\partial \varphi}{x_{1t}} - \lambda_3 q_{1t} = 0$$

$$(6.15) \quad \frac{\partial F}{\partial n_t} = -\lambda_1 + \lambda_2 = 0$$

$$(6.16) \quad \frac{\partial F}{\partial s_t} = -\lambda_2 + \lambda_3 p_t = 0$$

$$(6.17) \quad \frac{\partial F}{\partial x_{2t+1}} = \lambda_4 \frac{\partial \tilde{\varphi}}{x_{2t+1}} - \lambda_6 \tilde{q}_{2t+1} = 0$$

$$(6.18) \quad \frac{\partial F}{\partial X_{3t+1}} = -\lambda_3 q_{3t} + \lambda_4 \frac{\partial \tilde{\varphi}}{\partial X_{3t+1}} + \lambda_6 \tilde{q}_{3t+1\rho} = 0$$

$$(6.19) \quad \frac{\partial F}{\partial N_{t+1}} = -\lambda_2 + \lambda_4 \frac{\partial \tilde{\varphi}}{\partial N_{t+1}} + \lambda_5 = 0$$

$$(6.20) \quad \frac{\partial F}{\partial w_t} = \frac{\partial \psi}{\partial w_t} - \lambda_3 = 0$$

$$(6.21) \quad \frac{\partial F}{\partial \tilde{x}_{1t+1}} = \lambda_4 \frac{\partial \tilde{\varphi}}{\partial x_{1t+1}} - \lambda_6 \tilde{q}_{1t+1} = 0$$

$$(6.22) \quad \frac{\partial F}{\partial \tilde{n}_{t+1}} = -\lambda_4 + \lambda_5 = 0$$

$$(6.23) \quad \frac{\partial F}{\partial \tilde{s}_{t+1}} = -\lambda_5 + \lambda_6 \tilde{p}_{t+1} = 0$$

$$(6.24) \quad \frac{\partial F}{\partial \tilde{X}_{3t+2}} = \frac{\partial \psi}{\partial X_{3t+2}} - \lambda_6 \tilde{q}_{3t+1} = 0$$

$$(6.25) \quad \frac{\partial F}{\partial \tilde{N}_{t+2}} = \frac{\partial \psi}{\partial \tilde{N}_{t+2}} - \lambda_5 = 0$$

$$(6.26) \quad \frac{\partial F}{\partial \tilde{w}_{t+1}} = \frac{\partial \psi}{\partial \tilde{w}_{t+1}} - \lambda_6 = 0$$

## INTERPRETATION OF CONDITIONS

From (6.20), we have  $\lambda_3 = \partial\psi/\partial w_t$ , and, from (6.15) and (6.16),  $\lambda_1 = \lambda_2 = p_t(\partial\psi/\partial w_t)$ . From (6.26),  $\lambda_6 = (\partial\psi/\partial\tilde{w}_{t+1})$ , and, from (6.22) and (6.23),  $\lambda_4 = \lambda_5 = p_{t+1}(\partial\psi/\partial\tilde{w}_{t+1})$ . When the Lagrangian coefficients are eliminated, we have

$$(6.27) \quad \frac{\partial\varphi}{\partial x_{1t}} p_t = q_{1t} \quad (\text{from 6.14})$$

That is, for an input that can be decided on when both price of product and price of input are known, the condition of static theory that marginal-value product equals price of the input still holds.

$$(6.28) \quad \frac{\partial\tilde{\varphi}}{\partial x_{2t+1}} \tilde{p}_{t+1} = \tilde{q}_{2t+1} \quad (\text{from 6.17})$$

For an input that must be decided on before prices are known, expected marginal-value product equals anticipated price of the input.

$$(6.29) \quad \frac{\partial\tilde{\varphi}}{\partial X_{3t+1}} \tilde{p}_{t+1} + \rho\tilde{q}_{3t+1} = \frac{\partial\tilde{w}_{t+1}}{\partial w_t} q_{3t} \quad (\text{from 6.18})$$

$\frac{\partial\tilde{w}_{t+1}}{\partial w_t} = \frac{\partial\psi/\partial w_t}{\partial\psi/\partial\tilde{w}_{t+1}}$  is the marginal value to the producer of a dollar

of current withdrawals, in terms of dollars of prospective withdrawals in  $t + 1$ . The left side of (6.29) is the expected increase in value of product in period  $t + 1$ ,  $\left(\frac{\partial\tilde{p}}{\partial X_{3t+1}} \tilde{p}_{t+1}\right)$ , due to adding a marginal unit of  $X_{3t+1}$ , plus the expected worth of the unit at the end of the period ( $\rho\tilde{q}_{3t+1}$ ). This is the marginal contribution of a unit of  $X_{3t+1}$  to  $\tilde{w}_{t+1}$ . The right side expresses the subjective cost of a marginal unit of  $X_{3t+1}$ , in terms of expected withdrawals ( $\tilde{w}_{t+1}$ ). At the margin, these are equal.

$$(6.30) \quad \frac{\partial\tilde{p}}{\partial N_{t+1}} \tilde{p}_{t+1} + \tilde{p}_{t+1} = \frac{\partial\tilde{w}_{t+1}}{\partial w_t} p_t \quad (\text{from 6.19})$$

If the producer increases his ending inventory ( $N_{t+1}$ ) by one unit (holding production constant), he decreases his current sales by one unit and loses  $p_t$  in current withdrawals. To him this loss is worth as much as  $(\partial\tilde{w}_{t+1}/\partial w_t)p_t$  of planned withdrawals in the following period. An increase of one unit in  $N_{t+1}$  increases planned withdrawals by  $\tilde{p}_{t+1}$  directly and by  $(\partial\tilde{p}/\partial Y_{t+1})\tilde{p}_{t+1}$  through its marginal contribution to production in period  $t + 1$ . Thus the left side of (6.25) represents the increment of  $\tilde{w}_{t+1}$  due to an increase of one in  $N_{t+1}$ , and the right side represents the value (in terms of  $\tilde{w}_{t+1}$ ) of the reduction in  $w_t$  occasioned by adding a unit to  $N_{t+1}$ . For  $P_t$  to be a maximum, these must be equal.

$$(6.31) \quad \frac{\partial \bar{\varphi}}{\partial \bar{x}_{1t+1}} \bar{p}_{t+1} = \bar{q}_{1t+1} \quad (\text{from 6.21})$$

Expected marginal-value product of a planned input equals anticipated price.

$$(6.32) \quad \frac{\partial \bar{w}_{t+1}}{\partial \bar{X}_{3t+2}} = \bar{q}_{3t+1} \quad (\text{from 6.24})$$

The marginal rate of substitution between planned withdrawals and planned holdings of capital equipment is equal to the anticipated price of capital equipment.

$$(6.33) \quad \frac{\partial \bar{w}_{t+1}}{\partial \bar{N}_{t+2}} = \bar{p}_{t+1} \quad (\text{from 6.25})$$

This is similar to (6.32).

The 7 maximization conditions 6.27 to 6.33 and the 6 constraints determine the values of the 13 variables that the entrepreneur is to choose during  $t$ . 13 equations in 13 unknowns are assumed to be consistent and independent and to determine the unknowns uniquely. For brevity, we shall sometimes refer to the variables  $x_{2t}$ ,  $x_{3t}$ ,  $N_t$ ,  $p_t$ ,  $q_{1t}$ ,  $q_{2t}$ ,  $q_{3t}$  as a group under the heading "information variables," and similarly the anticipations ( $\bar{p}_{t+1}$ ,  $\bar{q}_{1t+1}$ ,  $\bar{q}_{2t+1}$ ,  $\bar{q}_{3t+1}$ ), the decisions ( $x_{1t}$ ,  $n_t$ ,  $s_t$ ,  $x_{2t+1}$ ,  $X_{3t+1}$ ,  $N_{t+1}$ ,  $w_t$ ), and the plans ( $\bar{y}_{1t+1}$ ,  $\bar{n}_{t+1}$ ,  $\bar{s}_{t+1}$ ,  $\bar{X}_{3t+1}$ ,  $\bar{N}_{t+2}$ ,  $\bar{w}_{t+1}$ ). An interesting consequence of the assumptions of the theory is that some of the decisions ( $x_{1t}$ ,  $n_t$ ) can be made independently of the anticipations. If we look at condition 6.27 and restraint 6.6, we have two equations in two unknowns. These can be solved for unknowns  $x_{1t}$ ,  $n_t$ , and the solutions will involve only information variables. In general, the other unknowns (decisions and plans) depend on both the information variables and the anticipations.

#### CONNECTIONS BETWEEN THEORETICAL AND EMPIRICAL MODELS

In our empirical model we have treated feed grains and protein feeds as variables whose use can be determined after prices of input and product are known. They are analogous to  $x_{1t}$  in the theoretical construction. This is not entirely realistic, as some feed is consumed well in advance of the time the product is marketed. To recognize this, we should have to recognize time intervals shorter than a year. Though this might be highly desirable on grounds of realism, it would complicate the model and would be difficult to carry through, so long as we are dealing with aggregates.<sup>5</sup> It seemed to us that the importance of marginal adjust-

<sup>5</sup> For individual commodities, it may be possible to isolate particular dates that have special significance for decision making.

ments of these marketable feeds made the assumption of quantity determination with price information a reasonable approximation. A consequence of this assumption is that anticipations do not appear in the relations expressing demand for feed.

To consider these relations further, we note that the variable  $x_{2t}$  in our illustrative theory can be roughly associated with labor. We assume that, within a year, there is so little flexibility in the amount of labor required for the livestock enterprise that it is closer to reality to regard labor as decided in advance, along with size of herd, rather than to assume that it is decided currently.

If we imagine that grains fed and protein feeds fed are substituted for  $x_{1t}$  in (6.6), then quantity consumed of each will be determined by a condition analogous to (6.27). Quantity of grains consumed will appear as a function of price of grains, quantity of protein feeds, and  $x_{2t}$ ,  $X_{3t}$ ,  $N_t$ ,  $p_t$ . Quantity of protein feeds would appear as a function of price of protein feeds, quantity of grains, and  $x_{2t}$ ,  $X_{3t}$ ,  $N_t$ ,  $p_t$ . These two equations could then be solved to yield each decision variable as a function of information variables. In the results, price of protein feed would replace quantity of protein feeds in the demand for grain equation, and quantity of grains would be replaced by price of grains in the demand for protein-feeds equation. The demand relations used in the empirical study (4.2), (4.3) differed from those described above in that roughage consumption was included, whereas  $x_{2t}$  and  $X_{3t}$  were excluded because they were not observable. No variable similar to roughage was put in the theoretical illustration because it is regarded as predetermined and would not have influenced the development of the conditions. If time had proved a good substitute for  $x_{2t}$ ,  $X_{3t}$  in the empirical production relation, there might have been grounds for also substituting time in the demand relations.

#### IMPLICATIONS OF ASSUMPTIONS ABOUT ANTICIPATIONS

In principle, all the anticipations should enter the empirical supply of livestock products relation 4.4. Since anticipated prices are not observable, some inquiries were made concerning variables that might be closely associated with anticipated prices. For an aggregate model, there were none in which much confidence was expressed. It is possible that such quests will be more helpful when applied to models representing markets for individual types of livestock. For the empirical supply relation that was used, anticipated prices were assumed to be functions of current prices. This made the empirical supply relation resemble the demand relations more closely than it would have done if other indicators of

anticipations had been available. Except for the price of labor,<sup>6</sup> prices enter (4.4) in a dual role: as indicators of current costs, and of anticipated costs. Current production ( $y_{1t}$ ) was entered in the supply equation for reasons that are not apparent in the illustrative theory. It was believed that sales could be adjusted after the effects of unobserved influences on production became at least partially known; i.e., producers in general would have some knowledge of the value of the disturbance  $U_{4t}$ . This recognizes some lapse of time between feeding and selling decisions. For the product of a given animal, such a lapse actually exists. Viewing the supply relation in this way suggests that feed prices, as well as price of labor, enter mainly as indicators of anticipations.

It may be of some interest to check the estimated supply equations to see if they are consistent with this interpretation. If input prices primarily reflect anticipations, and if anticipated price is an increasing function of current price, then a rise in price of an input, other variables being constant, should be associated with an increase in current marketings (and a decrease in inventory held for future production). The supply equations with limited-information 6.34 and least-squares 6.35 coefficients are given below.

$$(6.34) \quad Y_{4t} = \underset{(0.143)}{0.908}Y_{1t} + \underset{(0.032)}{0.149}Y_{2t} - \underset{(0.111)}{0.061}Y_{3t} - \underset{(0.121)}{0.191}Y_{5t} \\ + \underset{(0.136)}{0.0013}Z_{1t} + \underset{(0.045)}{0.099}Z_{3t} + \underset{(2.539)}{0.661}$$

$$(6.35) \quad Y_{4t} = \underset{(0.066)}{0.801}Y_{1t} + \underset{(0.020)}{0.135}Y_{2t} - \underset{(0.050)}{0.133}Y_{3t} - \underset{(0.052)}{0.136}Y_{5t} \\ + \underset{(0.074)}{0.083}Z_{1t} + \underset{(0.035)}{0.120}Z_{3t} + \underset{(0.568)}{0.993}$$

In both equations, the expected sign holds for coefficients of  $Y_{2t}$  (price of grains) and  $Z_{3t}$  (price of labor), but not for  $Y_{3t}$  (price of protein feeds). In view of our lack of information on anticipations, it is hard to say how much confidence should be placed in any specific interpretation of this result.<sup>7</sup> However, one fairly reasonable interpretation (first suggested to the authors by R. L. Anderson) is perhaps worth mentioning. In several regions, protein feeds are largely used to supplement grains in intensive feeding programs, designed to bring animals quickly to suitable market weights. A rise in the price of protein feeds could easily cause some producers to shift to longer feeding programs, involving more use

<sup>6</sup> The reason is that we assume that quantity of labor has been determined in a previous decision period.

<sup>7</sup> It may also be noted that the indication of sign of the coefficient of  $Y_3$  in (6.34) is not statistically significant.

of roughage. Such shifts would, for a time, decrease the rate at which animals became ready for market, even though producers did not wish to retain more animals for breeding purposes.

One question about livestock supply that has been the subject of much discussion<sup>8</sup> is whether an increase in current price tends to increase or decrease current marketings. It has often been asserted that, in the absence of offsetting factors, an increase in price (particularly of cattle) leads to favorable anticipations, to an attempt to build up inventories and lower current sales, and to a strengthening of the tendency for current price to rise. This is cited as a destabilizing tendency in livestock production. The coefficients of  $Y_5$  in both equations are consistent with this argument.

<sup>8</sup> Lorie, *op. cit.*, Ch. III.

Mordecai Ezekiel, The Cobweb Theorem, *Quarterly Journal of Economics*, Feb. 1938.

G. S. Shepherd, *Agricultural Price Analysis*, Ch. III, Iowa State College Press, Ames, 2d. ed. rev., 1947.



## CHAPTER VII

### THE DEMAND RELATION

#### DERIVED DEMAND FOR PERISHABLE COMMODITIES

Before additional calculations pertaining to the livestock demand relation are reported, it may be useful to examine some theoretical questions raised by the way in which the demand relation has been specified in this study. One such question concerns the nature of a demand relation in which the reactions of consumers and processors are combined. As in Chapter VI, it is simpler to introduce a new notation for the discussion of fairly general theoretical points than to employ the notation previously used in developing our economic and statistical models.

Consider a situation in which a commodity is generally sold by producers to intermediaries whom we call processors, and then resold by the processors to final consumers.<sup>1</sup> For given values of other relevant variables,<sup>2</sup> we may think of quantity sold by producers, quantity sold by processors, and prices at which the two kinds of exchanges take place as being determined by four relations. These would be the producers' supply relation, the processors' demand relation, the processors' supply relation, and the consumers' demand relation. If the commodity is perishable, or if storage is sufficiently expensive for any reason, it may be expected that the same quantity will be sold by processors as is sold by producers.<sup>3</sup> Then there is one less variable to be determined, and the behavior of processors can be represented by a single relation. The

<sup>1</sup> There are, of course, usually more than two stages in the complete production and marketing process. In general, the number of stages explicitly recognized by the investigator will depend on the detail with which he chooses to study the process.

<sup>2</sup> The other variables need not be predetermined, but it will do no harm to regard them temporarily as predetermined in the discussion of the theoretical interpretation of our derived demand relation. The existence of additional current endogenous variables in a particular application will mean that the equations considered in this discussion do not constitute a complete model in that application.

<sup>3</sup> Here it is implicitly assumed that net imports are negligible. Although this is approximately true for livestock products as a whole, foreign trade would have to be allowed for in a more refined model of an open economy.

processors' behavior relation thus shows the quantities that processors are willing to handle at various combinations of producer price and consumer price.

We are interested in the latter case since, for a period as long as a year, quantities of important livestock products taken by processors are equal or nearly equal to quantities taken by consumers, if consistent units of measurement are employed. Let the relevant relations be given by

$$(7.1) \quad \varphi(x, p, r) = 0 \quad (\text{producer supply relation})$$

$$(7.2) \quad \chi(x, p, q, w) = 0 \quad (\text{processor behavior relation})$$

$$(7.3) \quad \theta(x, q, y, z) = 0 \quad (\text{consumer demand relation})$$

where  $x$  = quantity exchanged

$p$  = price received by producers

$r$  = other factors influencing producer behavior

$q$  = price received by processors

$w$  = other factors affecting behavior of processors

$y$  = consumer income

$z$  = other factors affecting consumer behavior

Suppose that  $\chi$  and  $\theta$  are of such a form that it is possible to eliminate  $q$ , thus obtaining a relation among  $x, p, y, w, z$ . This may be called the derived demand relation.<sup>4</sup> It shows how much producers can sell at various prices when the behavior of both consumers and processors is taken into account. Let it be indicated by

$$(7.4) \quad \omega(x, p, y, w, z) = 0 \quad (\text{derived demand relation})$$

#### RELATIONS BETWEEN CORRESPONDING MARGINAL RESPONSES AND ELASTICITIES

Relations between various slopes and elasticities in  $\omega$  and in  $\theta$  are of some interest. Let  $(\partial x / \partial p)_\omega$ ,  $(\partial x / \partial q)_\theta$  be the limiting values of marginal

<sup>4</sup> To eliminate  $x$  from the two equations would yield a valid restriction among the remaining variables, but, since  $x$  appears elsewhere in the model, this would reduce the number of equations without reducing the number of endogenous variables, thus leaving the system indeterminate.

Equations obtained by simultaneously eliminating one or more equations and one or more endogenous variables from a model have been called partially reduced form equations in various discussions. In a certain fundamental sense, all equations we are likely to deal with may be regarded as partially reduced form relations. It is always possible to imagine a more fundamental explanation of the phenomena that we observe, involving more equations and more endogenous variables. If the model we use is a reasonable one, it should, in principle, be possible to derive it, either exactly or approximately, from the more fundamental model by successive elimination of variables.

responses of quantity to changes in price in (7.4) and (7.3), respectively, and let  $(\partial x/\partial y)_\omega$ ,  $(\partial x/\partial y)_\theta$  be the responses to changes in income. Let  $E_{\omega p}$ ,  $E_{\theta q}$ ,  $E_{\omega y}$ ,  $E_{\theta y}$  represent the price and income elasticities. Comparisons between corresponding slopes and elasticities depend on the properties of  $\chi$ . Relationships showing this dependence are developed below.

The total differentials of  $\chi$  and  $\theta$  may be written

$$(7.5) \quad d\chi = \frac{\partial \chi}{\partial x} dx + \frac{\partial \chi}{\partial q} dq + \frac{\partial \chi}{\partial p} dp + \frac{\partial \chi}{\partial w} dw = 0$$

$$(7.6) \quad d\theta = \frac{\partial \theta}{\partial x} dx + \frac{\partial \theta}{\partial q} dq + \frac{\partial \theta}{\partial y} dy + \frac{\partial \theta}{\partial z} dz = 0$$

By Cramer's rule we obtain the total differential of  $x$  as a function of the total differentials of  $p$ ,  $w$ ,  $y$ ,  $z$ .

$$(7.7) \quad dx = - \begin{vmatrix} \frac{\partial \chi}{\partial p} dp + \frac{\partial \chi}{\partial w} dw & \frac{\partial \chi}{\partial q} \\ \frac{\partial \theta}{\partial y} dy + \frac{\partial \theta}{\partial z} dz & \frac{\partial \theta}{\partial q} \\ \frac{\partial \chi}{\partial x} & \frac{\partial \chi}{\partial q} \\ \frac{\partial \theta}{\partial x} & \frac{\partial \theta}{\partial q} \end{vmatrix}$$

By principles of differentiation of implicit functions,<sup>5</sup>  $(\partial x/\partial p)_\omega$  is equal to the coefficient of  $dp$  in (7.7); i.e.

$$(7.8) \quad \left(\frac{\partial x}{\partial p}\right)_\omega = \frac{-\frac{\partial \chi}{\partial p} \frac{\partial \theta}{\partial q}}{\frac{\partial \chi}{\partial x} \frac{\partial \theta}{\partial q} - \frac{\partial \chi}{\partial q} \frac{\partial \theta}{\partial x}} = \frac{\left(\frac{\partial x}{\partial q}\right)_\theta \left(\frac{\partial q}{\partial p}\right)_x}{1 - \left(\frac{\partial q}{\partial x}\right)_x \left(\frac{\partial x}{\partial q}\right)_\theta}$$

The latter equality holds if  $\frac{\partial \chi}{\partial q} \frac{\partial \theta}{\partial x} \neq 0$ . This may generally be expected and is assumed in what follows. By finding the coefficient of  $dy$  in (7.7), we obtain

$$(7.9) \quad \left(\frac{\partial x}{\partial y}\right)_\omega = \frac{\frac{\partial \chi}{\partial q} \frac{\partial \theta}{\partial y}}{\frac{\partial \chi}{\partial x} \frac{\partial \theta}{\partial q} - \frac{\partial \chi}{\partial q} \frac{\partial \theta}{\partial x}} = \frac{\left(\frac{\partial x}{\partial y}\right)_\theta}{1 - \left(\frac{\partial q}{\partial x}\right)_x \left(\frac{\partial x}{\partial q}\right)_\theta}$$

<sup>5</sup> See, e.g., de La Vallée Poussin, Ch. J., *Cours d'analyse infinitésimale*, Vol. I, pp. 145-148, Dover Publications, New York, 1946.

Goursat, Edouard, *A Course in Mathematical Analysis*, translated by E. R. Hedrick, Ginn & Co., Vol. I, pp. 49-51, 1904.

Price elasticities for the derived and consumer demand relations are defined by

$$(7.10) \quad E_{\omega p} = \frac{p}{x} \left( \frac{\partial x}{\partial p} \right)_{\omega}, \quad E_{\theta q} = \frac{q}{x} \left( \frac{\partial x}{\partial q} \right)_{\theta}$$

From (7.10) and (7.8),

$$(7.11) \quad E_{\omega p} = \frac{p}{x} \frac{\left( \frac{\partial x}{\partial q} \right)_{\theta} \left( \frac{\partial q}{\partial p} \right)_x}{1 - \left( \frac{\partial q}{\partial x} \right)_x \left( \frac{\partial x}{\partial q} \right)_{\theta}} = \frac{E_{\theta q} \left[ \frac{p}{q} \left( \frac{\partial q}{\partial p} \right)_x \right]}{1 - \left( \frac{\partial q}{\partial x} \right)_x \left( \frac{\partial x}{\partial q} \right)_{\theta}}$$

The factor in brackets in the numerator of the expression on the right is (the limit of) the ratio of the relative change in retail price to the relative change in producers' price when other factors affecting processor behavior are held constant. This might be called the elasticity of price transmission.<sup>6</sup> Income elasticities for the two equations are defined by

$$(7.12) \quad E_{\omega y} = \frac{y}{x} \left( \frac{\partial x}{\partial y} \right)_{\omega}, \quad E_{\theta y} = \frac{y}{x} \left( \frac{\partial x}{\partial y} \right)_{\theta}$$

From the above and (7.9),

$$(7.13) \quad E_{\omega y} = \frac{y}{x} \frac{\left( \frac{\partial x}{\partial y} \right)_{\theta}}{1 - \left( \frac{\partial q}{\partial x} \right)_x \left( \frac{\partial x}{\partial q} \right)_{\theta}} = \frac{E_{\theta y}}{1 - \left( \frac{\partial q}{\partial x} \right)_x \left( \frac{\partial x}{\partial q} \right)_{\theta}}$$

The expressions on the right of (7.8), (7.9), (7.11), and (7.13) have the same denominator. In the class of cases in which  $x$  does not appear in  $\chi$ ,  $(\partial q/\partial x)_x$  is equal to zero, and this denominator is equal to one. We then have

$$(7.8') \quad \left( \frac{\partial x}{\partial p} \right)_{\omega} = \left( \frac{\partial x}{\partial q} \right)_{\theta} \left( \frac{\partial q}{\partial p} \right)_x$$

$$(7.9') \quad \left( \frac{\partial x}{\partial y} \right)_{\omega} = \left( \frac{\partial x}{\partial y} \right)_{\theta}$$

$$(7.11') \quad E_{\omega p} = E_{\theta q} \left[ \frac{p}{q} \left( \frac{\partial q}{\partial p} \right)_x \right]$$

$$(7.13') \quad E_{\omega y} = E_{\theta y}$$

The class of cases for which these latter relations hold includes such

<sup>6</sup> The term was suggested by Henry Houthakker.

processor behavior as constant percentage margins, constant dollar margins, or combinations of the two.<sup>7</sup>

If the processor behavior relation does contain  $x$ , then it seems to us that we may typically expect  $(\partial q/\partial x)_x$  to be positive, and therefore the denominator that appears in our earlier expressions may be expected to be greater than one. For a constant percentage margin, the elasticity of price transmission,  $\frac{p}{q} \left( \frac{\partial q}{\partial p} \right)_x$ , is equal to one; for constant dollar margin or the combined case, it is less than one. It seems reasonable to suppose that, in general, this elasticity rarely exceeds one.<sup>8</sup> This means that, if producers' price rises while quantity processed and such other factors as prices of inputs used by processors remain fixed, the relative change in consumer price will not exceed the relative change in producers' price. This would certainly be true if effective competition existed in processing, and might be expected to be typical of other instances as well.

If the two presumptions

$$(7.14) \quad \left( \frac{\partial q}{\partial x} \right)_x \geq 0, \quad \frac{p}{q} \left( \frac{\partial q}{\partial p} \right)_x \leq 1$$

are accepted, then we have

$$(7.8'') \quad \left( \frac{\partial x}{\partial p} \right)_\omega \geq \left( \frac{\partial x}{\partial q} \right)_\theta \left( \frac{\partial q}{\partial p} \right)_x$$

$$(7.9'') \quad \left( \frac{\partial x}{\partial y} \right)_\omega \leq \left( \frac{\partial x}{\partial y} \right)_\theta$$

$$(7.11'') \quad E_{\omega p} \geq E_{\theta q}$$

$$(7.13'') \quad E_{\omega y} \leq E_{\theta y}$$

The directions of the inequalities correspond to the usual case where

<sup>7</sup> These particular assumptions have been extensively used by the Bureau of Agricultural Economics in comparing farm and retail prices of agricultural food products. See

Price Spreads between Farmers and Consumers for Food Products 1913-44, *USDA Miscellaneous Publication 576*, Bureau of Agricultural Economics, Sep. 1945. Also Been, Richard O., Price Spreads between Farmers and Consumers, *Agricultural Information Bulletin 4*, Bureau of Agricultural Economics, U. S. Department of Agriculture, Nov. 1949.

<sup>8</sup> Relations between farm and retail prices for food livestock products have been discussed and analyzed statistically by Karl A. Fox. See Fox, Factors Affecting Farm Income, Farm Prices, and Food Consumption, *Agricultural Economics Research*, Vol. III, pp. 72, 73. His results tend to confirm our supposition for the important products with which we are concerned.

$(\partial x/\partial p)_w$ ,  $(\partial x/\partial q)_\theta$  are negative and  $(\partial x/\partial y)_w$ ,  $(\partial x/\partial y)_\theta$  are positive. If the above presumptions are correct, consumer demand is at least as elastic with respect to both price and income as is the derived demand.

#### LONG- AND SHORT-RUN RESPONSES OF QUANTITY CONSUMED

The derived livestock demand relation fitted in Chapter IV did not contain any variables corresponding to  $w$  in (7.4), though variables reflecting costs of inputs used in processing would have been included if satisfactory data had been available. Variables corresponding to the  $z$  in (7.4) were general price level ( $z_{7t}$ ), population ( $z_{6t}$ ), lagged population ( $z_{6t-1}$ ), and lagged consumption ( $z_{4t}$  or  $y_{4t-1}$ ). The presence of lagged consumption raises questions about the stability of the system and about the long-run responses of consumption to other variables. Questions of stability are not investigated here because they would be rather involved, and because the incompleteness of our model and our uncertainties about various aspects of the specification would make such an analysis almost purely illustrative.<sup>9</sup> Some simple observations about the long-run behavior of consumption that are helpful in interpreting our results, however, are indicated below.

Suppose the demand relation is linear in the observed variables. In the notation introduced in this chapter, it can then be written

$$(7.15) \quad x_t = \xi_0 + \xi_1 p_t + \xi_2 y_t + \xi_3 z_t + \xi_4 x_{t-1}$$

To consider long-run reactions of consumers, we assume that  $p_t$ ,  $y_t$ ,  $z_t$ , remain constant indefinitely at levels  $\bar{p}$ ,  $\bar{y}$ ,  $\bar{z}$ . (7.15) becomes

$$(7.16) \quad x_t = \xi_0 + \xi_1 \bar{p} + \xi_2 \bar{y} + \xi_3 \bar{z} + \xi_4 x_{t-1}$$

Given any initial value  $x_0$  for consumption, we have

$$(7.17) \quad \begin{aligned} x_1 &= \xi_0 + \xi_1 \bar{p} + \xi_2 \bar{y} + \xi_3 \bar{z} + \xi_4 x_0 \\ x_2 &= (1 + \xi_4)(\xi_0 + \xi_1 \bar{p} + \xi_2 \bar{y} + \xi_3 \bar{z}) + \xi_4^2 x_0 \\ &\quad \cdot \\ &\quad \cdot \\ &\quad \cdot \\ &\quad \cdot \\ &\quad \cdot \\ x_t &= (1 + \xi_4 + \xi_4^2 + \cdots + \xi_4^{t-1})(\xi_0 + \xi_1 \bar{p} + \xi_2 \bar{y} + \xi_3 \bar{z}) + \xi_4^t x_0 \end{aligned}$$

If  $-1 < \xi_4 < 1$ , we may write

<sup>9</sup> Some of the stability properties of a simpler model have been considered by Richard J. Foote. See article in *Journal of Farm Economics*, Feb. 1953.

$$(7.18) \quad \bar{x} = \lim_{t \rightarrow \infty} x_t = \frac{1}{1 - \xi_4} (\xi_0 + \xi_1 \bar{p} + \xi_2 \bar{y} + \xi_3 \bar{z})$$

Long-run responses of consumption to unit changes in price, income, and other factors are given by the coefficients  $\frac{\xi_1}{1 - \xi_4}$ ,  $\frac{\xi_2}{1 - \xi_4}$ ,  $\frac{\xi_3}{1 - \xi_4}$ .

As before, let logarithms of observed variables be denoted by capital letters. Suppose the demand relation is linear in these logarithms and is given by

$$(7.19) \quad X_t = \eta_0 + \eta_1 P_t + \eta_2 Y_t + \eta_3 Z_t + \eta_4 X_{t-1}$$

For constant values  $\bar{P}$ ,  $\bar{Y}$ ,  $\bar{Z}$  of  $P_t$ ,  $Y_t$ ,  $Z_t$  and  $-1 < \eta_4 < 1$ , we have

$$(7.20) \quad \bar{X} = \lim_{t \rightarrow \infty} X_t = \frac{1}{1 - \eta_4} (\eta_0 + \eta_1 \bar{P} + \eta_2 \bar{Y} + \eta_3 \bar{Z})$$

In this case, the coefficients  $\eta_1$ ,  $\eta_2$ ,  $\eta_3$  may be regarded as short-run elasticities of demand, and the coefficients  $\eta_1/(1 - \eta_4)$ ,  $\eta_2/(1 - \eta_4)$ ,  $\eta_3/(1 - \eta_4)$  as corresponding long-run elasticities.

#### A LINEAR DEMAND RELATION

Estimates of parameters in livestock-demand equations have been obtained, both under the assumption that the equation is linear in the observed variables and under the assumption that it is linear in the logs. Estimates obtained under the latter assumption were given in Table X, Chapter IV. Estimates obtained for the linear equation are given in Table XVIII. In the notation used in Chapter IV, the linear demand relation is given by

$$(7.21) \quad \beta''_{64} y'_{4t} + y_{6t} + \gamma''_{54} z'_{4t} + \gamma''_{55} z'_{6t} + \gamma''_{57} z_{7t} + \gamma''_{50} = u_{6t}$$

where, as before,

$y'_{4t} = y_{4t}/z_{6t}$  = per-capita consumption of livestock products

$y_{6t}$  = price of livestock products

$z'_{4t} = y_{4t-1}/z_{6t-1}$  = lagged per-capita consumption of livestock products

$z'_{6t} = z_{6t}/z_{6t}$  = disposable income per capita

$z_{7t}$  = index of nonfarm wholesale prices

$z_{1t}$ ,  $z_{3t}$ ,  $z_{6t}$ ,  $z_{8t}$ ,  $z_{9t}$ ,  $z_{10t}$  were used as  $z_{**}$  in obtaining the limited-information estimates given in Table XVIII.  $y_{6t}$  was treated as dependent in the application of least squares. In the calculations underlying the estimates,  $y'_4$  and  $z'_4$  were measured in dollars worth of product (at average

TABLE XVIII  
ESTIMATES FOR LINEAR VERSION OF DEMAND FOR  
LIVESTOCK PRODUCTS

Method	Estimates of Coefficients				
	$\beta_{44}''$	$\gamma_{54}''$	$\gamma_{55}''$	$\gamma_{57}''$	$\gamma_{50}''$
Limited information	0.0184(0.0061)	-0.0077(0.0063)	-0.0012(0.00014)	-0.0044(0.00095)	-0.283(0.546)
Least squares	0.0174(0.0057)	-0.0074(0.0062)	-0.0012(0.00014)	-0.0044(0.00094)	-0.248(0.355)

$R^2 = 0.980$

prices) per person,  $y_5$  is a price index which takes the value 1 when products are valued at their average 1920-49 prices,  $z'_5$  is in current dollars per person, and  $z_7$  is an index whose average value 1935-39 equals 100.

#### COMPARISONS OF ALTERNATIVE ESTIMATES

Some interpretations of these results and the results previously presented in Chapter IV can be facilitated by solving each equation containing a particular set of estimated coefficients for the quantity variable or its logarithm. (7.22) below was obtained by inserting the limited-information coefficients from Table X into (4.5') and solving. (7.23) was obtained from (4.5'), using the least-squares coefficients. (7.24), (7.25) follow from (7.21) when limited-information and least-squares estimates, respectively, from Table XVIII are inserted.

$$(7.22) \quad Y'_4 = 3.417 - 0.758Y_5 + 0.748Z'_5 + 0.373Z_7 - 0.031Z'_4$$

$$(7.23) \quad Y'_4 = 3.501 - 0.765Y_5 + 0.753Z'_5 + 0.378Z_7 - 0.035Z'_4$$

$$(7.24) \quad y'_4 = 15.38 - 54.25y_5 + 0.068z'_5 + 0.237z_7 + 0.419z'_4$$

$$(7.25) \quad y'_4 = 14.20 - 57.35y_5 + 0.072z'_5 + 0.255z_7 + 0.425z'_4$$

The constant terms in (7.22) and (7.23) have been adjusted to be consistent with the units of measurement used in the linear equations.

Coefficients in the logarithmic equations seem plausible, except for the coefficient of  $Z'_4$ . A negative coefficient for lagged consumption implies a tendency for years of relatively high consumption to be followed by years of relatively low consumption, and conversely. By relatively high and low, we mean as compared with the long-run equilibrium quantities that consumers would take if prices and incomes were stable. One way of interpreting this result is to note that it implies that, if consumers start from a position of long-run equilibrium and encounter a change in income or prices, they tend to overrespond in the first year. In the next



year, they more than correct for the initial overresponse, and so forth, gradually approaching the new equilibrium consumption in an oscillatory fashion, provided the coefficient of  $Z'_4$  is greater than  $-1$ . This would introduce a kind of cobweb effect on the demand side of the market.

Although such behavior is conceivable, it is contrary to the usual presumption of initial underadjustments of consumption to changes in prices and incomes, and would be regarded as implausible by most economists. The negative sign of this coefficient could easily have occurred by chance; it results from the positive estimates of  $\gamma'_{54}$  in Table X, and these are clearly not statistically significant. So far as these results go, we have no basis for saying whether or not it is reasonable to allow for the influence of past events on present consumption through a lagged consumption term. It did not turn out to be a very useful procedure in estimating these two equations.

#### COMPARISONS WITH OTHER DEMAND STUDIES

The price and income elasticities indicated by the coefficients of  $Y_5$  and  $Z'_5$  in (7.22) and (7.23) seem reasonable, so far as our a priori ideas as to the general orders of magnitude are concerned. They are a little higher (in absolute value) than might be expected from the results of some previous studies, but a number of economists have expressed suspicions that most statistical analyses have tended to understate price and income elasticities. The studies most directly comparable to the present one were by Karl A. Fox.<sup>10</sup> Using annual data, 1922-41, and first differences of logarithms of observed variables, he obtained a price elasticity of  $-0.41$  and an income elasticity of  $0.50$  for a relation representing demand for all food livestock products at the farm level. This makes his relation conceptually very close to our derived demand relation. For demand for all food livestock products at retail, he obtained price elasticities of  $-0.52$ ,  $-0.56$ , and  $-0.61$  and income elasticities of  $0.40$ ,  $0.47$ , and  $0.51$ , using aggregate data for the same period. From family budget data he estimated income elasticity at  $0.33$ .<sup>11</sup>

<sup>10</sup> Karl A. Fox, *op. cit.* As Mr. Fox has emphasized in correspondence, part of the difference in results between these two trials undoubtedly results from different time series used to represent the quantity variable. Fox used the BAE series of civilian consumption per capita of all food livestock products, as published in *National Food Situation*. Aside from weighting differences, this series differs from  $y_{4t}$  in excluding government purchases for military use, net exports, and net increases in commercial stocks.

<sup>11</sup> The figures  $-0.61$  for price elasticity and  $0.51$  for income elasticity are calculated from the data in Fox, *op. cit.*, Table 3, p. 71. The cross-section elasticity is from Table 9, p. 82. It includes only food consumed at home and may be an underestimate for this reason.

There have been a number of recent studies of demand for all food. These deal with demand for food at retail. From our arguments at the beginning of this chapter, which tend to be confirmed by Fox's results, we expect retail demand to exhibit price and income elasticities (in absolute value) equal to or higher than our derived demand. On the other hand, we would expect both income and price elasticity of demand for livestock products to be higher than the corresponding elasticities of demand for food, since food includes a number of commodities (e.g. cereals) for which the elasticities are known to be very low. If quantity effects on processor behavior are negligible, then income elasticity of the derived demand is negligibly different from income elasticity in the retail demand. This is consistent with Fox's results cited above. Under these circumstances, income elasticity of demand for all food should be less than income elasticity of either retail or derived demand for livestock products. Although other comparisons depend on additional and even more hazardous assumptions,<sup>12</sup> some readers may be willing to make additional assumptions and may thus be interested in looking at various results on price and income elasticities of demand for all food in this connection.

Using both family budget data for 1941 and time series, Tobin<sup>13</sup> obtained estimates of  $-0.53$  and  $0.45$  for price and income elasticity, respectively. In his discussion of Tobin's study, Professor Stone<sup>14</sup> reports estimates obtained by himself and jointly with Tobin that vary between  $-0.39$  and  $-0.90$  for price elasticity and between  $0.53$  and  $0.83$  for income elasticity. Three of the six estimates of price elasticity are within  $0.06$  of  $-0.57$ , and four of the six estimates of income elasticity are within  $0.05$  of  $0.54$ . Girshick and Haavelmo<sup>15</sup> have estimated price elasticity at  $-0.25$  and income elasticity at  $0.25$ , using time series, 1922-41, and limited-information methods. From family budget data for 1948, Fox<sup>16</sup> obtained an income elasticity of  $0.42$  for demand for all food.

The signs of coefficients in the linear equations (7.24 and 7.25 above)

<sup>12</sup> If we assume that processor behavior is given by an equation of the form  $(q_t/q_{t-1}) = \alpha_0(p_t/p_{t-1})^{\alpha_1}$ , where  $q_t$  is retail price,  $p_t$  is farm price, and  $\alpha_0, \alpha_1$  are constants, then the elasticity of price transmission is equal to  $\alpha_1$ , and Fox's figure of 1.47 (Table 4, p. 72) for the percentage response of farm price to a 1% change in retail price is a least-squares estimate of  $1/\alpha_1$ . This would make  $\alpha_1$  approximately 0.68. Price elasticity of retail demand would be 1.47 times price elasticity of the derived demand.

<sup>13</sup> James Tobin, A Statistical Demand Function for Food in the U. S. A., *Journal Royal Statistical Society*, Vol. CXIII, Part II.

<sup>14</sup> Tobin, *op. cit.*, p. 142.

<sup>15</sup> M. A. Girshick and Trygve Haavelmo, *op. cit.*

<sup>16</sup> *Op. cit.*, pp. 79-82.

TABLE XIX  
POINTS AT WHICH ELASTICITIES ARE COMPUTED

Date	Observed Values					Computed Values of $y_4$	
	$y_5$	$z_5'$	$z_7$	$z_4'$	$y_4'$	(7.24)	(7.25)
1933	0.444	358	69.0	54.2	55.1	54.9	55.0
Avg. 1920-49	0.922	643	97.7	55.7	55.9	55.9	55.9
1949	1.685	1249	152.4	60.5	61.7	70.9	71.4

TABLE XX  
ELASTICITIES OF ESTIMATED LINEAR RELATIONS  
AT SELECTED POINTS

Method of Estimation	Date of Selected Point	Elasticity of Quantity with Respect to:			
		Price, $y_5$	Income, $z_5'$	General, prices, $z_7$	Lagged consumption, $z_4$
Limited information (7.24)	1933	-0.44	0.22	0.30	0.41
	Avg. 1920-49	-0.89	0.78	0.41	0.42
	1949	-1.29	1.20	0.51	0.36
Least squares (7.25)	1933	-0.46	0.47	0.32	0.42
	Avg. 1920-49	-0.95	0.83	0.45	0.41
	1949	-1.35	1.26	0.54	0.36

TABLE XXI  
APPLICATION OF DURBIN-WATSON TEST TO DEMAND EQUATIONS  
5 % Level of Significance

Equation No.	Method of Estimation	No. of Predetermined Variables, $k'$	Calculated Statistic, $d$	Limits of Relevant Boundary	
				Lower limit, $d_L$	Upper limit, $d_U$
4.5	Limited information	3	1.107	1.04	1.54
4.5	Least squares	4	1.102	0.96	1.65
7.21	Limited information	3	1.329	1.04	1.54
7.21	Least squares	4	1.313	.96	1.65

$n = 26$

are all plausible. In attempting to judge the plausibility of the magnitudes and form some notion of the appropriateness of the linear form for this relation, it seems convenient to convert the slopes of (7.24) and (7.25) to elasticities. The elasticities given by either linear equation vary, of course, with the point at which the elasticities are computed. To get calculations for both representative and extreme values of the variables and still stay within the general range of the observations, three sets of values for the variables on the right in (7.24), (7.25) were chosen. These were the average values for the whole period, the observed values for 1933, and the observed values for 1949. They are given in Table XIX, along with the observed values of  $y'_4$  and the values of  $y'_4$  computed from (7.24) and (7.25). The latter were used in computing the elasticities given in Table XX. The extreme increases in price and income elasticities as real income increases seem rather implausible and tend to cast some doubt on the appropriateness of the linear form for this relation.

#### TESTS OF HYPOTHESES FOR DEMAND RELATIONS

For both the linear and logarithmic forms, the hypothesis of overidentification is not rejected by the Rubin-Anderson test. For the logarithmic equation, the calculated value of  $T \log (1 + 1/\lambda)$  is 11.89, and for the linear version it is 11.73. The critical value of  $\chi^2$  with 6 degrees of freedom and an 0.05 level of significance is 12.6.

The Durbin-Watson test of the hypothesis of serially independent disturbances is inconclusive in all four cases: i.e., for both the linear and logarithmic forms and both limited-information and least-squares calculations. The application of this test is shown in Table XXI.

## CHAPTER VIII

### PREDICTION TESTS

#### PURPOSES OF AVAILABLE TESTS

Tests like those that have thus far been applied to various fitted equations have been described by Christ<sup>1</sup> as tests of internal consistency. The Durbin-Watson and the Rubin-Anderson tests check the extent to which the sample data are consistent with some of the assumptions on which estimates of parameters are based. When we check the plausibility of coefficients, we are checking implications of the data and a priori assumptions used in estimation against a priori information that was not employed in the estimation process.

The ultimate purpose of deriving economic relations is useful prediction, and success in prediction is the final test of any structure. Structures or equations that are internally consistent do not necessarily lead to useful predictions. On the other hand, it is conceivable that a prediction scheme, initially unrelated to any consistent theory, could meet with considerable success in prediction. In the latter case, we would naturally try to incorporate a rationalization for the success of the scheme in our theories of the phenomena predicted. This would be done partly to improve our theories, possibly enabling us to extend successful prediction to related areas, and partly to see whether the revised theory indicated circumstances in which the scheme might be expected to fail. If the latter were true, we would be cautious in applying the scheme in these circumstances, until a reasonable number of observations were available. These would either tend to confirm our revised theory or lead to new revisions.

Though a successful prediction formula would be valuable, regardless of its source, the fact that we cannot investigate every possible formula means that some discretion concerning sources must be exercised. In the simultaneous-equations approach, an effort is made to use existing theory, knowledge of institutional arrangements, and historical data as sources, and to arrive at promising methods of prediction through formal

<sup>1</sup> Carl Christ, *A Test of an Econometric Model for the United States, 1921-47*, Conference on Business Cycles, National Bureau of Economic Research, New York, 1951.

statistical procedures. Unfortunately limitations imposed by both limited sources of data and available statistical procedures typically force an investigator to include some dubious information in his formal procedure and to exclude some potentially useful information. Tests of internal consistency are efforts to get clues about the validity of the doubtful assumptions, on which formal procedures have been partly based. They thus offer some opportunity for selecting, and possibly improving, structures before or in connection with more direct tests of predictive usefulness.

The more direct tests of predictive usefulness presented here involve the use of data for the year 1950. These data became available while the computations already reported were in process. The tests applied follow much the same procedures as those used by Christ and Marshall.<sup>2</sup> They involve calculating residuals from fitted equations, using observations for 1950 and parameters estimated from observations for the period 1920-49. The residuals are used in a formal test of significance and are also compared with residuals from certain "naïve" relations.<sup>3</sup>

#### AN ACCEPTANCE REGION FOR A CALCULATED RESIDUAL

The test of significance is straightforward when applied to an equation estimated by single-equation least-squares methods. Certain complications, to be discussed below, arise if one makes the assumptions appropriate to simultaneous-equations methods. The test is directed toward substantially the same purpose as the Christ-Marshall tolerance interval test,<sup>4</sup> namely, to check whether the estimated relation fits data outside the sample period as well as should be expected from its fit during the sample period.<sup>5</sup> Let the equation in question be written

<sup>2</sup> Christ, *op. cit.*

Andrew W. Marshall, A Test of Klein's Model III for Changes of Structure, unpublished, M. A. Thesis, University of Chicago, 1949.

<sup>3</sup> Marshall, *op. cit.*, p. 21-25.

Christ, *op. cit.*, pp. 56-59.

<sup>4</sup> There are some conceptual difficulties with the way in which tolerance intervals were obtained by Christ and Marshall. The tables they used were constructed to obtain tolerance intervals from successive independent observations from a stable normal population. Successive calculated residuals are not independent.

<sup>5</sup> Of course, the fit during the outside period should not be expected to be so good as the fit during the sample period. In the single-equation case, and in the notation developed in the text,

$$E(\hat{u}_i^2) = \sigma^2(1 - x_i M_{xx}^{-1} x_i')$$

if  $t$  is part of the sample period, and

$$E(\hat{u}_i^2) = \sigma^2(1 + x_i M_{xx}^{-1} x_i')$$

$$(8.1) \quad y_t - \alpha x'_t = u_t$$

where  $y_t$  is the value of the dependent variable at time  $t$ ,  $x'_t$  is a column vector of the values of independent variables at time  $t$ ,  $\alpha$  is a row vector of constant coefficients.  $u_t$  is the value of an unobserved random disturbance at time  $t$ ; it is assumed to be normally distributed, nonautocorrelated, and independent of  $x_t$ . Ordinarily the equation contains a constant term, which we allow for by letting one of the components of  $x_t$  take the value unity at each time point. The constant term is then the coefficient of this particular independent variable.

The least squares (also maximum-likelihood under appropriate assumptions) estimates of the coefficients based on observations of  $y_t, x_t$  for  $t = 1 \dots T$  are given by

$$(8.2) \quad \tilde{\alpha} = M_{yx} M_{xx}^{-1}$$

where  $M_{yx}$  is a vector of sums of cross products of the dependent variable and each of the independent variables in turn.  $M_{xx}$  is a matrix whose elements are sums of squares and cross products of the independent variables. The residual for time  $t$  is given by

$$(8.3) \quad \tilde{u}_t = y_t - \tilde{\alpha} x'_t$$

Let the subscript  $r$  denote an interval of time outside the sample period. The variance of the residual for the  $r$ th period is indicated by<sup>6</sup>

$$(8.4) \quad E(\tilde{u}_r^2) = \sigma^2(1 + x_r M_{xx}^{-1} x'_r)$$

where  $\sigma^2$  is the variance of  $u_t$  and is unknown. However, let

$$(8.5) \quad s^2 = \sum_{t=1}^T \tilde{u}_t^2$$

Then, if  $k$  is the number of elements in  $x_t$ ,

$$(8.6) \quad t = \frac{\tilde{u}_r(T - k)}{\sqrt{s^2(1 + x_r M_{xx}^{-1} x'_r)}}$$

has the  $t$  distribution, with  $T - k$  degrees of freedom, and contains no unknown parameters. This relation can be used to construct confidence intervals for predictions or tests of significance for calculated residuals. The latter are more suitable for our present purposes.

The null hypothesis under which the quantity given in (8.6) has the  $t$  distribution consists of two parts:

if  $t$  is outside the sample period.  $\tilde{u}_t$  is the calculated residual for time  $t$ , and  $\sigma^2$  is the variance of the true disturbance.

<sup>6</sup> Mood *op cit.*, p. 304.

- (a) The assumption of the appropriateness of the statistical specification for the sample period.
- (b) The assumption of no structural change between the sample period and the test period.

An acceptance region of desired size for  $t$  can be found from standard tables and converted to an acceptance region for  $\bar{u}_t$  by (8.6). The occurrence of a value of  $\bar{u}_t$  outside the acceptance region (in the critical region) can be explained in one or a combination of three ways:

- (a) As an indication that an inappropriate specification was used during the sample period.
- (b) As an indication of structural change.
- (c) As a statistical accident.

An investigator can never know for sure which explanation is true in a particular case. He can exercise some control over the probability that  $c$  is the sole factor, by his choice of the size of the acceptance region. He can usually exercise some judgment between  $a$  and  $b$ . Important structural changes are likely to have manifold effects and to leave various kinds of evidence behind. Indications of inappropriate specification may be supported by the results of tests of internal consistency or by the results of experiments with alternative specifications. To the extent that the investigator bases his initial specification on peculiarities of the data for the sample period, he increases the chance that the test will reject the null hypothesis for the reason of inappropriate specification.

#### TESTS BASED ON 1950 DATA

Intervals of 95% acceptance for the first five relations of the original statistical model of Chapter IV and for the linear versions of demand for feed grains and demand for livestock products are given in the fourth column of Table XXII. The linear version of demand for feed grains 6.1 was included because it was rejected by none of the tests of internal consistency of Chapter VI and should thus be regarded as a possibly useful equation. Demand for livestock products in linear form 7.21 was included because the logarithmic version 4.5 fits the 1950 data so badly. If the linear version had fit the 1950 and subsequent data well, there would have been an incentive to re-examine the grounds on which some of the implications of this equation were labeled implausible in the preceding chapter. However, it can be seen from the last row of the table that this is not the case.

No attempt was made to compute an acceptance interval under the simultaneous-equations assumptions as no satisfactory procedure has



been developed that could be applied to our case. Rubin<sup>7</sup> has developed an approximation formula for the variance of a residual, but it involves all of the predetermined variables of a system and thus does not apply to an incomplete model. We conjecture that, if a reasonable procedure were found for our case, it would lead to larger intervals than our least-squares intervals.<sup>8</sup> If this is true and if we regard the occurrence of a limited-information residual outside the least-squares interval as grounds for questioning the limited-information equation, we shall be applying a more stringent test to the limited-information equations than to the

<sup>7</sup> Herman Rubin, *The Approximate Distribution of Calculated Disturbances, Cowles Commission Discussion Paper, Statistics 318*, Oct. 1947 (hectographed). Rubin's formula is given in Christ, *op. cit.*, p. 55.

<sup>8</sup> Let  $\tilde{u}_r$  be the calculated residual for time  $r$  (outside the sample period) under least-squares procedures and  $\hat{u}_r$  the calculated residual from a limited-information equation.

$$(i) \quad \hat{u}_r = y_r + \hat{\alpha}x'_r = (\hat{\alpha} - \alpha^*)x'_r + u_r^*$$

where  $y_r$  is the variable whose coefficient has been normalized in limited information. It is assumed that this variable was treated as dependent in least squares.  $\hat{\alpha} = (\hat{\beta} \hat{\gamma})$  is the vector of estimated limited-information coefficients.  $\alpha^*$  is the vector of true coefficients,  $u_r$  is the true disturbance.  $x_r = (w_r z_r)$  is the vector of other variables in the equation;  $w_r$  is a vector of other current endogenous variables. Let  $w'_r = \pi s'_r + v'_r$  be the reduced form equations for  $w_r$ , with  $s_r$  being all the predetermined variables and  $v_r$  the reduced-form disturbances. We compare  $E(\tilde{u}_r^2)$  and  $E(\hat{u}_r^2)$ . The latter depends on two sets of random variables, the  $\hat{\alpha}$  and the  $v_r$ , which are independent. We can write

$$(ii) \quad E(\hat{u}_r^2) = E_v[E(\hat{u}_r^2 | v_r)]$$

where the expectation sign outside the square brackets refers to expected value over the distribution of  $v_r$ .

$$(iii) \quad E(\hat{u}_r^2 | v_r) = x_r \psi x'_r + 2\hat{a}x'_r v_r + u_r^{*2}$$

where  $\psi = E(\hat{\alpha} - \alpha^*)(\hat{\alpha} - \alpha^*)'$ , and  $\hat{a} = E(\hat{\alpha}) - \alpha$  is a vector of biases in the limited information estimates.

$$(iv) \quad E(\hat{u}_r^2) = E_v(x_r \psi x'_r) + 2\hat{b}E_v(v'_r u_r^*) + \sigma^{*2}$$

where  $\sigma^{*2}$  is the variance of  $u^*$  and  $\hat{b} = E(\hat{\beta}) - \beta$ . We compare this with

$$(v) \quad E(\tilde{u}_r^2) = x_r \theta x'_r + \sigma^2 \quad \text{where} \quad \theta = E(\hat{\alpha} - \alpha)'(\hat{\alpha} - \alpha)$$

Estimates of diagonal elements of  $\psi$  and  $\theta$  were obtained when coefficients were estimated, and the estimates of diagonal elements of  $\psi$  were uniformly larger than estimates of corresponding elements of  $\theta$ . Estimates of  $\sigma^{*2}$  obtained in the calculations were uniformly larger than estimates of  $\sigma^2$ . The sign of the middle term of (iv) is not known, but its absolute value becomes arbitrarily small as the sample size increases.

TABLE XXII  
TESTS BASED ON OBSERVATIONS FOR 1950

Equation No.	Variable of Normalization	1950 Value of Variable of Normalization	95% Acceptance Interval for Least-Squares Residual	Least-Squares Residual	Limited-Information Residual	Residual I	Residual II
4.1	$Y_1$	9.978	$\pm 0.023$	-0.007	-0.005	0.007	-0.018
4.2	$Y_2$	1.500	$\pm 0.108$	0.001	-0.010	0.050	0.225
6.1	$y_2$	31.63	$\pm 4.53$	-2.04	-1.82	3.41	17.42
4.3	$Y_3$	1.766	$\pm 0.056$	0.017	0.007	-0.011	0.057
4.4	$Y_4$	9.970	$\pm 0.035$	0.028	0.021	0.006	-0.021
4.5	$Y_5$	0.239	$\pm 0.057$	-0.078	-0.078	0.013	0.087
7.21	$y_5$	1.736	$\pm 0.147$	-0.270	-0.271	0.051	0.365

IDENTIFICATION OF EQUATIONS AND VARIABLES OF NORMALIZATION. 4.1—production relation,  $Y_1$ : log of index of livestock production. 4.2—demand for feed grains,  $Y_2$ : log of price of feed grains. 6.1—linear version of demand for feed grains,  $y_2$ : price of feed grains. 4.3—demand for protein feeds,  $Y_3$ : log of price of protein feeds. 4.5—demand for livestock products,  $Y_5$ : log of price of livestock products. 7.21—linear version of demand for livestock products,  $y_5$ : price of livestock products.

least squares. In the absence of an appropriate limited-information test, we elected to follow this procedure. This is a crude makeshift which we hope can be improved on in later studies through investigations of the small-sample properties of limited-information procedures.

The variables of normalization in the second column of Table XXII are the variables whose coefficients were set equal to unity in the limited-information estimation of the indicated equations and were treated as dependent variables in the least-squares procedures.

Residual I in the next to last column of the table was obtained by subtracting the 1949 value of the variable of normalization from the 1950 value. It is the residual obtained by applying naïve model I,<sup>9</sup> which arbitrarily postulates that this year's value of any variable is equal to last year's value plus a random element. Residual II was obtained by adding to the 1949 value the algebraic difference obtained by subtracting the 1948 value from the 1949 value. The result was then subtracted from the observed 1950 value. This corresponds to the application of naïve model II, which arbitrarily assumes that the change in any variable from last year to this will be, except for a random disturbance, the same as the

<sup>9</sup> Christ, *op. cit.*, pp. 56-59, 69.

Marshall, *op. cit.*, pp. 21-22.

change in that variable from year before last to last year. When applied to logs of observed variables, this model projects the most recently observed proportionate change; when applied directly to observed variables, it projects the latest absolute change.

The use of naïve model tests is largely intuitive. As yet there is no formal theory of just how the tests are to be interpreted or when they may significantly reject an equation.<sup>10</sup> Nevertheless, their intuitive appeal is strong, and they may easily detect difficulties against which other tests are relatively weak. Of course, we obtain only a few fragments of evidence from a single year's comparisons in any case. With more data it may often happen in practice that the implications of the tests will be rather apparent and convincing, even in the absence of a formal theory.

The most striking features of Table XXII are the very large residuals from the fitted equations for demand for livestock products. They are uniformly large whether one looks at the linear or logarithmic version,

<sup>10</sup> For special structures, it could happen that accurate structural estimates could be obtained; yet the calculated residual from a given structural equation could be consistently larger (in absolute value) than the residuals from one or the other of the naïve models. As an example consider the following.

Let

$$(i) \quad \beta y'_t + \Gamma z'_t = u_t$$

be a structural system, and let

$$(ii) \quad y'_t = -\beta^{-1}\Gamma z'_t + \beta^{-1}u_t = \pi z'_t + v'_t$$

be the reduced form. Let

$$(iii) \quad y_{kt} + bw'_{kt} + gz'_t = u_{kt}$$

represent the  $k$ th structural equation normalized on  $y_{kt}$ .  $w_{kt}$  is a vector of other current endogenous variables;  $u_{kt}$  is the random disturbance.  $b$  and  $g$  are vectors of coefficients. The  $k$ th equation of the reduced form might be written

$$(iv) \quad y_{kt} = \pi^{(k)}z'_t + v_{kt}$$

Now suppose that, for each predetermined variable  $z_{it}$ , one of the two following statements holds: (a) The year to year variation in  $z_{it}$  is small, or (b) the coefficient  $\pi_i^{(k)}$  of  $z_{it}$  in (iv) is small. If, in addition, the variance of  $v_{kt}$  is small, then year-to-year variation in  $y_{kt}$  will be small, and residuals from applying naïve model I will be small. However, the variance of  $u_{kt}$  might be large, relative to that of  $v_{kt}$ , and the calculated residuals from the  $k$ th structural equation might thus be larger than the calculated residuals from naïve model I. Nothing in the above assumptions prevents the obtaining of accurate estimates of  $b$  and  $g$ . The illustration is admittedly highly special, but it does suggest the need for further examination of the interpretations of naïve model tests of structural relations. This possibility would not arise in naïve model tests of reduced-form relations. Such tests were not undertaken in this study because of the incompleteness of the original model.

the least-squares or limited-information residuals, or whether one compares them with the acceptance interval or with residual I. Possible interpretations of these residuals are offered below. Other aspects of the tabulated results perhaps deserve passing mention. From 1948 to 1949 the changes in all of the listed variables were quite large. From 1949 to 1950 they were either modest in size or opposite in direction to the 1948-49 changes. Thus residuals II are uniformly large. The slight tendency for limited-information residuals to be less in absolute value than the least-squares residuals is interesting, but attempts to interpret this might well be deferred until evidence for other years is available.

Residuals for equation 4.4, supply of livestock products, are rather large. It will be recalled from Chapter VI that anticipations of producers enter critically into this relation and that our search for indicators of anticipations had been unrewarding. Also, the Rubin-Anderson and Durbin-Watson tests tended to reject the limited-information version of this relation, and the latter test was inconclusive when applied to the least-squares calculations. Thus the various scraps of evidence relating to this equation tend to suggest that it may not be very useful in its present form. The authors believe strongly that better handling of anticipations is necessary here. We think that the prospects for this will be much better when particular types of livestock are studied individually. There will then be better opportunities for relating such things as surveys of intentions and outlook information for particular crops to producers' plans and behavior.

#### POSTWAR DEVELOPMENTS IN DEMAND FOR LIVESTOCK

To return to the livestock demand relation, the price calculated from the limited-information estimate of equation 4.5 is compared with the observed price for each year 1920-42 and 1947-50 in Figure 2. The comparison would be negligibly altered if the least-squares parameters had been used instead. The badness of fit of the relation in 1950 is also apparent in 1947 and 1949.

It was suggested earlier that a tested residual could fall in the critical region because of a statistical accident, inappropriate specification, or structural change. The probability that the 1950 result was due to statistical accident was 0.05 in the formal test, and the similar discrepancy for 1949 makes this explanation even less acceptable. The generally good fit in the prewar period suggests structural change as a strong possibility. In this instance, there are a number of reasons for believing that a structural change has taken place in the demand for some livestock products. Though there is no reason to believe that other factors may not also be at work, it is possible to account rather well for the postwar

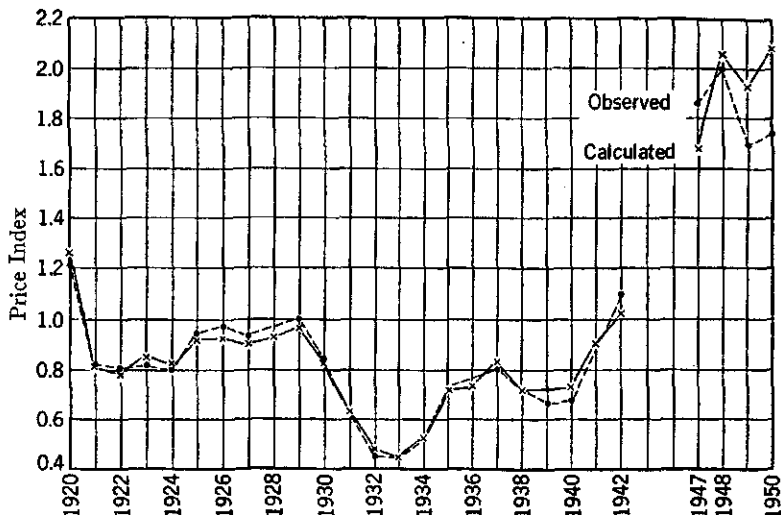


FIG. 2. Observed and calculated price of livestock products.

behavior of our calculated and observed livestock prices with the hypotheses that a significant change in the substitutability of other materials for animal fats and oils took place during World War II.

Oleomargarine was widely substituted for butter during the war. Many consumers became acquainted with the product for the first time, and, in the postwar period, restrictions on the sale of oleomargarine were gradually removed. Detergents were developed which compete with soap in many uses, and various vegetable oils came into wider use for cooking. If the above factors account for a significant part of the apparent decline in demand for livestock products, then prices of hogs and milk should be most affected since animal fats account for a larger fraction of the total value of these products than of the others. Farm prices and per-capita consumption of individual and aggregate livestock products are shown as ratios of their levels in four postwar years to 1920-49 averages in Table XXIII. In 1949 and 1950 prices of both hogs and milk are below the calculated price for all livestock products while per capita consumption of each of these products is relatively low.

Prices of poultry products are also low, but this can be fairly adequately explained by increased production.<sup>11</sup> The data on price and consumption of individual products for 1949 and 1950 are thus roughly

<sup>11</sup> The supply curve for poultry seems to have moved rather steadily to the right during much of our period. New breeds were developed, eggs per hen increased substantially; the broiler industry developed along with new techniques of specialized poultry production, pushing poultry off many general farms.

TABLE XXIII  
POSTWAR PRICE AND CONSUMPTION OF LIVESTOCK PRODUCTS  
Figures in Table Are Ratios to 1920-49 Averages

Commodity	1947		1948		1949		1950	
	Price	Per-capita consumption	Price	Per-capita consumption	Price	Per-capita consumption	Price	Per-capita consumption
Aggregate livestock								
(a) Observed	1.84	1.14	2.00	1.08	1.69	1.10	1.74	1.12
(b) Calculated	1.67	....	2.06	....	1.92	....	2.08	....
Milk	1.67	1.02	1.88	1.00	1.54	0.99	1.50	0.94
Cattle	2.09	1.24	2.51	1.08	2.24	1.07	2.60	1.05
Calves	1.92	1.35	2.30	1.19	2.14	1.07	2.49	0.96
Hogs	2.41	1.02	2.31	0.97	1.81	1.03	1.73	1.03
Sheep & lambs	1.80	0.93	2.01	0.85	2.01	0.69	2.31	0.59
Eggs	1.62	1.19	1.68	1.17	1.61	1.17	1.30	1.23
Chicken & broilers	1.40	1.16	1.61	1.09	1.32	1.25	1.26	1.33
Turkeys	1.43	1.45	1.83	1.32	1.38	1.75	1.29	1.82

TABLE XXIV  
POSTWAR DEVELOPMENTS IN FATS AND OILS  
Quantity Figures Are in Millions of Pounds; Price Indices, 1935-39 = 100

Year	Domestic Disappearance of Oleomargarine	Use of Fats and Oils in Soap	Total Vegetable Fats and Oils				Total Animal Fats and Oils		
			Domestic production	Domestic disappearance	Index of wholesale prices		Domestic production	Domestic disappearance	Index of wholesale prices
					Domestic	Foreign			
1947	719	2347	3520	4658	318	295	6431	5856	244
1948	892	2122	4163	4901	303	278	6093	5580	248
1949	854	1833	4820	4825	177	224	6695	5512	183
1950	930	1873	4844	5358	200	200	6935	5960	188

Data from *Agricultural Statistics, 1951.*

consistent with the structural change hypothesis. The fact that observed price of the aggregate was well above calculated price in 1947 is as yet unexplained.

In 1947 liquid assets held by individuals were abnormally high, consumers' durable goods were scarce and informally rationed, and rationing and price control of livestock products had just been abandoned in the fall of 1946. These are all factors that were not taken into account in our equation. Therefore they did not influence calculated price but undoubtedly did influence observed price. In addition, many of the fats and oils that compete with animal fats were in short supply in 1947 so that the increased postwar substitutability may have been largely ineffective. Regulations affecting sale of oleomargarine were gradually liberalized during the observed postwar period, and detergents became better known. Evidence of these developments is shown in Table XXIV.

There thus exists substantial evidence of a structural change in the postwar demand for various animal fats. The fact that this and other changes (such as technical developments in poultry) affect the various products differently furnishes an additional incentive to construct fairly complete models for individual products in future studies.

## APPENDIX<sup>1</sup>

The computational procedures used to obtain the numerical results of the foregoing chapters are illustrated in this section by a presentation of the detailed steps involved in all of the calculations pertaining to equation 4.1. Limited-information results were obtained by the limited-information single-equation (LISE) method which was developed by Rubin and Anderson<sup>2</sup> and has been explained in some detail by Chernoff and Divinsky<sup>3</sup> and by Klein.<sup>4</sup> Least squares procedures are well known; the calculations are included here mainly for completeness.

Insofar as possible, the notation of the Chernoff-Divinsky paper is also used here, so that the reader may easily refer to their exposition of the method. The relation to be estimated was given in Chapter 4.

$$(4.1) \quad Y_{1t} + \beta_{16}Y_{6t} + \beta_{17}Y_{7t} + \gamma_{11}Z_{1t} + \gamma_{12}z_{2t} + \gamma_{13}Z_{3t} + \gamma_{10} = U_{1t}$$

In the following, the time subscript will be omitted wherever practicable. The variables that enter the limited-information calculation are classified as:

(i) Current endogenous or jointly dependent variables entering the relation, denoted by  $y_{\Delta} = (Y_1 Y_6 Y_7)$ .

(ii) Predetermined variables entering the relation and denoted by  $z_{*} = (Z_1 z_2 Z_3)$ .

(iii) Predetermined variables absent from the relation but present in the model, denoted by  $z_{**} = (Z_4 Z_5 Z_7 Z_8 Z_9 Z_{10})$ .

It will also be convenient sometimes to let

$$(iv) \quad z = (z_{*} z_{**}) \quad \text{and}$$

$$(v) \quad x = (y_{\Delta} z) = (x_1 x_2 \cdots x_{12}); \quad \text{i.e.,}$$

$$x_1 = Y_1, \quad x_2 = Y_6, \quad x_3 = Y_7$$

$$x_4 = Z_1, \quad x_5 = z_2, \quad x_6 = Z_3, \quad \text{etc.}$$

<sup>1</sup> This appendix was prepared by Mrs. Jagna Zahl and Mr. Francis Bobkoski with the assistance of the authors.

<sup>2</sup> See T. W. Anderson and H. Rubin, Estimation of the Parameters of a Single Equation in a Complete System of Stochastic Equations of *Annals, Mathematical Statistics*, Vol. 20, pp. 46-63, 1949.

<sup>3</sup> Herman Chernoff and Nathan Divinsky, The Computation of Maximum Likelihood Estimates of Linear Structural Equations, Ch. X of *Studies in Econometric Method*, Cowles Commission Monograph 14, William C. Hood and T. C. Koopmans, editors, John Wiley & Sons, 1953.

<sup>4</sup> Lawrence R. Klein, *A Textbook of Econometrics*, Row Peterson & Co., 1953. See pp. 169-183.



The values taken by these variables in our sample period and their arithmetic means,  $m^{(0)}$ , are given in Table 1.1.<sup>5</sup> The data are given to six significant figures. The computations were carried out to nine decimal places; for brevity, the numbers appearing in later tables in this article have been rounded to four places, which will mean that a reader who checks particular steps in the computations may obtain results that differ slightly from those presented here.

The first step in the computation is to calculate the matrix of moments of these variables,  $\bar{M}_{xx}^{(0)}$ , as defined in the Chernoff-Divinsky article in Section 2. A sample element of this matrix, say the moment involving  $Y_6$  and  $Z_3$ , is given by

$$(1) \quad \bar{m}_{Y_6 Z_3}^{(0)} = T \sum_t Y_{6t} Z_{3t} - \left( \sum_t Y_{6t} \right) \left( \sum_t Z_{3t} \right)$$

where  $T = 30$ , the number of observations.

These moments<sup>6</sup> are given in Table 1.2, together with the row sums,  $\sum_{j=1}^{12} \bar{m}_{x_i x_j}^{(0)}$  (for row  $i$ ). The sums include those terms under the main diagonal that need not be written down because the matrix is symmetric. This convention will be followed throughout this appendix. The calculation of this moment matrix is checked by the identity  $\sum_{j=1}^{12} \bar{m}_{x_i x_j}^{(0)} = \bar{m}_{x_i x_i}^{(0)}$ , where  $x_i = \sum_{j=1}^{12} x_j$  for each  $t$ , which is the column headed  $\sum$  in Table 1.1. In other words, the sum of the  $i$ th row of the moment matrix must be equal to the moment of  $x_i$  and  $\sum$ .

Next the moment adjustments are determined (Section 2, Chernoff-Divinsky) and entered in Table 2.1. In the same table are the means of each variable, copied from Table 1.1, and the adjusted means, which are the products of the means and the corresponding adjustment factors.

Next is the computation of a matrix of residuals,  $\bar{W}_{\Delta\Delta}$ , presented in Table 3.<sup>7</sup>  $\bar{M}_{xx}$  and  $\bar{M}_{x\Delta}$  are formed. These are blocks of the adjusted

<sup>5</sup> We deviate here from the Chernoff-Divinsky notation by keeping our symbols for the variables throughout the exposition rather than relabeling them in standard notation. Since our original notation already distinguishes between jointly dependent and predetermined variables, this should not cause confusion and will facilitate comparisons between the appendix and our text.

<sup>6</sup> The three preliminary calculations illustrated by Chernoff-Divinsky for each moment given in Table 1.2 are here omitted and only the final moments entered.

<sup>7</sup> This represents a deviation from the Chernoff-Divinsky article; in preparation for the solution of the other equations of the model, their Table 3 is the computation of  $\bar{W}_{yy} = \bar{M}_{yy} - \bar{M}_{y\Delta} \bar{M}_{\Delta\Delta}^{-1} \bar{M}_{\Delta y}$ , which includes the moments of all the variables in the model. If LISE estimates only are desired, it is sufficient to find the submatrix  $W_{\Delta\Delta}$ , which excludes the  $y$ 's that do not occur in the equation under consideration.

$\bar{M}_{zz}^{(0)}$  matrix corresponding to the variables  $z$ ,  $z$  and  $z$ ,  $y_{\Delta}$ , respectively. A sample element of  $\bar{M}_{z\Delta}$  is the adjusted moment of  $Y_6$  and  $z_2$  which is  $\bar{m}_{Y_6 z_2} = \bar{m}_{Y_6 z_2}^{(0)} k_{Y_6} k_{z_2} = \bar{m}_{Y_6 z_2}^{(0)} \cdot 1 \cdot 10^{-2}$ . Then  $\bar{M}_{\Delta z} \bar{M}_{zz}^{-1} \bar{M}_{z\Delta}$  is computed by the Doolittle method. The row sums of  $\bar{M}_{\Delta z} \bar{M}_{zz}^{-1} \bar{M}_{z\Delta}$  are obtained and copied in the column headed  $\sum_1$ ; these are used in the sum check for the computation of  $\bar{W}_{\Delta\Delta}$ . The column headed  $\sum_2$  is a sum check for

TABLE 1.1  
TIME SERIES FOR VARIABLES  
Part I

Year	$Y_1$	$Y_6$	$Y_7$	$Z_1$	$z_2$	$Z_3$
1920	9.748084	8.068389	6.856327	9.774997	1	8.379150
1921	9.765719	8.105650	6.822490	9.769460	2	8.363938
1922	9.796348	8.102376	6.834405	9.778758	3	8.382668
1923	9.810109	8.114904	8.871142	9.796037	4	8.383129
1924	9.809222	8.098474	6.802960	9.776513	5	8.372399
1925	9.802213	8.072969	6.929392	9.748548	6	8.354120
1926	9.815773	8.098248	6.957628	9.742323	7	8.356553
1927	9.829126	8.109833	6.964877	9.747289	8	8.370315
1928	9.828799	8.105673	6.956671	9.751579	9	8.375821
1929	8.833779	8.100195	6.975930	9.754352	10	8.369334
1930	9.838925	8.091019	6.978998	9.755504	11	8.335463
1931	9.850433	8.082708	6.953046	9.758315	12	8.337076
1932	9.852929	8.130834	6.933601	9.782538	13	8.352973
1933	9.861783	8.127156	6.958915	9.802675	14	8.354061
1934	9.820362	8.012879	6.943129	9.802718	15	8.342169
1935	9.801663	7.971032	6.952794	9.744170	16	8.358042
1936	9.830960	8.064606	7.009480	9.755272	17	8.336144
1937	9.823768	7.996518	7.026471	9.745581	18	8.339627
1938	9.846623	8.094853	7.026197	9.748278	19	8.376573
1939	9.877329	8.118528	7.053128	9.773336	20	8.368374
1940	9.883906	8.124569	7.082682	9.791397	21	8.403127
1941	9.897390	8.142696	7.123580	9.789017	22	8.435870
1942	9.955978	8.218703	7.188769	9.824941	23	8.458267
1943	9.991382	8.295371	7.218770	9.865975	24	8.455265
1944	9.969697	8.235548	7.216875	9.864963	25	8.452174
1945	9.969946	8.244758	7.217291	9.831898	26	8.467254
1946	9.954799	8.233562	7.185296	9.821342	27	8.441883
1947	9.952042	8.201303	7.197108	9.810597	28	8.437942
1948	9.946647	8.171542	7.213443	9.792172	29	8.424071
1949	9.971076	8.236400	7.249224	9.799996	30	8.425557
$\Sigma$	295.936790	243.771596	210.802519	293.498541	465	251.609339
$m^{(0)}$	9.864560	8.125720	7.020751	9.783285	15.5	8.386978

$Y_1$  = logarithm of production of livestock products in dollars.

$Y_6$  = logarithm of the quantity of feed grains fed in 1000 lb. TDN.

$Y_7$  = logarithm of the quantity of protein feeds fed in 1000 lb. TDN.

$Z_1$  = logarithm of beginning inventory of livestock in dollars.

$Z_3$  = logarithm of the quantity of roughages fed in 1000 lb. TDN.

## Part II

Year	Z <sub>4</sub>	Z <sub>5</sub>	Z <sub>7</sub>	Z <sub>8</sub>	Z <sub>9</sub>	Z <sub>10</sub>	Σ
1920	9.769387	1.840106	2.189771	1.763428	8.150955	6.866952	74.427546
1921	9.752454	1.740363	2.000434	1.574031	8.219585	6.878292	74.992716
1922	9.755776	1.771587	1.988113	1.568202	8.156801	6.870803	76.005837
1923	9.780621	1.834421	2.003891	1.628389	8.143769	6.903104	77.269716
1924	9.810186	1.839473	1.987219	1.638489	8.127418	6.949356	78.311714
1925	9.819966	1.863917	2.006038	1.643453	8.102729	6.988243	79.331488
1926	9.807323	1.879689	2.000000	1.648360	8.166857	7.017608	80.490342
1927	9.816910	1.881385	1.975891	1.648360	8.138763	7.023753	81.506502
1928	9.820507	1.892651	1.976808	1.648360	8.148182	6.990449	82.495500
1929	9.822525	1.916454	1.969882	1.653213	8.145911	7.006210	83.549785
1930	9.824031	1.867467	1.933993	1.623248	8.115113	6.994172	84.357934
1931	9.826662	1.799341	1.872739	1.505150	8.092900	6.978083	85.056453
1932	9.833841	1.679428	1.834421	1.380211	8.174903	6.975705	85.931384
1933	9.829947	1.655138	1.838849	1.332438	8.171566	6.979986	86.912511
1934	9.839861	1.712650	1.885926	1.380211	8.026765	6.957531	87.724221
1935	9.851221	1.763428	1.904174	1.423246	7.998172	6.960296	88.728238
1936	9.793733	1.820201	1.906874	1.454845	8.074944	7.012236	90.059295
1937	9.829708	1.851870	1.935507	1.505150	8.029853	7.035188	91.119241
1938	9.820828	1.816241	1.906335	1.491362	8.173991	7.046625	92.347906
1939	9.835368	1.846337	1.900367	1.491362	8.198353	7.054429	93.516911
1940	9.855662	1.879096	1.907411	1.498311	8.206962	7.064997	94.698150
1941	9.879714	1.963788	1.945961	1.585461	8.223555	7.121547	96.108579
1942	9.880941	2.067071	1.986772	1.698970	8.266015	7.160933	97.707360
1943	9.936738	2.121888	1.994317	1.819544	8.282105	7.215318	99.196653
1944	9.976315	2.167317	1.998259	1.897627	8.239977	7.215042	100.235794
1945	9.987021	2.179264	2.003461	1.944483	8.255596	7.211921	101.312893
1946	9.977232	2.201124	2.060320	1.982271	8.259415	7.190973	102.308217
1947	9.961137	2.229170	2.162863	2.008600	8.241154	7.209551	103.411467
1948	9.964474	2.275981	2.205577	2.029364	8.229313	7.223097	104.472801
1949	9.948048	2.272770	2.182985	2.012247	8.325092	7.256021	105.679416
Σ	295.608387	57.628701	59.463158	49.478407	245.088714	211.380421	2679.266573
m <sup>(0)</sup>	9.853613	1.920957	1.982105	1.649280	8.169624	7.046014	

Z<sub>4</sub> = logarithm of the quantity of livestock products sold in dollars, lagged one year, i.e., Y<sub>t-1</sub>.

Z<sub>5</sub> = logarithm of disposable personal income in the United States in billions of current dollars.

Z<sub>7</sub> = logarithm of wholesale price index, excluding farm products (1935-39 = 100).

Z<sub>8</sub> = logarithm of cost of farm labor in cents per hour.

Z<sub>9</sub> = logarithm of supply of feed grains in 1000 lb. TDN.

Z<sub>10</sub> = logarithm of production of protein feeds in 1000 lb. TDN.

the computation of  $\bar{M}_{\Delta z} \bar{M}_{zz}^{-1} \bar{M}_{z\Delta}$ .<sup>8</sup>  $\bar{M}_{\Delta\Delta}$  is written down, analogously to  $\bar{M}_{zz}$ , and the difference  $\bar{W}_{\Delta\Delta} = \bar{M}_{\Delta\Delta} - \bar{M}_{\Delta z} \bar{M}_{zz}^{-1} \bar{M}_{z\Delta}$  formed.

<sup>8</sup>  $\bar{M}_{zz}$  is the matrix  $\bar{M}_{zy}$  with rows written as columns. This is, in general, the meaning of an interchange of subscripts of a matrix.

This particular product of three matrices can be computed directly by a variation of the standard Doolittle method. Although the general Doolittle procedure is explained in statistics literature, we will sketch the technique for this com-

putation, as well as another that will be needed later in this discussion of LISE. To change the notation, suppose  $A$  is an  $n \times n$  nonsingular symmetric matrix and  $B$  and  $n \times m$  matrix. We desire  $B'A^{-1}B$  (as in this case) and also  $A^{-1}B$  (this will be used later).

(a) *Forward solution* (same for both cases). Let the element  $(i, j)$  of  $A$  be  $a_{ij}$  and of  $B$  be  $a_{i,n+j}$ . The two matrices are written side by side, and added across:

$$\begin{array}{ccccccc}
 & & & A & & & B & & \Sigma \\
 a_{11} & a_{12} & \cdots & a_{1n} & a_{1,n+1} & \cdots & a_{1,n+m} & a_{1.} \\
 a_{21} & a_{22} & \cdots & a_{2n} & a_{2,n+1} & \cdots & a_{2,n+m} & a_{2.} \\
 \cdot & & & & \cdot & & & \\
 \cdot & & & & \cdot & & & \\
 \cdot & & & & \cdot & & & \\
 a_{n1} & a_{n2} & \cdots & a_{nn} & a_{n,n+1} & \cdots & a_{n,n+m} & a_{n.}
 \end{array}$$

Define

$$\alpha_{ri} = a_{ri} - \sum_{k=1}^{r-1} \frac{\alpha_{kr}}{\alpha_{kk}} \alpha_{ki}$$

Let  $\beta_{kr}$  be the coefficient  $\alpha_{kr}/\alpha_{kk}$ .

Then we may write

$$\alpha_{ri} = a_{ri} - \sum_{k=1}^{r-1} \beta_{kr} \alpha_{ki}$$

All the elements  $\alpha_{ri}$  form a triangular matrix, as well as the elements  $\beta_{ri}$ . This means that they are zero under the main diagonal. They are computed alternately, in the order and layout indicated below. After an entire row has been computed, it is checked by the relation, for a row of  $\alpha$ 's,

$$\alpha_{r.} = a_{r.} - \sum_{k=1}^{r-1} \beta_{kr} \alpha_{rk} \quad \text{and for a row of } \beta\text{'s, } \beta_{r.} = \frac{\alpha_{r.}}{\alpha_{rr}}$$

(We have used the notation  $\alpha_{r.} = \sum_{i=1}^{n+m} \alpha_{ri}$ ,  $\beta_{r.} = \sum_{i=1}^{n+m} \beta_{ri}$ ,  $a_{r.} = \sum_{i=1}^{n+m} a_{ri}$ ):

$$\begin{array}{ccccccc}
 & & & & & & \Sigma \\
 \alpha_{11} & \alpha_{12} & \cdots & \alpha_{1n} & \alpha_{1,n+1} & \cdots & \alpha_{1,n+m} & \alpha_{1.} \\
 1 & \beta_{12} & \cdots & \beta_{1n} & \beta_{1,n+1} & \cdots & \beta_{1,n+m} & \beta_{1.} \\
 0 & \alpha_{22} & \cdots & \alpha_{2n} & \alpha_{2,n+1} & \cdots & \alpha_{2,n+m} & \alpha_{2.} \\
 0 & 1 & \cdots & \beta_{2n} & \beta_{2,n+1} & \cdots & \beta_{2,n+m} & \beta_{2.} \\
 \cdot & & & & & & & \\
 \cdot & & & & & & & \\
 \cdot & & & & & & & \\
 0 & 0 & \cdots & \alpha_{nn} & \alpha_{n,n+1} & \cdots & \alpha_{n,n+m} & \alpha_{n.} \\
 0 & 0 & \cdots & 1 & \beta_{n,n+1} & \cdots & \beta_{n,n+m} & \beta_{n.}
 \end{array}$$

(b) *Computation of  $B'A^{-1}B$  (backward solution)*: Let

$$\begin{array}{ccc}
 & & \begin{array}{cc} \Sigma_1 & \Sigma_2 \end{array} \\
 B'A^{-1}B = \begin{pmatrix} c_{11} & c_{12} & \cdots & c_{1m} \\ \cdot & & & \\ \cdot & & & \\ \cdot & & & \\ c_{m1} & c_{m2} & \cdots & c_{mn} \end{pmatrix} & \begin{array}{c} c_{1\cdot} \\ \\ \\ c_{m\cdot} \end{array} & \begin{array}{c} c_{1\cdot} + a_{\cdot, n+1} \\ \\ \\ c_{m\cdot} + a_{\cdot, n+m} \end{array}
 \end{array}$$

It is symmetric. Adjoining the matrix, as indicated, are two sum columns; for row  $k$  the first sum is  $c_{k\cdot} = \sum_{i=1}^m c_{ki}$ , the second  $c_{k\cdot} + a_{\cdot, n+k} = c_{k\cdot} + \sum_{i=1}^n a_{i, n+k} =$  the sum of the  $k$ th row of  $B'A^{-1}B$  and the  $k$ th column of  $B$ .

Then

$$c_{ij} = \sum_{k=1}^n \alpha_{k, n+i} \beta_{k, n+j} = \alpha_{1, n+i} \beta_{1, n+j} + \alpha_{2, n+i} \beta_{2, n+j} + \cdots + \alpha_{n, n+i} \beta_{n, n+j}$$

Each row of  $B'A^{-1}B$ , say, the  $k$ th, is checked by the relation

$$c_{k\cdot} + a_{\cdot, n+k} = \sum_{i=1}^n \alpha_{1, n+k} \beta_{i\cdot}$$

(c) Computation of  $A^{-1}B$  (alternate backward solution): Let

$$A^{-1}B = \begin{pmatrix} d_{11} & d_{12} & \cdots & d_{1m} \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ d_{n1} & d_{n2} & \cdots & d_{nm} \end{pmatrix} \begin{array}{c} \Sigma \\ d_{1\cdot} \\ \cdot \\ \cdot \\ d_{n\cdot} \end{array}$$

The last row is computed first, then the preceding row, etc., as follows:

$$d_{ni} = \beta_{n, n+i}$$

$$\text{Check: } d_{n\cdot} = \beta_{n\cdot} - 1$$

$$d_{n-1, i} = \beta_{n-1, n+i} - \beta_{n-1, n} d_{ni}$$

$$\text{Check: } d_{n-1\cdot} = \beta_{n-1\cdot} - \beta_{n-1, n} d_{n\cdot} - 1$$

$$d_{n-2, i} = \beta_{n-2, n+i} - \beta_{n-2, n} d_{ni} - \beta_{n-2, n-1} d_{n-1, i}$$

$$\text{Check: } d_{n-2\cdot} = \beta_{n-2\cdot} - \beta_{n-2, n} d_{n\cdot} - \beta_{n-2, n-1} d_{n-1\cdot} - 1$$

⋮  
⋮  
⋮

$$d_{1, i} = \beta_{1, n+i} - \beta_{1n} d_{ni} - \beta_{1, n-1} d_{n-1, i} - \cdots - \beta_{12} d_{2i}$$

$$\text{Check: } d_{1\cdot} = \beta_{1\cdot} - \beta_{1n} d_{n\cdot} - \beta_{1, n-1} d_{n-1\cdot} - \cdots - \beta_{12} d_{2\cdot} - 1$$

The inversion of  $A$  alone is a special case of either method; it is obtained by letting  $B$  be the identity matrix. Method  $b$  is the simpler. These computations become clear if we observe that the matrix consisting of the elements  $\alpha_{i, n+j}$ , and the matrix whose elements are  $\beta_{i, n+j}$  correspond to matrices  $(B'Q)'$  and  $PB$  where  $PAQ = I$ , the identity matrix, and  $PA$  is a triangular matrix whose elements on the main diagonal are ones. The other elements of the matrix  $PA$  are the  $\beta_{ij}$ . Method  $b$  of the backward solution corresponds to the matrix product  $B'QPB = B'A^{-1}B$ , and method  $c$  corresponds to  $PB + [I - PA]A^{-1}B = A^{-1}B$ .

TABLE 1.2  
MOMENTS  $\bar{M}_{zz}^{(0)}$

	$Y_1$	$Y_6$	$Y_7$	$Z_1$	$z_2$	$Z_3$	$Z_4$	$Z_5$	$Z_7$	$Z_8$	$Z_9$	$Z_{10}$	$\Sigma$
$Y_1$	4.0764	3.8946	7.4685	1.5631	472.8347	2.0532	3.8039	8.9707	1.6522	7.7248	3.4573	6.5324	524.0218
$Y_6$		5.0023	6.4885	1.8208	347.5748	2.3808	3.3439	9.2228	2.6585	9.6122	4.8047	5.8858	402.6797
$Y_7$			14.8409	2.5258	949.3906	3.7841	7.3076	17.9673	3.9648	15.2824	5.9185	12.9399	1047.8789
$Z_1$				1.0237	149.1434	.9936	1.4395	3.1355	.7041	3.0561	1.5462	2.1357	169.0875
$z_2$					67425.0000	212.6307	472.1944	1027.6748	161.8000	762.2073	317.8088	811.3831	73109.6426
$Z_3$						1.5190	1.9769	5.4554	1.7948	5.5140	2.3362	3.3131	243.7518
$Z_4$							4.1498	9.4261	2.3008	8.7767	2.8286	6.4832	524.0314
$Z_5$								28.3392	10.6826	29.5819	8.6868	16.2892	1175.6323
$Z_7$									8.1978	14.6186	3.2235	3.9762	215.7739
$Z_8$										35.2625	9.2871	14.4666	915.3902
$Z_9$											5.3842	5.4046	370.8865
$Z_{10}$												11.5024	900.3122

TABLE 2.1  
TRANSLATION PAGE

Variable	Adjustment Factor, $k$	Mean, $m^{(0)}$	Adjusted Mean, $m = km^{(0)}$
$Y_1$	1	9.8646	9.8646
$Y_6$	1	8.1257	8.1257
$Y_7$	$10^{-1}$	7.0268	0.7027
$Z_1$	1	9.7833	9.7833
$z_2$	$10^{-2}$	15.5	0.1550
$Z_3$	1	8.3870	8.3870
$Z_4$	1	1.920957	1.920957
$Z_5$	$10^{-1}$	9.853613	0.9853613
$Z_7$	1	1.982105	1.982105
$Z_8$	$10^{-1}$	1.649280	0.1649280
$Z_9$	1	8.169624	8.169624
$Z_{10}$	$10^{-1}$	7.046014	0.7046014

In Table 4.1,  $\bar{M}_{**}$ , the submatrix of  $\bar{M}_{zz}$  corresponding to the variables  $Z_1, z_2, Z_3$ , is given, and its inverse,  $\bar{M}_{**}^{-1}$  is computed. Since  $\bar{M}_{zz}$  was formed so that  $\bar{M}_{**}$  constitutes its upper left corner, half of the Doolittle forward solution in Table 4.1 is the same as in Table 3 and could conveniently be copied.

Table 4.2 consists of three matrices:

1.  $\bar{M}_{*\Delta}$  which is the matrix of adjusted moments of  $Z_1, z_2, Z_3$ , each with  $Y_1, Y_6, Y_7$ ; they are copied from  $\bar{M}_{z\Delta}$  in Table 3.
2.  $P_{\Delta*}^{*'} = \bar{M}_{**}^{-1} \bar{M}_{*\Delta}$ , product of the two matrices,<sup>9</sup> and
3.  $\bar{M}_{\Delta*} P_{\Delta*}^{*'}.$

The matrices  $\bar{M}_{**}^{-1}$  and  $P_{\Delta*}^{*'}$  are used later in the computation; consequently, the method of computing  $\bar{M}_{\Delta*} \bar{M}_{**}^{-1} \bar{M}_{z\Delta}$  is not used in computing  $\bar{M}_{\Delta*} P_{\Delta*}^{*'}$ .

From the above we compute  $R_{\Delta\Delta} = \bar{M}_{\Delta*} \bar{M}_{**}^{-1} \bar{M}_{z\Delta} - \bar{M}_{\Delta*} P_{\Delta*}^{*'}$ ; the first matrix is in Table 3, the second in Table 4.2. Our equation is overidentified, and so the procedure outlined in Section 3 of Chernoff-Divinsky is applicable. Next to  $R_{\Delta\Delta}$  is written  $\bar{W}_{\Delta\Delta}$  from Table 3. Then  $Q_{\Delta\Delta} = R_{\Delta\Delta}^{-1} \bar{W}_{\Delta\Delta}$  is computed.<sup>10</sup> This is the content of Table 4.3.

Since the equation for the largest latent root  $k_1^1$  of  $R_{\Delta\Delta}^{-1} \bar{W}_{\Delta\Delta}$  is of the third degree, the iterative method of solution described in Section 3 of Chernoff-Divinsky was used. To conserve space we omit the first few iterations from our original arbitrary vector. Also the iterations were

<sup>9</sup> Matrix multiplication and its check are described in Section 2 of Chernoff-Divinsky.

<sup>10</sup> Cf. footnote 6.

TABLE 3  
MATRIX  $W_{\Delta\Delta}$

	$\bar{M}_{\Sigma\Sigma}$									$\bar{M}_{\Sigma\Delta}$			$\Sigma$
	$Z_1$	$Z_2$	$Z_3$	$Z_4$	$Z_5$	$Z_7$	$Z_8$	$Z_9$	$Z_{10}$	$Y_1$	$Y_6$	$Y_7$	
$Z_1$	1.0237	1.4914	0.9936	1.4395	0.3136	0.7041	0.3056	1.5462	0.2136	1.5631	1.8208	0.2526	11.6678
$Z_2$		0.7425	2.1263	4.7219	1.0277	1.6180	0.7622	3.1781	0.8114	4.7283	3.4757	0.9494	31.6330
$Z_3$			1.5190	1.9769	0.5455	1.7948	0.5514	2.3362	0.3313	2.0532	2.3808	0.3784	16.9875
$Z_4$				4.1498	0.9426	2.3008	0.8777	2.8286	0.6483	3.8039	3.3439	0.7308	27.7647
$Z_5$					0.2834	1.0883	0.2958	0.8687	0.1629	0.8971	0.9223	0.1797	7.5274
$Z_7$						8.1978	1.4619	3.2285	0.3978	1.6522	2.6585	0.3965	25.4939
$Z_8$							0.3526	0.9287	0.1447	0.7725	0.9612	0.1528	7.5671
$Z_9$								5.3842	0.5405	3.4573	4.8047	0.5919	29.6886
$Z_{10}$									0.1150	0.6532	0.5886	0.1294	4.7365
	1.0237	1.4914	0.9936	1.4395	0.3136	0.7041	0.3056	1.5462	0.2136	1.5631	1.8208	0.2526	11.6678
1.		1.4569	0.9706	1.4061	0.3063	0.6878	0.2985	1.5104	0.2086	1.5269	1.7786	0.2467	11.3974
		4.5697	0.6788	2.6248	0.5709	0.5922	0.3170	0.9255	0.5002	2.4511	0.8231	0.5814	14.6346
		1.	0.1485	0.5744	0.1249	0.1296	0.0694	0.2025	0.1095	0.5364	0.1801	0.1272	3.2025
			0.4538	0.1899	0.1564	1.0234	0.2077	0.6980	0.0497	0.1721	0.4913	0.0469	3.4893
			1.	0.4184	0.3447	2.2550	0.4577	-1.5381	0.1096	0.3791	1.0826	0.1033	7.6885
				0.5386	0.1084	0.5424	0.1790	-0.1891	0.0399	0.1261	1.053	0.0220	1.4926
				1.	0.2012	1.0071	0.3323	-0.3140	0.0740	0.2342	0.1955	0.0409	2.7711
					0.0403	0.3367	0.0550	0.0729	0.0098	0.0274	0.0713	0.0091	0.6226
					1.	8.3515	1.3645	1.8087	0.2437	0.6804	1.7671	0.2253	15.4410
						1.9703	0.1025	0.0273	-0.0484	-0.4847	-0.5095	-0.0564	1.0011
						1.	0.0520	0.0138	-0.0246	-0.2460	-0.2586	-0.0286	0.5081
							0.0045	0.0388	-0.0007	0.0030	0.0300	-0.0011	0.0744
							1.	8.6667	-0.1527	0.6614	6.7094	-0.2568	16.6281
								1.2666	0.0414	0.3064	0.7835	0.0217	2.4916
								1.	0.0327	0.2419	0.6186	0.0171	1.9103
									0.0023	0.0025	0.0060	0.0018	0.0128
									1.	1.0875	2.6716	0.7976	5.5567



TABLE 3 (Continued)

$$\bar{M}_{\Delta z} \bar{M}_{zz}^{-1} \bar{M}_{z\Delta}$$

	$Y_1$	$Y_6$	$Y_7$	$\Sigma_1$	$\Sigma_2$
$Y_1$	4.0128	3.8224	0.7470	8.5822	28.1631
$Y_6$		4.8990	0.6502	9.3716	30.3282
$Y_7$			0.1478	1.5450	5.3063

$$\bar{M}_{\Delta\Delta}$$

	$Y_1$	$Y_6$	$Y_7$	$\Sigma$
$Y_1$	4.0764	3.8846	0.7469	8.7078
$Y_6$		5.0023	0.6488	9.5357
$Y_7$			0.1484	1.5441

$$\bar{W}_{\Delta\Delta} = \bar{M}_{\Delta\Delta} - \bar{M}_{\Delta z} \bar{M}_{zz}^{-1} \bar{M}_{z\Delta}$$

	$Y_1$	$Y_6$	$Y_7$	$\Sigma$
$Y_1$	0.0636	0.0622	-0.0001	0.1257
$Y_6$		0.1033	-0.0014	0.1641
$Y_7$			0.0006	-0.0009

TABLE 4.1

	$\bar{M}_{..}$			$I$			$\Sigma$
	$Z_1$	$z_2$	$Z_3$				
$Z_1$	1.0237	1.4914	0.9936	1.			4.5088
$z_2$		6.7425	2.1263		1.		11.3602
$Z_3$			1.5190			1.	5.6389
	1.0237	1.4914	0.9936	1.			4.5088
	1.	1.4569	0.9706	0.9768			4.4043
		4.5697	0.6788	-1.4569	1.		4.7916
		1.	0.1485	-0.3188	0.2188		1.0486
			0.4538	-0.7542	-0.1485	1.	0.5511
			1.	-1.6618	-0.3273	2.2035	1.2144

	$\bar{M}_{..}^{-1}$			$\Sigma$
	$Z_1$	$z_2$	$Z_3$	
$Z_1$	2.6946	-0.0720	-1.6618	0.9608
$z_2$		0.2674	-0.3273	-0.1318
$Z_3$			2.2035	0.2144

TABLE 4.2

	$\bar{M}_{\Delta\Delta}$			$\Sigma$
	$Y_1$	$Y_6$	$Y_7$	
$Z_1$	1.5631	1.8208	0.2526	3.6365
$z_2$	4.7283	3.4757	0.9494	9.1535
$Z_3$	2.0532	2.3808	0.3784	4.8125

$$P_{\Delta\Delta}' = \bar{M}_{..}^{-1} \bar{M}_{\Delta\Delta}$$

	$Y_1$	$Y_6$	$Y_7$	$\Sigma$
$Z_1$	0.4596	0.6997	-0.0166	1.1427
$z_2$	0.4801	0.0193	0.1119	0.6113
$Z_3$	0.3791	1.0826	0.1033	1.5651
$\Sigma$	1.3187	1.8017	0.1987	3.3190

$$\bar{M}_{\Delta\Delta} P_{\Delta\Delta}'$$

	$Y_1$	$Y_6$	$Y_7$	$\Sigma$
$Y_1$	3.7667	3.4079	0.7153	7.8900
$Y_6$		3.9187	0.6047	7.9314
$Y_7$			0.1411	1.4612

TABLE 4.3

$Q_{\Delta\Delta}$

	$R_{\Delta\Delta}$			$\bar{W}_{\Delta\Delta}$			$\Sigma$
	$Y_1$	$Y_6$	$Y_7$	$Y_1$	$Y_6$	$Y_7$	
$Y_1$	0.2462	0.4144	0.0317	0.0636	0.0622	-0.0001	0.8179
$Y_6$		0.9803	0.0455		0.1033	-0.0014	1.6043
$Y_7$			0.0067			0.0006	0.0829
	0.2462	0.4144	0.0317	0.0636	0.0622	-0.0001	0.8179
1.		1.6834	0.1286	0.2582	0.2527	-0.0004	3.3224
		0.2827	-0.0078	-0.0448	-0.0014	-0.0012	0.2274
		1.	-0.0277	-0.1585	-0.0050	-0.0042	0.8046
			0.0024	-0.0095	-0.0094	0.0006	-0.0160
			1.	-4.0033	-3.9515	0.2458	-6.7090

$$Q_{\Delta\Delta} = R_{\Delta\Delta}^{-1} \bar{W}_{\Delta\Delta}$$

	$Y_1$	$Y_6$	$Y_7$	$\Sigma$
$Y_1$	1.2263	0.9533	-0.0365	2.1431
$Y_6$	-0.2692	-0.1143	0.0026	-0.3809
$Y_7$	-4.0033	-3.9515	0.2458	-7.7090

carried further than is indicated in the table where convergence of the elements of  $k_{(n)}^\dagger$  to only four places was required. In the actual computation, convergence to nine places required twelve steps instead of the five shown here.

The final vector of the iteration  $Q_{(8)}$  is a characteristic vector  $b'_\Delta$  of  $R_{\Delta\Delta}^{-1} \bar{W}_{\Delta\Delta}$  associated with the largest root  $k_{(8)}^\dagger = k_1^\dagger$ . Next is the product  $c'_* = -P_{\Delta*}^{*'} b'_\Delta$ , where  $P_{\Delta*}^{*'}$  comes from Table 4.2. The iteration for  $b'_\Delta$  and the calculation of  $c'_*$  are in Table 4.4. The sum check for matrix multiplication is used in checking  $c'_*$ .

$$\begin{aligned}
 c'_* &= -P_{\Delta*}^{*'} b'_\Delta \\
 Z_1 & 1.1216 \\
 Z_2 & 0.2645 \\
 Z_3 & -0.6850 \\
 \Sigma & 0.7011
 \end{aligned}$$

Table 4.5 concludes the estimation of coefficients. First the adjusted means are copied for each variable from Table 2.1. In the second column headed  $[b_\Delta c_* c]$  the first three numbers are the components of  $b_\Delta$ , the second three are the components of  $c_*$ , the last number  $e$  is the estimate of  $\gamma_{10}$ . This estimate is equal to  $-[b_\Delta c_*]m'$ , the inner product of the two

TABLE 4.4  
CALCULATION OF  $b'_\Delta$  AND  $c'_\Delta$

	$Q'_{(0)}$	$Q'_{(1)}$	$Q'_{(2)}$	$Q'_{(3)}$	$Q'_{(4)}$	$Q'_{(5)}$	$Q'_{(6)}$	$Q'_{(7)}$	$10^{-4}Q'_{(8)}$ $= b'_\Delta$
$Y_1$	-10.6465	-12.1964	-13.9311	-15.9135	-18.1784	-20.7659	-23.7216	-27.0980	-3.0955
$Y_6$	2.2819	2.7004	3.0867	3.5257	4.0275	4.6007	5.2556	6.0036	0.6858
$Y_7$	36.0835	42.4739	48.5957	55.5189	63.4220	72.4493	82.7615	94.5415	10.7998

	$k_{(1)}^\dagger$	$k_{(2)}^\dagger$	$k_{(3)}^\dagger$	$k_{(4)}^\dagger$	$k_{(5)}^\dagger = k_1^\dagger$
$Y_1$	1.1456	1.1422	1.1423	1.1423	1.1423
$Y_6$	1.1834	1.1430	1.1422	1.1423	1.1423
$Y_7$	1.1771	1.1441	1.1425	1.1423	1.1423

columns so far formed. The third column is the vector of adjustment factors  $k'$ ; this also is copied from Table 2.1, with an adjustment 1 for  $e$ . The fourth column is the product of the preceding two; these are the estimates with the effect of the adjustments removed. The last column is obtained by dividing each element of the fourth by  $-3.0955$ , the coefficient of  $Y_1$ ; these are the normalized estimates.

The next computation is for the estimate of the sampling variance-covariance matrix. We introduce the following notation: If  $y$  is a vector  $(y_1 y_2 \dots y_n)$  then  ${}_1y$  shall mean the vector  $y$  with the first element omitted:  ${}_1y = (y_2 \dots y_n)$ . If  $A$  is a matrix, then  ${}_{10}A$  is the matrix  $A$  with the first row deleted,  ${}_{01}A$  is  $A$  with the first column deleted,  ${}_{11}A$  is  $A$  with both the first row and first column deleted.

In Table 5.1 appear the vector  $b'_\Delta$  copied from Table 4.4,  $W_{\Delta\Delta}$  copied from Table 3, and the products indicated in the table.

Table 5.2 shows the calculation of two constants,  $\hat{l}_1/b_\Delta W_{\Delta\Delta} b'_\Delta$  and  $C^*$ , that are used in subsequent calculations. In Table 5.3, the matrices

TABLE 4.5  
ESTIMATES OF THE COEFFICIENTS

	$m'$	$[b_\Delta c_{*e}]'$	$k'$	$[b_\Delta^{(0)} c_{*e}^{(0)} e^{(0)}]'$	$[b_\Delta^{(0)} c_{*e}^{(0)} e^{(0)}]'$ (norm)
$Y_1$	9.8646	-3.0955	1	-3.0955	1
$Y_6$	8.1257	0.6858	1	0.6858	-0.2216
$Y_7$	0.7027	10.7998	$10^{-1}$	1.0800	-0.3489
$Z_1$	9.7833	1.1216	1	1.1216	-0.3623
$z_2$	0.1550	0.2645	$10^{-2}$	0.002645	-0.0009
$Z_3$	8.3870	-0.6850	1	-0.6850	0.2213
$e$		12.1051	1	12.1051	-3.9105

TABLE 5.1

	$b'_\Delta$	$\bar{W}_{\Delta\Delta}$			$\Sigma$
		$Y_1$	$Y_6$	$Y_7$	
$Y_1$	-3.0955	0.0636	0.0622	-0.0001	0.1257
$Y_6$	0.6858		0.1033	-0.0014	0.1641
$Y_7$	10.7998			0.0006	-0.0009
$b_\Delta \bar{W}_{\Delta\Delta}$					
		$Y_1$	$Y_6$	$Y_7$	$\Sigma$
		-0.1553	-0.1364	0.0059	-0.2858
					-0.1305 (sum excluding $Y_1$ )
$b_\Delta \bar{W}_{\Delta\Delta} b'_\Delta = 0.4511$					
${}_{11}[(b_\Delta \bar{W}_{\Delta\Delta})' (b_\Delta \bar{W}_{\Delta\Delta})] = ({}_{1b_\Delta} \bar{W}_{\Delta\Delta})' ({}_{1b_\Delta} \bar{W}_{\Delta\Delta})$					
			$Y_6$	$Y_7$	$\Sigma$
$Y_6$			0.0186	-0.0008	0.0178
$Y_7$				0.0000	-0.0008

$F_{\beta\beta}$ ,  $F_{\beta\gamma}$ , and  $F_{\gamma\gamma}$  are computed. The necessary steps should be clear from the table.

In Table 5.4, blocks of the variance-covariance matrix of the adjusted normalized coefficients are computed. These are

$$\bar{F} = C^* \begin{bmatrix} F_{\beta\beta} & -F_{\beta\gamma} \\ -F_{\gamma\beta} & F_{\gamma\gamma} \end{bmatrix} \bar{F}_1 m', \quad {}_1 m \bar{F}_1 m' + \frac{C^*}{T^2}$$

The submatrices  $F_{\beta\beta}$ ,  $F_{\beta\gamma}$ ,  $F_{\gamma\gamma}$  are from Table 5.3,  $C^*$  is from Table 5.2,

TABLE 5.2

$k_1^{\dagger}$	= 1.1423 (copied from Table 4.4)
$l_1^{\dagger}$	= $1/k_1^{\dagger}$ = 0.8754
$(l_1^{\dagger}/b_\Delta \bar{W}_{\Delta\Delta} b'_\Delta)$	= 1.9406 (the denominator comes from Table 5.1)
$C$	= $(1 + l_1^{\dagger}) b_\Delta \bar{W}_{\Delta\Delta} b'_\Delta$ = 0.8460
$T$	= 30 = the number of observations
$F$	= 6 = the number of $y_\Delta$ 's and $z_*$ 's
$k_1$	= 1 = the adjustment on $Y_1$
$C^*$	= $C/(T - F)$ $(b_1 k_1)^2$ = 0.003679

## APPENDIX

 TABLE 5.3  
 ${}_{11}R_{\Delta\Delta}$  (from Table 4.3)

	$Y_6$	$Y_7$	$\Sigma$
$Y_6$	0.9803	0.0455	1.0258
$Y_7$		0.0067	0.0521

$${}_{11}H = {}_{11}R_{\Delta\Delta} - \frac{I_1^{\dagger}}{b_{\Delta}W_{\Delta\Delta}b_{\Delta}'} {}_{11}[(b_{\Delta}W_{\Delta\Delta})' (b_{\Delta}W_{\Delta\Delta})]$$

	$Y_6$	$Y_7$	$\Sigma$
$Y_6$	0.9442	0.0470	0.9913
$Y_7$		0.0066	0.0536

$$F_{\beta\beta} = ({}_{11}H)^{-1}$$

	$Y_6$	$Y_7$
$Y_6$	1.6425	-11.7107
$Y_7$		235.0580

 $({}_{01}P_{\Delta\Delta}^*)'$  (from Table 4.2)

	$Y_6$	$Y_7$	$\Sigma$
$Z_1$	0.6997	-0.0166	0.6832
$z_2$	0.0193	0.1119	0.1312
$Z_3$	1.0826	0.1033	1.1859
$\Sigma$	1.8017	0.1987	

$$F_{\beta\gamma} = F_{\beta\beta}({}_{01}P_{\Delta\Delta}^*)'$$

	$Z_1$	$z_2$	$Z_3$	$\Sigma$
$Y_6$	1.3434	-1.2785	0.5680	0.6329
$Y_7$	-12.0900	26.0730	11.6130	25.5960

$$F_{\gamma\gamma}^{\dagger} = ({}_{01}P_{\Delta\Delta}^*)' F_{\beta\gamma}$$

	$Z_1$	$z_2$	$Z_3$	$\Sigma$
$Z_1$	1.1404	-1.3267	0.2050	0.0187
$z_2$		2.8924	1.3103	2.8760
$Z_3$			1.8150	3.3303

$$F_{\gamma\gamma} = F_{\gamma\gamma}^1 + \bar{M}_{**}^{-1} \quad (\text{the last from Table 4.1})$$

	Z <sub>1</sub>	z <sub>2</sub>	Z <sub>3</sub>	Σ
Z <sub>1</sub>	3.8350	-1.3987	-1.4568	0.9795
z <sub>2</sub>		3.1599	0.9830	2.7442
Z <sub>3</sub>			4.0185	3.5447

and  ${}_1m'$  is the vector of adjusted means with the mean of  $Y_1$  deleted.  $\bar{F}$  corresponds to  $V^*(b_\Delta, c_*)^{(norm)}$  in the Chernoff-Divinsky paper.

The variance-covariance matrix  $V^*(b_\Delta^{(0)}, c_*^{(0)}, e^{(0)})^{(norm)}$  of the normalized, unadjusted coefficients is computed in Table 5.5. If we let  $D$  be a diagonal matrix whose diagonal elements are  $k_{Y_6}, k_{Y_7}, k_{Z_1}, k_{z_2}, k_{Z_3}, 1$  then

TABLE 5.4

$$F = C^* \begin{bmatrix} F_{\beta\beta} & -F_{\beta\gamma} \\ -F_{\gamma\beta} & F_{\gamma\gamma} \end{bmatrix}$$

	Y <sub>6</sub>	Y <sub>7</sub>	Z <sub>1</sub>	z <sub>2</sub>	Z <sub>3</sub>	Σ
Y <sub>6</sub>	0.0060	-0.0431	-0.0049	0.0047	-0.0021	-0.0394
Y <sub>7</sub>		0.8647	0.0445	-0.0959	-0.0427	0.7274
Z <sub>1</sub>			0.0141	-0.0051	-0.0054	0.0431
z <sub>2</sub>				0.0116	0.0036	-0.0811
Z <sub>3</sub>					0.0148	-0.0318
		${}_1m'$ (from Table 2.1)			$\bar{F}({}_1m')$	
Y <sub>6</sub>		8.1257			-0.0463	
Y <sub>7</sub>		0.7027			0.3195	
Z <sub>1</sub>		9.7833			0.0834	
z <sub>2</sub>		0.1550			-0.0474	
Z <sub>3</sub>		8.3870			0.0251	
Σ					0.3343	

${}_1m[V^*(b_\Delta, c_*) / (b_1^{(0)})^2]_1m'$ , the inner product of the two columns above, = 0.8670

$$\frac{C^*}{T^2} = 0.000004087$$

$${}_1m\bar{F}_1m' + \frac{C^*}{T^2} = 0.8670$$

TABLE 5.5  
 $V^*(b_{\Delta}^{(0)}, c_*^{(0)}, e^{(0)})^{(norm)}$

	$Y_6$	$Y_7$	$Z_1$	$z_2$	$Z_3$	1
$Y_6$	0.0060	-0.00431	-0.0049	0.00047	-0.0021	0.0463
$Y_7$		0.008647	0.00445	-0.0000959	-0.00427	-0.03195
$Z_1$			0.0141	0.000051	-0.0054	-0.0834
$z_2$				0.00000116	0.00036	0.000474
$Z_3$					0.0148	-0.0251
1						0.8670

$$V^*(b_{\Delta}^{(0)}, c_*^{(0)}, e^{(0)})^{(norm)} =$$

$$D \begin{pmatrix} \bar{F} & \bar{F}_1 m' \\ {}_1 m' \bar{F} & {}_1 m' \bar{F}_1 m' + \frac{C^*}{T^2} \end{pmatrix} D$$

where the matrix consisting of the blocks  $\bar{F}$ ,  $\bar{F}_1 m'$ ,  ${}_1 m' \bar{F}$  and  ${}_1 m' \bar{F}_1 m' + C^*/T^2$  is the variance-covariance matrix of the adjusted normalized coefficients. This is equivalent to multiplying the estimated covariance between two adjusted coefficients by the adjustments on those coefficients.

The last step is to test for serial independence, using the Durbin-Watson statistic.<sup>11</sup>

$$d = \frac{\sum_{t=2}^T v_{1t}^2}{\sum_{t=1}^T \tilde{u}_{1t}^2},$$

where  $\tilde{u}_{1t}$  is the calculated residual, and  $v_{1t} = \tilde{u}_{1t} - \tilde{u}_{1,t-1}$ . A simplification is given by the identity

$$\sum_t \tilde{u}_{1t}^2 = C^* \left( \frac{T - F}{T} \right), \quad \text{so that} \quad d = \frac{T \sum v_{1t}^2}{C^*(T - F)}.$$

This yields 1.11 for the value of the Durbin-Watson statistic.

The calculated residuals are sometimes useful for casual inspections of the results (such as seeing how closely they correspond to movements in omitted variables or whether striking historical events have occasioned particularly large residuals) and are therefore shown in Table 6.

<sup>11</sup> J. Durbin and G. S. Watson, Testing for Serial Correlation in Least Squares Regression I, *Biometrika*, Vol. 37, Parts 3 and 4, pp. 409-428, Dec. 1950.

J. Durbin and G. S. Watson, Testing for Serial Correlation in Least Squares Regression II, *Biometrika*, Vol. 38, Parts 1 and 2, pp. 159-178, June 1951.

This statistic was not used in our other references, Chernoff-Divinsky and Klein.



TABLE 6  
CALCULATED RESIDUALS BY LISE

Year	Calculated Residual ( $\bar{u}_i$ )
1920	-0.030655
1921	-0.011751
1922	0.015434
1923	0.006589
1924	0.002086
1925	-0.003226
1926	-0.003212
1927	0.005435
1928	0.007704
1929	0.003884
1930	0.001225
1931	0.022113
1932	0.014616
1933	0.007543
1934	-0.006553
1935	0.004520
1936	-0.016416
1937	-0.011023
1938	-0.002789
1939	0.000803
1940	-0.003977
1941	-0.001526
1942	0.008565
1943	0.000108
1944	-0.009510
1945	0.003713
1946	-0.000433
1947	0.002002
1948	0.000254
1949	-0.005531

Equation 4.1 was also estimated by the least squares method, with  $Y_1$  chosen as the dependent variable, and  $v = (Y_6 Y_7 Z_1 z_2 Z_3)$  the independent variables. The adjustment factors are the same as before.

First, the matrix of the adjusted moments of the independent variables,  $\bar{M}_{vv}$ , is formed, analogously to Table 3, and inverted (we omit presenting the intermediate forward solution).

TABLE 7

	$Y_6$	$Y_7$	$Z_1$	$z_2$	$Z_3$	$\Sigma$
$\bar{M}_{vv}$						
$Y_6$	5.0023	0.6488	1.8208	3.4757	2.3808	14.3285
$Y_7$		0.1484	0.2526	0.9494	0.3784	3.3776
$Z_1$			1.0237	1.4914	0.9936	6.5822
$z_2$				6.7425	2.1263	15.7854
$Z_3$					1.5190	8.3981
$\bar{M}_{vv}^{-1}$						
$Y_6$	1.2256					-6.9426
$Y_7$	-7.4375	182.6998				152.3685
$Z_1$	-0.9809	8.2322	3.5173			8.3440
$z_2$	0.8085	-20.2975	-0.9741	2.5228		-17.0453
$Z_3$	-0.5583	-10.8284	-1.4506	0.8950	3.9268	-8.0154

Table 8 gives the computation of the coefficient estimates; first  $\bar{M}_{Y_1v}$  is formed, which is the column of the adjusted moments of  $Y_1$  with each of the  $v$ 's. Then the adjusted means  $m'$  are copied for each variable from Table 2.1. The third column, headed  $[\beta, e]'$ , consists of two calculations;  $\beta' = \bar{M}_{vv}^{-1} \bar{M}_{vY_1}$ , a column of five numbers, and  $e = m_{Y_1} - \sum_{i=1}^5 \beta_{v_i} m_{v_i}$ , where we consider  $v_1 = Y_6$ ,  $v_2 = Y_7$ ,  $v_3 = Z_1$ , etc. The sum indicated is of  $\beta'$  only and is used for a check of the first calculation. The next column is  $k'$ , the vector of adjustment factors copied from Table 2.1. The last column consists of the desired estimates with adjustments canceled; it is obtained by multiplying each element of column 3 by its corresponding adjustment and dividing by the adjustment on  $Y_1$ , here 1. From the form in which equation 4.1 is written, we actually find  $-\beta_{16} = \beta_{Y_6}$ ,  $-\beta_{17} = \beta_{Y_7}$ , etc.

TABLE 8  
ESTIMATION OF COEFFICIENTS

	$\bar{M}_{Y_1v}$	$m'$	$[\beta e]'$	$k'$	$[\beta^{(0)}e^{(0)}]$
$Y_1$		9.8646		1	
$Y_6$	3.8846	8.1257	0.3495	1	0.3495
$Y_7$	0.7469	0.7027	2.2191	$10^{-1}$	0.2219
$Z_1$	1.5631	9.7833	0.2518	1	0.2518
$z_2$	4.7283	0.1550	0.2250	$10^{-2}$	0.002250
$Z_3$	2.0532	8.3870	-0.2286	1	-0.2286
$e$			4.8846	1	4.8846

TABLE 9  
CALCULATED RESIDUALS  $\hat{u}_t$  BY LEAST SQUARES ESTIMATES

Year	$\hat{u}_t$
1920	-0.025691
1921	-0.018010
1922	0.010916
1923	0.005649
1924	0.003658
1925	0.000330
1926	-0.001360
1927	0.005980
1928	0.006856
1929	0.005047
1930	0.002435
1931	0.020018
1932	0.005293
1933	0.002743
1934	-0.000214
1935	0.009688
1936	-0.016350
1937	-0.002530
1938	-0.007963
1939	-0.002446
1940	-0.003390
1941	0.000515
1942	0.011898
1943	0.000561
1944	-0.002921
1945	0.003981
1946	-0.005544
1947	-0.000094
1948	0.000504
1949	-0.009554

The residuals  $\hat{u}_t$  calculated from the least squares estimates are in Table 9.

The Durbin-Watson statistic, using the formula given previously, is  $d = 1.28$ .

To obtain estimates of the sampling covariance matrix, we find (Table 10A).

TABLE 10A

1.  $\bar{M}_{Y_1v} \bar{M}_{vv}^{-1} \bar{M}_{vY_1} = \bar{M}_{Y_1v\beta'} = 4.0033$
2.  $\bar{M}_{Y_1Y_1} = 4.0764$
3.  $\bar{W}_{Y_1Y_1} = 2. - 1. = 0.0732$
4.  $\bar{W}_{Y_1Y_1}/T - F = 0.003048$ ;  $T$  and  $F$  have the same meaning and value as in LISE.

$$m_v \bar{M}_{vv}^{-1}$$

$Y_6$	$Y_7$	$Z_1$	$z_2$	$Z_3$	$\Sigma$
-9.4201	154.5174	19.9083	-9.3254	6.7363	162.4164

and  $m_v \bar{M}_{vv}^{-1} m_v' = 281.8506$  where  $m_v$  is the vector of means of  $Y_6, Y_7, Z_1, z_2, Z_3$ .

TABLE 10B  
ADJUSTED COVARIANCE MATRIX

$$V^*(\beta, e) =$$

$$\frac{\bar{W}_{Y_1Y_1}}{T - F} \begin{pmatrix} \bar{M}_{vv}^{-1} & -\bar{M}_{vv}^{-1} m_v' \\ \hline & m_v \bar{M}_{vv}^{-1} m_v' + \frac{1}{T^2} \end{pmatrix}$$

	$Y_6$	$Y_7$	$Z_1$	$z_2$	$Z_3$	$e$
$Y_6$	0.003736	-0.02267	-0.002990	0.002464	-0.001702	0.02871
$Y_7$		0.5569	0.02509	-0.06187	-0.03301	-0.4710
$Z_1$			0.01072	-0.002969	-0.004422	-0.06068
$z_2$				0.007690	0.002728	0.02843
$Z_3$					0.01197	-0.02053
$e$						0.8591

To obtain the final covariance matrix, multiply each element of  $V^*(\beta, e)$  by the two adjustment factors corresponding to the row and column in which it is located, and divide by  $(k_{Y_1})^2$ , in this case 1.

TABLE 10C  
COVARIANCE MATRIX

$$V^*(\beta^{(e)}, e^{(e)})$$

	$Y_6$	$Y_7$	$Z_1$	$z_2$	$Z_3$	$e$
$Y_6$	0.003736	-0.002267	-0.002990	0.00002464	-0.001702	0.02871
$Y_7$		0.005569	0.002509	-0.00006187	-0.003301	-0.04710
$Z_1$			0.01072	-0.00002969	-0.004422	-0.06068
$z_2$				0.000007690	0.00002728	0.0002843
$Z_3$					0.01197	-0.02053
$e$						0.8591

We also obtain the estimated variance  $s^2$  of the residual  $\tilde{u}$ , and the correlation coefficient  $R$ :

$$s^2 = \frac{\bar{W}_{Y_1 Y_1}}{T(T-F)} \cdot \frac{1}{(k_{Y_1})^2} = 0.001016; \quad s = 0.03187$$

$$R^2 = \frac{\bar{M}_{Y_1 Y_1} \bar{M}_{Y_1 Y_1}^{-1} \bar{M}_{Y_1 Y_1}}{\bar{M}_{Y_1 Y_1}} = 0.9821; \quad R = 0.9910$$



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